Differential Equations

FOR ENGINEERS AND APPLIED SCIENTISTS

W D Morris

Differential equations for engineers and applied scientists

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To Pamela, Janet, and Nigel

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Preface

The ability to accurately predict the behaviour of physical systems prior to their manufacture is one of the ultimate objectives of engineers and applied scientists for such facility would permit the feasibility of new design concepts to be assessed with confidence and the need for expensive experimentation and prototype testing minimized.

The language of prediction is mathematics, and predictive techniques involve the construction of equations, based on the mathematical embodiment of the physical laws appropriate to the analysis, which, hopefully, model the behaviour of the system being investigated.

These modelling equations are often differential in nature, that is, they involve differential coefficients of the required performance parameters, and the next step in the analysis involves the mathematical gymnastics necessary to unravel the desired information. In other words, a solution of the modelling equations must be determined.

It should be clear from the foregoing remarks that the branch of mathematics which deals with the solution of these so-called differential equations is of particular importance for engineers, and the present text has been designed to present the elements of differential equation theory within the context of its engineering application.

It must be borne in mind that, although a solution to a modelling differential equation has been found which, in the mathematical sense, is correct, it is not necessarily an adequate prediction of the performance of the system being studied. The prediction is only as good as the modelling equation itself, which, in turn, is only as good as the assumptions made during its construction. This point is emphasized throughout the text.

This book has been deliberately pitched at a level suitable for students who have completed a basic course in pure mathematics involving differential and integral calculus with perhaps a little complex number theory, such as is found in a traditional university first course in pure mathematics. A knowledge of a pre-university physics course is also assumed. In view of this, the book is suitable for students of engineering or applied science in the second half of the first year of a three-year degree syllabus or its equivalent timing for syllabuses leading to other qualifications. The material is, of course, sufficient to spread into subsequent years as well.

It should be pointed out that the text makes no claim to place between one set of covers all aspects of differential equation theory, for the subject is indeed vast. But, at the level contemplated, a suitable compromise has been attempted between the basic underlying theory and its engineering application which should stimulate the urge for even further study and hence the ability to attempt even more sophisticated predictions.

I should like to record my appreciation to my colleagues and students at the School of Applied Sciences, University of Sussex, for the many helpful discussions we have had during the writing of this book. Finally, I should like to thank my wife, Pamela, for all her effort in meticulously typing the manuscript and for helping in numerous ways.

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1. Some strategic features of technical analysis

1.1 Introduction

An engineer may be broadly defined as a person who, having made been aware that a piece of equipment would constitute a benefit to mankind, attempts a systematic application of his knowledge of science, natural resources, social and economic factors for the conception, evaluation, and eventual construction of a device which fulfills this need.

Whereas the pure scientist or mathematician is often interested, and justifiably so, in the acquisition of knowledge for its own sake, this same knowledge is merely a means to an end from the viewpoint of the practising engineer or technologist. Similar arguments may be made, of course, for knowledge in the field of economics, social science, or any other discipline which relates to his effective functioning as a professional engineer.

A significant difference in the attitude of mind of an engineer compared to that of the pure scientist results from the fact that, for an engineer, more than one solution to his problem usually exists. For example, if a number of companies are requested to tender proposals for the design of a vehicle to be used for the exploration of the moon's surface their design teams will suggest many seemingly feasible solutions. Which, we may enquire, is correct? Clearly, in principle, all may be equally correct. Further, the engineer's problems are open in that as new information and techniques become available his original solution is improved and modified accordingly.

The arrival at a viable design solution is not a random process. Indeed the engineer should be trained to proceed along an ordered sequence of thoughts to ensure that any fundamental flaws present in his basic conceptual solution are discovered at an early stage. Although the present text has been designed to introduce engineering students to one particular aspect of the activity of an engineer (namely, the application of known scientific facts and mathematics for predicting the performance of a system or sub-system) it is instructive at this point to elaborate on the concept of 'an ordered sequence of thoughts' mentioned above.

Figure 1.1 attempts to illustrate the basic thought processes and operations which are necessary to steer an ordered path beginning at society's recognition of a need for a particular piece of equipment and leading to the eventual marketing of a viable solution to the consumer. This block diagram is, in essence, a structure which could be adopted by a typical company for many of the blocks represent departmental activities. Although it is not the only structure which could be devised it is nevertheless ideally suited for the purpose of illustrating the typical activities of an engineer.

The awareness of society that it requires a machine, device, or commodity to fulfil a certain function may be self generated, or indeed, artificially created by the advertising and marketing departments of industry at large. The manner in which this awareness arises and the associated ethics are not, however, the concern of the present text. Let it suffice to say that a commercial enterprise decides, by some means or another, that there exists a potential market and hence profit for a certain piece of equipment and decides to produce a design which is aimed at this market. Because society's ideas on its requirements are often in themselves vague, block 1 in Fig. 1.1 has been drawn as a closed curved line to differentiate between the rectangular blocks used to represent more clear cut functions or processes in the flow diagram.

As the result of a thorough marketing investigation it will be possible to translate this requirement of society into a detailed, definitive specification which the final design must satisfy; see block 2 in the flow diagram. This detailed design specification should include considerations of economic and social factors as well as the actual technical performance required. In fact all features likely to influence the commercial success of the product should be well thought out at this stage.

The design department now attempts to conceive ideas on the form which the equipment should take in order to satisfy the constraints of the specification. Conceptual thinking of this type is rather intangible and many authorities subscribe to the idea that the teaching of this form of thought is not possible. It is for this reason that block 3 in the flow diagram is also shown as a closed curved line. Although the manner in which a conceptual idea for the design arises may be intangible this is certainly not true for the manner in which the viability of the concept is assessed.

Before expending too much time and hence finance on the details of a particular design concept it is advisable to check, in broad terms, the feasibility of the idea. This usually takes the form of a technical analysis and is shown as block 4. To achieve this it is necessary to have knowledge of physical laws, advanced mathematics, and sometimes material properties. This information is shown assembled in block 5 and is injected as required into block 4. If, at this stage, the concept is shown to be defective then the concept is rejected (block 6) and another concept (block 3a) substituted in its place. The closed loop linking blocks 3, 4, 6, and 3a is repeated continuously until the current concept being assessed is shown to pass the initial technical analysis.

When the initial technical assessment demonstrates that the concept is seemingly feasible the time and associated finance may be justified on a more thorough detailed analysis. This is shown in block 7 on the flow chart. Now factors other than those of a direct technical nature must be considered. Estimation of the cost

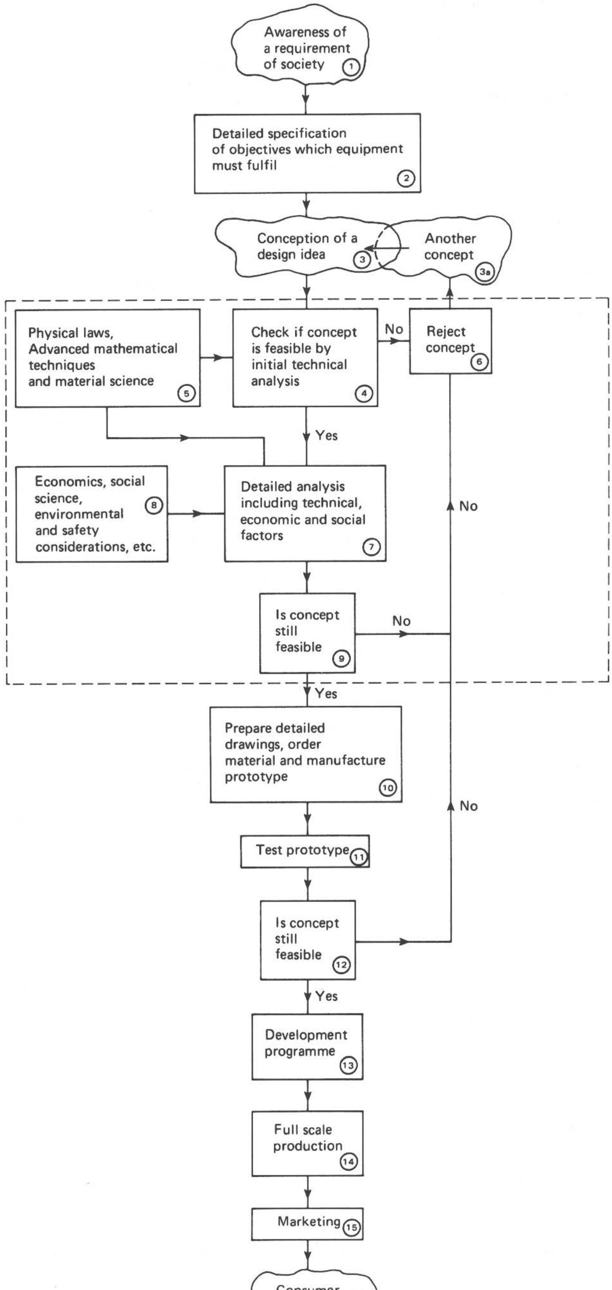

Fig. 1.1 Flow diagram typifying the processes required for the design, production, and marketing of an engineering product

of the product in relation to its competitors, the degree of environmental pollution expected, ease of servicing and maintenance are now all factors which are of crucial importance in deciding if the concept is still viable on an overall basis. To successfully complete the analysis shown in block 7 it is necessary to have available knowledge, for example, of economic principles, social science, environmental and safety factors shown in block 8. This is additional to the technical knowledge of block 5.

The decision as to whether or not the concept is still commercially feasible must now be taken again and this is shown in block 9. If an affirmative decision is made then the design proceeds to the next stage in the flow chart. However if some defect is detected in the concept at this stage then the concept must be rejected (block 6) and a quest for yet another idea made (block 3a). It should be self evident that the further along the flow chart that the decision is made to reject the concept the more costly has been the exercise to the company concerned.

If the design appears to be technically and economically sound then the construction of a prototype may be undertaken. To achieve this detailed drawings of each component must be prepared, all the raw material ordered and each of the components manufactured. The prototype may then be assembled and tested to check if its performance does indeed satisfy the desired specification. These phases are shown as blocks 10 and 11 on the flow chart.

In the harsh reality of an engineering environment, tests conducted on a prototype device usually result in a variety of teething problems which were not anticipated at the design stage. Hopefully, from the standpoint of the company, these problems would not be sufficiently serious to warrant the complete rejection of the design for such a decision would be costly in terms both of hard finance and time. Nevertheless if the design is shown to be seriously defective then this unpleasant decision must be taken (at block 12 in the flow chart) and another idea examined.

The teething troubles present in a basically sound prototype may be removed by a carefully planned programme of development (see block 13 in flow chart). To this end a number of revised prototypes or early production models may be used. When the company is satisfied that a reliable product having adequate performance has been made, full scale production and marketing to the eventual consumer may be organized. These functions are shown in the final three blocks of the flow chart.

It is not difficult to appreciate from the above discussion that there are many problems which must be solved when a piece of engineering equipment is being designed, produced, and marketed. Of these problems, the present book is restricted to some aspects of those which occur within the blocks encompassed by the dotted boundary shown in Fig. 1.1. Specifically the text is designed to introduce students to the marriage which engineers must effect between physical laws and mathematics in order to model a system or sub-system and hence by solving the modelling equation to predict its performance prior to manufacture.

As a result of the application of a mathematical formulation of the pertinent physical laws, the performance or behaviour of the system is often found to be governed by a differential equation. That is by an equation which relates the gradients of some performance parameter to an independent variable. We expect consequently that the solution of differential equations is a task which is frequently going to confront engineers when the analysis of a piece of equipment is being made and it is with this branch of mathematics that the present text is concerned.

The engineer is not, or should not be, content with obtaining a mathematical solution to the equation which controls the behaviour of his system for he must then assess the implication of the solution on his original design ideas and determine whether or not his solution is feasible in engineering terms.

Having now taken cognizance of the general aspects of an engineer's activities and specified which of these are to be introduced in this book we are in a position to actually itemize the objectives which the text sets out to fulfil. These are:

Primary objectives

1. To demonstrate how the mathematical representation of the physical laws pertinent to a given system may be assembled to produce a mathematical model of the system taking the form of a differential equation.
2. To present, via the engineering application, the rudiments of mathematical methods available for solving differential equations.

Secondary objectives

1. To emphasize as a result of the above two objectives that many apparently different systems (e.g., mechanical, electrical, thermal, etc.) behave in an analogous manner. This is achieved by showing that their respective modelling equations have a similar mathematical structure.
2. Finally, to use these strategic objectives as a vehicle to develop in the student an awareness for an engineer's thought process.

1.2 A basic strategy for technical analysis

In Fig. 1.1 a flow chart was used to demonstrate the ordered breakdown of the functions and decision taking actions involved in the design and manufacture of an engineering device. Separate flow charts showing details of each of the individual blocks in Fig. 1.1 may also be devised. In this respect Fig. 1.2 represents the systematic plan which must be executed in order to attempt a technical analysis of a given problem situation. Since the implementation of this plan is one of the objectives of this text it is crucial that a clear understanding of its strategic implication is grasped by the student.

Let us suppose that the performance of a piece of equipment is to be predicted and that someone, presumably higher in the executive chain of command, has delegated the task to the analytical department of the company. Very often the

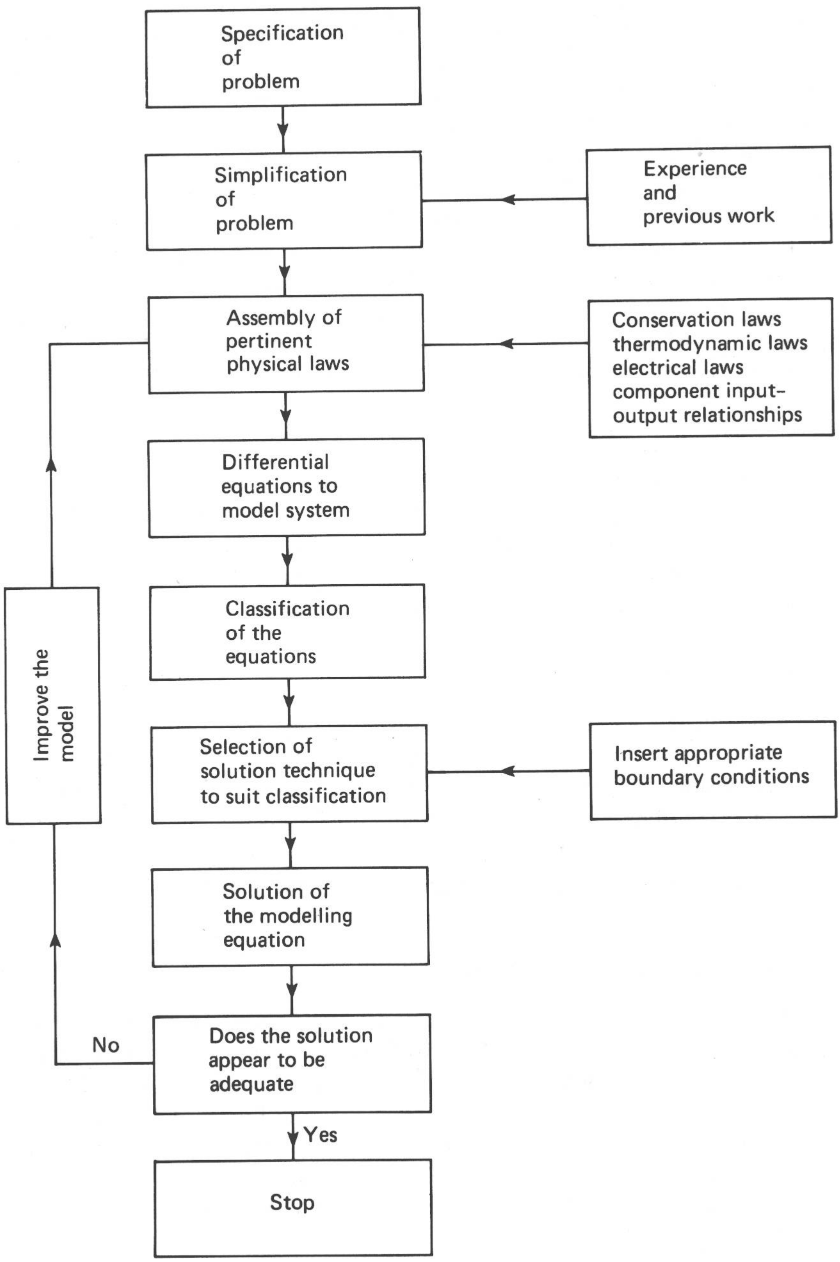

Fig. 1.2 Flow chart illustrating the typical thought processes for technical analysis

specification of the problem will be vague and the analyst must seek for himself the relevant information which will permit him to complete the task. This, incidentally, is in complete contrast to the problem situations encountered by engineering students during their studies where sufficient information is usually given to permit a solution of the problem to be obtained. In an industrial environment problems seldom occur having the traditional format of an examination question! To exemplify, a real life problem may arise as follows.

Typical problem

As the result of market research the directors of a motor firm have identified a potential market for a light truck capable of carrying a load of 1000 kg for use by sheep farmers in the Australian outback. The design department of the company have decided to use conventional independent suspension systems on the front and rear wheels comprising a coil spring and damper. Select a suitable combination of spring and damper characteristics so that a tolerably smooth ride may be achieved when negotiating the typical terrain.

Reverting back to Fig. 1.2 we note that the first step which must be taken by the analyst in his quest for a solution is to identify the parameters most likely to have a direct influence on the performance of the system being assessed. In other words the real problem must be simplified. To achieve this the analyst must make decisions based on his previous experience and also from the experience of other workers via a survey of technical literature relevant to the area of concern. At this stage it is necessary, in order to make an initial appraisal, to select only those which are the most significant parameters. Those which have a second order influence on the performance of a system may be ignored until such time as a more sophisticated approach is required. For example, taking our motor vehicle suspension problem, the analyst may decide on the following breakdown of priorities.

Parameters having first order of importance

1. The typical load–deflection characteristic of the spring system envisaged by the designers.
2. The typical resistance characteristic of the damping element envisaged by the designers.
3. The proportion of the total weight of the vehicle and its load supported by the suspension unit.
4. The nature of the undulations of the typical surface to be negotiated.

Parameters having second order of importance

1. The springiness and the inherent internal damping of the pneumatic tyres.
2. The effect of changes in ambient temperature on the performance of the main spring and damper.