MECHANICS OF FLUIDS AND TRANSPORT PROCESSES

M. Lesieur

Turbulence in Fluids

MARTINUS NIJHOFF PUBLISHERS

Turbulence in fluids

Stochastic and numerical modelling

By

Marcel Lesieur

National Polytechnic Institute
School of Hydraulics and Mechanics
Grenoble, France

1987 **MARTINUS NIJHOFF PUBLISHERS**
a member of the KLUWER ACADEMIC PUBLISHERS GROUP
DORDRECHT / BOSTON / LANCASTER

Distributors

for the United States and Canada: Kluwer Academic Publishers, P.O. Box 358, Accord Station, Hingham, MA 02018-0358, USA
for the UK and Ireland: Kluwer Academic Publishers, MTP Press Limited, Falcon House, Queen Square, Lancaster LA1 1RN, UK
for all other countries: Kluwer Academic Publishers Group, Distribution Center, P.O. Box 322, 3300 AH Dordrecht, The Netherlands

Library of Congress Cataloging in Publication Data

ISBN 90-247-3470-3

PRINTED IN THE NETHERLANDS

Turbulence in fluids

à mes Parents,

à Stéphanie et Juliette,

Preface and Acknowledgements

Turbulence is a dangerous topic which is often at the origin of serious fights in the scientific meetings devoted to it since it represents extremely different points of view, all of which have in common their complexity, as well as an inability to solve the problem. It is even difficult to agree on what exactly is the problem to be solved.

Extremely schematically, two opposing points of view have been advocated during these last ten years: the first one is "statistical", and tries to model the evolution of averaged quantities of the flow. This community, which has followed the glorious trail of Taylor and Kolmogorov, believes in the phenomenology of cascades, and strongly disputes the possibility of any coherence or order associated to turbulence.

On the other bank of the river stands the "coherence among chaos" community, which considers turbulence from a purely deterministic point of view, by studying either the behaviour of dynamical systems, or the stability of flows in various situations. To this community are also associated the experimentalists who seek to identify coherent structures in shear flows.

My personal experience in turbulence was acquired in the first group, since I spent several years studying the stochastic models of turbulence, applied to various situations such as helical or two dimensional turbulence and turbulent diffusion. These techniques were certainly not the ultimate solution to the problem, but they allowed me to get acquainted with various disciplines such as astrophysics, meteorology, oceanography and aeronautics, which were all, for different reasons, interested in turbulence. It is certainly true that I discovered the fascination of Fluid Dynamics through the somewhat abstract studies of turbulence.

This monograph is then an attempt to reconcile the statistical point of view and the basic concepts of fluid mechanics which determine the evolution of flows arising in the various fields envisaged above. It is true that these basic principles, accompanied by the predictions of the instability theory, give valuable information on the behaviour of turbulence and of the structures which compose it. But a statistical analysis of these structures can, at the same time, supply information about strong nonlinear energy transfers within the flow.

I have tried to present here a synthesis between two graduate courses given in Grenoble during these last few years, namely a "Turbulence" course and a "Geophysical Fluid Dynamics" course. I would like to thank my colleagues of the Ecole Nationale d'Hydraulique et Mécanique and Université Scientifique et Médicale de Grenoble, who offered me the opportunity of giving these two courses. The students who attended these classes were, through their questions and remarks, of great help. I took advantage of a sabbatical year spent at the Department of Aerospace Engineering of the University of Southern California to write the first draft of this monograph: this was rendered possible by the generous hospitality of John Laufer and his collaborators. Finally, I am grateful to numerous friends around the world who encouraged me to undertake this work.

I am greatly indebted to Frances Métais who corrected the English style of the manuscript. I am uniquely responsible for the remaining mistakes, due to last minute modifications. I ask for the indulgence of the English speaking reader, thinking that he might not have been delighted by a text written in perfect French. I hope also that this monograph will help the diffusion of some French contributions to turbulence research.

Ms Van Thai was of great help for the drawings. I am also extremely grateful to Jean-Pierre Chollet, Yves Gagne and Olivier Métais for their contribution to the contents of the book and their help during its achievement, and to Sherwin Maslowe who edited several Chapters.

This book was written using the TEX system. This would not have been possible without the constant help of Evelyne Tournier, of Grenoble Applied Mathematics Institute, and of Claude Goutorbe, of the University computing center.

Finally I thank Martinus Nijhoff Publishers for offering me the possibility of presenting these ideas.

Grenoble, October 1986 Marcel Lesieur

Contents

Chapter I

INTRODUCTION TO TURBULENCE IN FLUID MECHANICS

1 Is it possible to define turbulence?

Everyday life gives us an intuitive knowledge of turbulence in fluids: the smoke of a cigarette or over a fire exhibits a disordered behaviour characteristic of the motion of the air which transports it. The wind is subject to abrupt changes in direction and velocity, which may have dramatic consequences for the seafarer or the hang-glider. During air travel, one often hears the word turbulence generally associated with the fastening of seat-belts. Turbulence is also mentioned to describe the flow of a stream, and in a river it has important consequences concerning the sediment transport and the motion of the bed. The rapid flow of any fluid passing an obstacle or an airfoil creates turbulence in the boundary layers and develops a turbulent wake which will generally increase the drag exerted by the flow on the obstacle (and measured by the famous C_x coefficient): so turbulence has to be avoided in order to obtain better aerodynamic performance for cars or planes. The majority of atmospheric or oceanic currents cannot be predicted accurately and fall into the category of turbulent flows, even in the large planetary scales. Small scale turbulence in the atmosphere can be an obstacle towards the accuracy of astronomic observations, and observatory locations have to be chosen in consequence. The atmospheres of planets such as Jupiter and Saturn, the solar atmosphere or the Earth outer core are turbulent. Galaxies look strikingly like the eddies which are observed in turbulent flows such as the mixing layer between two flows of different velocity, and are, in a manner of speaking, the eddies of a turbulent universe. Turbulence is also produced in the Earth's outer magnetosphere, due to the development of instabilities caused by the interaction of the solar wind with the magnetosphere. Numerous other examples of turbulent flows arise in aeronautics, hydraulics, nuclear and chemical engineering, oceanography, meteorology, astrophysics and internal geophysics.

It can be said that a turbulent flow is a flow which is disordered in time and space. But this, of course, is not a precise mathematical definition. The flows one calls "turbulent" may possess fairly different dynamics, may be three-dimensional or sometimes quasi- two-dimensional, may exhibit well organized structures or otherwise. A common property which is required of them is that they should be able to mix transported quantities much more rapidly than if only molecular diffusion processes were involved. It is this latter property which is certainly the more important for people interested in turbulence because of its practical applications: the engineer, for instance, is mainly concerned with the knowledge of turbulent heat diffusion coefficients, or the turbulent drag (depending on turbulent momentum diffusion in the flow). The following definition of turbulence can then be tentatively proposed and may contribute to avoiding the somewhat semantic discussions on this matter:

-Firstly, a turbulent flow must be unpredictable, in the sense that a small uncertainty as to its knowledge at a given initial time will amplify so as to render impossible a precise deterministic prediction of its evolution.

-Secondly it has to satisfy the increased mixing property defined above.

Such a definition allows in particular an application of the term "turbulent" to some two-dimensional flows. It also implies that certain non dimensional parameters characteristic of the flow should be much greater than one: indeed, let l be a characteristic length associated to the large energetic eddies of turbulence, and v a characteristic fluctuating velocity; a very rough analogy between the mixing processes due to turbulence and the incoherent random walk allows to define a turbulent diffusion coefficient proportional to $l\ v$. As will be seen later on, l is also called the mixing length. Then, if ν and κ are respectively the molecular diffusion coefficients[1] of momentum (called below the kinematic molecular viscosity) and heat (the molecular conductivity), the increased mixing property for these two transported quantities implies that the two dimensionless parameters lv/ν and lv/κ should be much greater than one. The first of these parameters is called the Reynolds number, and the second one the Peclet number.

A turbulent flow is by nature unstable: a small perturbation will generally, due to the nonlinearities of the equations of motion, amplify.

[1] These coefficients will be precisely defined in Chapter II.

The contrary occurs in a "laminar" flow, as can be seen on Figure 1, where the streamlines, perturbed by the small obstacle, reform downstream. The Reynolds number of this flow, defined as

$$Re = [\text{fluid velocity}] \times [\text{size of the obstacle }]/\nu$$

is in this experiment equal to $2.26\ 10^{-2}$. This Reynolds number is different from the turbulent Reynolds number introduced above, but it will be shown in chapter III that they both characterize the relative importance of inertial forces over viscous forces in the flow. Here the viscous forces are preponderant and will damp any perturbation, preventing the turbulence from developing.

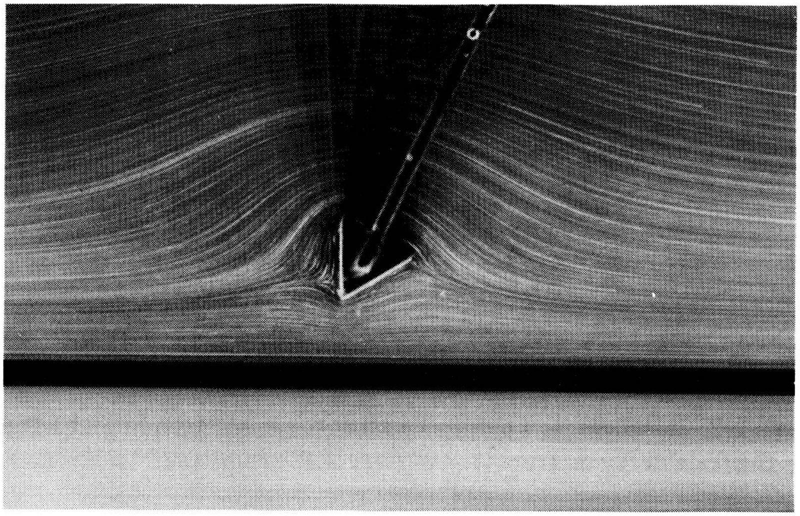

Figure I-1: Stokes flow of glycerin past a triangular obstacle (picture by S.Taneda, Kyushu University; from [1], courtesy S. Taneda and "La Recherche")

It may be interesting to ask oneself how turbulence does in fact arise in a flow. For a vast ensemble of flows, it is the presence of boundaries or obstacles, which create vorticity (the vorticity is the velocity curl: $\underline{\omega} = \nabla \times \underline{u}$) inside a flow which was initially irrotational (i.e. with a zero-vorticity). The vorticity produced in the proximity of the boundary, and due to the zero velocity condition imposed on the boundary[2], will diffuse throughout the flow which will generally become turbulent in the rotational regions. Production of vorticity will then be increased, due to the vortex filaments stretching mechanism, to be described later. Turbulence is thus associated with vorticity, and it is impossible to imagine

[2] in a viscous fluid

a turbulent irrotational flow. In what is called grid turbulence for instance, which is produced in the laboratory by letting a flow go through a fixed grid, the rotational "vortex streets" behind the grid rods interact together and degenerate into turbulence. Notice that the same effect would be obtained by pulling a grid through a fluid initially at rest. In some situations, the vorticity is created in the interior of the flow itself through some external forcing or rotational initial conditions (as in the example of the mixing layer presented later on).

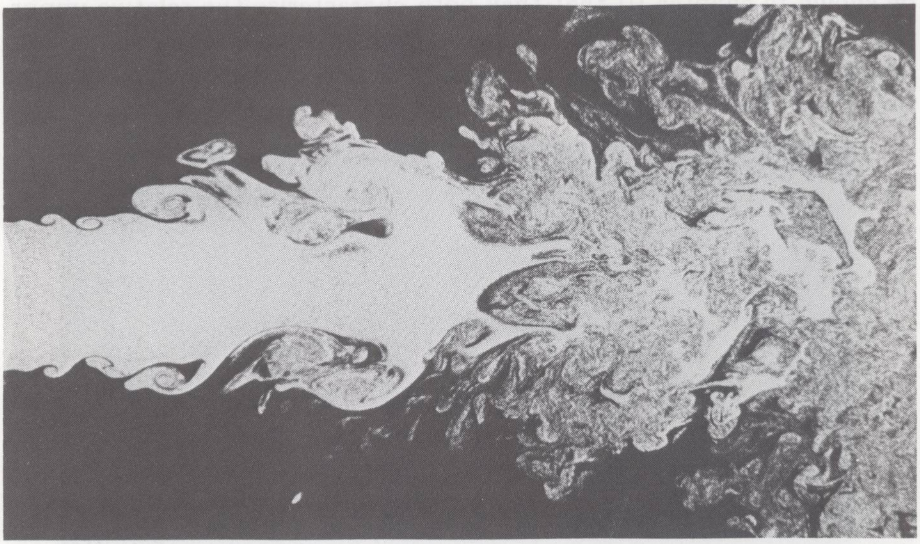

Figure I-2: turbulent jet (picture by J.L. Balint, M. Ayrault and J.P. Schon, Ecole Centrale de Lyon; from [1], courtesy J.P. Schon and "La Recherche")

2 Examples of turbulent flows

To illustrate the preceding considerations, it may be useful to display some flows which come under our definition of turbulence. Figure 2 shows a turbulent air jet marked by incense smoke and visualized thanks to a technique of laser illumination.

Figure 3 shows a "grid turbulence" described above.

Figure 4 shows a mixing layer between two flows of different velocities [2], which develop at their interface a Kelvin-Helmholtz type instability responsible for the large quasi-two-dimensional structures. Upon these structures are superposed three-dimensional turbulent small scales which seem to be more active when the Reynolds number is increased.

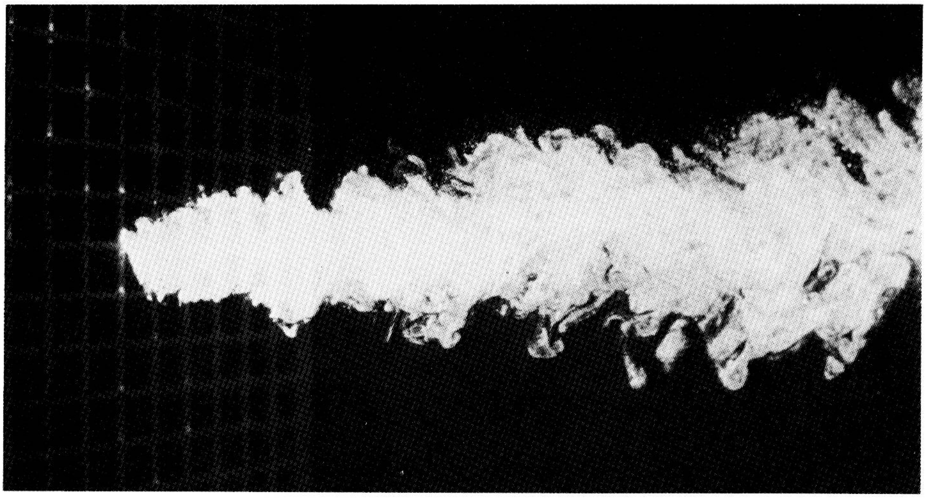

Figure I-3: turbulence created in a wind tunnel behind a grid. Here turbulence fills the whole apparatus, and a localized source of smoke has been placed on the grid to visualize the development of turbulence (picture by J.L. Balint, M. Ayrault and J.P. Schon, Ecole Centrale de Lyon; from [1], courtesy "La Recherche")

A

B

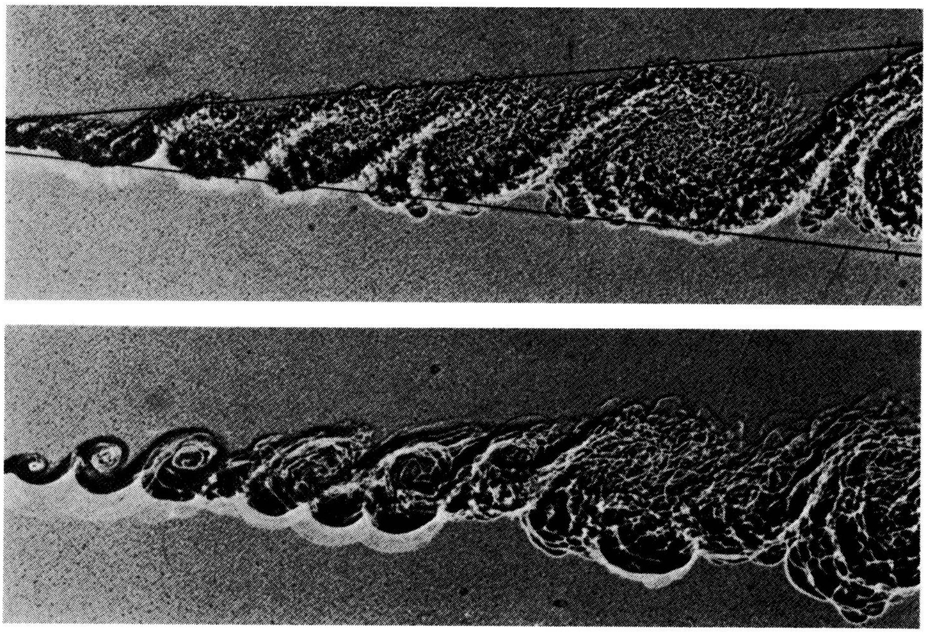

Figure I-4: turbulence in a mixing layer [2]. In Figure 4A, The Reynolds number (based on the velocity difference and the width of the layer at a given downstream position) is twice Figure 4B's (courtesy A.Roshko and J. Fluid Mech.)

Figure I-5: isovorticity regions (corresponding to fluid particles having the same vorticity) in the two-dimensional numerical simulation of the mixing layer reported in [3] (courtesy P. Leroy, Institut de Mécanique de Grenoble)

Figure 5 shows a two-dimensional numerical calculation of the vorticity of the large structures, in a numerical resolution of the equations of the flow motion in the particular case of the mixing layer [3]. As already stressed, the resemblance to the spiral galaxies is striking.

The latter structures are often called "coherent" because they can be found extremely far downstream. But it is possible for them to become irregular and unpredictable, and constitute then a quasi-two-dimensional turbulent field. Evidence for that is shown in Figure 6, corresponding to the same calculation as that presented in Figure 5: the evolution of the flow after 30 characteristic dynamic initial times is presented for four independent initial small random perturbations superimposed upon the basic inflexional velocity shear: the structures display some important differences, since there are for instance four eddies in Figure 6-d and only three eddies in Figure 6-b. They therefore show some kind of unpredictability.

In the mixing layer experiment of Figure 4, the turbulence in the small scales could be called fully developed turbulence, because it might have forgotten the mechanisms of generation of turbulence, i.e. the basic inflexional shear. On the contrary, the large structures depend crucially on the latter, and the terminology of "developed" cannot be used for them.

Similar large structures can be found in the turbulence generated in a rapidly rotating tank by an oscillating grid located at the bottom of the tank. Figure 7 shows a section of the tank perpendicular to the axis of rotation.

Here, the effect of rotation is to induce two-dimensionality in the flow, and to create strongly concentrated eddies with axes parallel to the axis of rotation [4]. These eddies could have some analogy with tornadoes in the atmosphere.

As already mentioned earlier, atmospheric and oceanic flows are highly unpredictable and fall into the category of turbulent flows. Their dynamics in the large scales is strongly influenced by their shallowness (the ratio of vertical scales to the horizontal extension of planetary scales is of the order of 10^{-2} in the Earth's atmosphere), by the Earth's sphericity and rotation, by differential heating between the equator and the poles, and by topography. Figure 8 shows for instance the eddy field which can be seen from satellites in the Alboran sea.

The simplified model of two-dimensional and quasi- geostrophic turbulence will be considered in chapter IX so as to study the particular dynamics associated with these flows.

On a planet such as Jupiter which, like the Earth, is rapidly rotating (this concept of rapid rotation can be defined with respect to the smallness of a dimensionless parameter, the Rossby number, which will