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Fluid Mechanics

S. NAGARATNAM

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FLUID MECHANICS

[M.K.S. AND S.I. UNITS]

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PREFACE TO THE THIRD EDITION

In this edition, additional materials have been presented at places where they should appear, unlike in the previous edition.

Due to requests from students and faculty members a few more additional worked examples and problems have been added in all chapters.

The author expresses his heart felt thanks for many useful suggestions, and hopes to receive continued support in future.

August, 1976 S. NAGARATNAM

PREFACE TO THE SECOND EDITION

A few more worked examples and problems adopted from University, Public Service Commission and Institution of Engineers examination papers have been added in all chapters to meet the requirements of students.

A chapter on "Hydraulic Machines" has been added to meet the syllabus requirements of some of the universities. The treatment on the subject of hydraulic machines has been kept at an elementary level. Students who need advanced level of instruction may refer the author's book on Fluid Machines and Systems.

It is indeed gratifying to receive appreciation of the book from so many teachers. All helpful suggestions have been incorporated in this present edition. Many thanks to those who had sent their suggestions and comments about the book. The author would always welcome further suggestions towards improvement of the book.

December, 1974 S. NAGARATNAM

PREFACE TO THE FIRST EDITION

This book is intended as a text-book at the under-graduate level for all branches of engineering students. An attempt has been made to present the principles and concepts of fluid mechanics in simple, coherent and logical manner so that a student new to the subject will be able to comprehend and understand the subject with the minimum of external guidance. A large number of worked examples are presented in each chapter to help students to have easy understanding of the applications. Problems at the end of each chapter would help the students to prepare for various examinations.

This book has been written both in MKS and SI units. This will help the staff and students to familiarise themselves with the SI units which is gaining increasing international acceptance. A brief note on the SI system and conversion tables are given in the Appendix. The abbreviation kgf is used to denote the unit of force in the MKS system, while kg would indicate the measure of mass only in both MKS and SI systems.

Adoption of uniform notations throughout a book of this nature is difficult. Though every effort has been made to maintain uniformity, yet wherever clarity demands deviations have been adopted. However notations have been explained wherein it appears for the first time in their particular usage.

It is the aim of the author to present the subject material of fluid mechanics as an entity so that beginner will have a comprehensive idea of the subject. It is to serve this purpose chapters on Ideal Fluid Flow and Compressible Fluid Flow have been added in this text. The chapter on Dimensional Analysis and Model Study, sandwiched between chapters on Frictionless Fluid Flow and the Chapters on Real Fluid Flow, would serve as a bridge in the application of a refined mathematical tool in the analysis of engineering problems.

In the preparation of this text, the author has greatly benefited from the many works listed in the Bibliography.

The materials in the text were used during the past few years in the classes by the author and his colleagues. The helpful suggestions of the students and colleagues have been incorporated. The author acknowledges with thanks the help he had received from Mr. N. Rajagopalan of the Mechanical Engineering Department in the final preparation of the text on the chapter on Compressible Fluid Flow. My thanks are due to Prof. P. S. Mani Sundaram, Principal, for the facilities provided during the preparation of the book. The author would always welcome helpful suggestions towards improvement of the book.

June, 1973 S. NAGARATNAM

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1

Properties of Fluids

1·1. Introduction. Fluid mechanics is a branch of mechanics which deals with the static, kinematic and dynamic aspects of fluids. Fluids are at rest when there is no external unbalanced force and this aspect of study of fluids is called fluid statics. Kinematics refers to the study of fluids in motion where pressure forces are not considered, and if the pressure forces are also considered for the fluid in motion, it is called fluid dynamics.

This is one of the basic subjects in the field of engineering with wide applications. Early development of this subject called by the name hydraulics was essentially empirical in its approach arising out of engineering necessity to build hydraulic works. At the time of Mohan-jo-daro and Harrappa drainage systems were built and during the times of Roman and Greek civilizations about the beginning of the Christian era large water supply systems were built. These works could not have been built without the basic understanding of the principles of liquid motion. It is evident from the writings of the *Vedas* and the writings of Greek and Roman builders that they did not have correct and clear concepts of the laws of motion of liquids. However, they were aware of the need to have a better understanding of the fluid motion in the construction and maintenance of engineering works. They gained knowledge by the process of trial and error and their approach was empirical. The advent of industrial civilization in the West had a great impetus for the development of this subject. Lacking the application of mathematical analysis the subject continued to be empirical till the beginning of the eighteenth century. It was shown by physicists and mathematicians that mathematical solutions to many fluid problems can be obtained with reasonable accuracy if viscosity of fluid can be ignored. Hence a new branch of study of ideal fluids called hydrodynamics came into being. Till the beginning of this century, hydraulics essentially empirical and experimental, and hydrodynamics essentially mathematical existed side by side in parallel without making use of the techniques and advantages of each other. The study of fluids made a significant progress with the application of mathematical techniques when it was established that the influence of viscosity is predominant in the case of real fluids only near the boundary. The development of boundary layer theory served as a bridge between the mathematical and the empirical approaches in the study of fluids and this new science of fluid mechanics slowly gained increasing importance. Modern study of mechanics of fluids combines both theoretical and experimental aspects and the principles of fluid mechanics are used in many fields of engineering such as astronautics, aeronautics, hydronautics, meteorology, rate processes, fluid machines, gas dynamics, etc. Fundamental principles of fluid flow which could be applied to various fields of engineering are presented in this book.

1·2·1. Fluid. Any substance which is capable of flow from the point of view of internal molecular structure can be defined as a fluid. Fluids can exist in solid, liquid or gaseous states. But large number of solids are not fluids as they cannot flow. Some solids of non-crystalline structure such as plastics and glassy substances can be made to flow when subjected to high pressure for a long time. They can be considered as fluids as relative motion occurs between molecules under high stress. Such substances when heated do not melt suddenly at a fixed temperature but gradually become softer and more pliable. Though fluid can exist in any of the three states, the important distinction between a fluid and a solid is that a fluid begins to flow even if it is imperceptible due to the action of the small net shear force and it continues to flow as long as shear force is applied. But a solid does not deform (or flow) continuously due to an application of shear force. A solid will undergo a shear strain, *i.e.* angular deformation due to a shear force and it will disappear when the shear force is removed before the elastic limit of the substance is reached. Thus a solid body regains its original shape on the removal of the shear force. In the case of fluids, upon

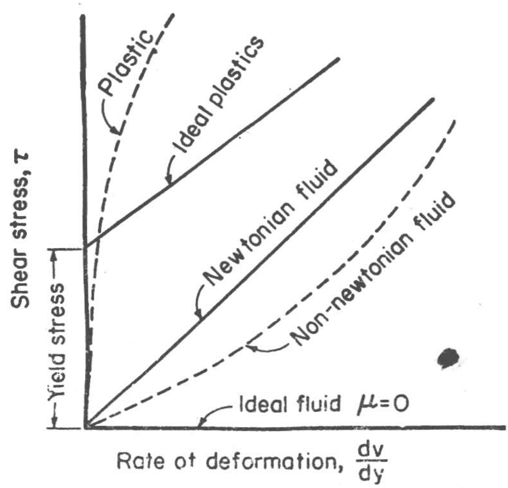

Fig. 1·1. Relationship between shear stress and rate of angular deformation.

removal of shear force the continuous shear strain ceases, but the body retains its new shape and position. A solid can resist shear force at rest, but a fluid cannot resist shear force at rest. Shear stresses occur in fluids when they are in motion only.

In some fluids the rate of angular deformation is directly proportional to shear stress and the constant of proportionality is called dynamic viscosity of the fluid. Such fluids are called Newtonian fluids. The fluids where this relationship is non-linear or with a definite yield stress are called non-Newtonian fluids. Though most of the fluids of common occurrence such as air, water, etc., are Newtonian, yet vast number of fluids such as plastics, bituminous materials, honey, dilatents, etc., are non-Newtonian in nature. With increasing use of plastics in engineering industries the study of non-Newtonian fluids has become important and the study of this subject is called rheology. The relationship between the time rate of angular deformation and shear stress for various substances are shown in Fig. 1·1. It may be noted that the non-linearity of non-Newtonian fluids is not uniform. We shall deal about Newtonian fluids only.

All liquids and gases are fluids as they undergo deformation continuously when subjected to even slightest shear force. We shall hereafter refer liquids and gases only as fluids.

1·2·2. Liquids and Gases. A liquid has a definite volume and it takes the shape of the vessel containing it. It would occupy the vessel fully or partially depending on its content and it can have a free surface. But a gas has no definite shape and it would expand and occupy the vessel fully and it cannot have a free surface. The volume of a liquid varies very slightly due to change in temperature and pressure. This variation is so small that for all practical purposes it is negligible and hence a liquid is considered as incompressible. But a gas undergoes considerable change in volume due to change in temperature and pressure. Hence gases are compressible fluids.

1·3. Continuum. A fluid consists of innumerable molecules of the same kind and any two adjacent molecules are separated by an empty space. The number of molecules in a given volume of liquid is much more than in an equal volume of gas. As the distance between two adjacent molecules in a gas is more than the distance between two adjacent molecules in a liquid the volume of a gas can be reduced by relatively a smaller pressure. Molecules are in constant movement and they are held together because of an attractive force between any two molecules. This molecular movement increases with the increase in temperature or decrease in pressure.

The molecular attraction is larger in a solid state than in a liquid state and hence a larger shear force is required to deform a solid body. In the case of liquids even a slightest shear force can move the molecules one past the other and this process of displacement continues till the force is removed. Hence continuous shear deformation (angular deformation) takes place.

It is nearly impossible to trace the absolute motion of a fluid molecule. Even the quantum theory provides only a satistical motion of a fluid molecule. It would be adequate for us to have an average value of velocity and pressure of a fluid at a point. Absolute motion of a fluid molecule at a point is beyond the scope of normal engineering activity. We, therefore, assume that there is no empty space between any two adjacent fluid molecules. As the entire space is considered fully filled up with fluid molecules, fluid motion is continuous and hence fluid can be taken as a *continuum*. This assumption is valid for most of the engineering application except in the field of rarefied flow. In a continuum, all physical properties of a fluid including velocity, acceleration and pressure are considered a varying continuously with respect to time and space.

1·4·1. Physical Properties. Property is a characteristic of a substance which is invariant when the substance is in a particular state under a particular condition. Various properties of fluids will be discussed in this article after a brief introduction to the concept of pressure.

It is the usual engineering practice to refer pressure intensity, that is, the force per unit area as pressure. It is denoted by the symbol p and it has a dimension of FL^{-2} or $ML^{-1}T^{-2}$, where F, M, L, T refer to force, mass, length and time respectively. It is a scalar quantity unlike force. It is expressed in multiples of newtons per square metre in SI units or in kilogram force per square centimetre in MKS units.

Any pressure measuring instrument indicates the difference between two pressures. If a pressure is measured with respect to the surrounding atmosphere, it is called *gauge pressure* which is positive if it is above atmospheric pressure and negative if below atmospheric pressure. The sum of gauge pressure and local atmospheric pressure is called *absolute pressure*. Zero absolute pressure would refer to the condition of perfect vacuum.

The atmospheric pressure slightly varies from place to place and under normal temperature of 15°C at sea-level at 45° latitude it can be taken as 101·325 kN/m² or 1·033 kgf/cm². A pressure of 10^5 N/m² or 100 kN/m² is called one bar and one thousandth part of a bar is called a millibar which is commonly used by meteorologists to express atmospheric pressure.

Pressures below atmospheric pressure can be expressed as negative gauge pressure or with reference to absolute pressure. Pressure expressed in absolute values are always positive.

It is customary to take the standard atmospheric pressure as 1 kgf/cm² in MKS units and similarly for the convenience of engineering calculations the standard atmospheric pressure can be taken as 100 kN/m² or 1 bar. Pressure can also be expressed in equivalent height of fluid column. The standard atmospheric pressure is taken as 10 metres of fresh water.

1·4·2. Density, Relative Density, Specific Volume and Specific Weight. Considering mass is uniformly distributed, the ratio of mass of a given substance to the volume the mass occupies is defined as the mean density or simply density of that substance. The symbol ρ (rho) shall be used to denote the density of a substance per unit volume. Density at a point is expressed mathematically,

$$\rho = \lim_{\forall \to 0} \frac{\Delta M}{\Delta \forall}$$

where M is the mass occupying the volume $\forall$. It has a dimension M/L^3. If the mass is expressed in kilogram (kg), the density will be in kilogram per cubic metre. The density of water at 4°C and at standard atmospheric pressure is 100 kg/m³ or 1 tonne/m³.

The relative density, often called specific gravity, is defined as the ratio between the density of a substance to the density of water at 4°C and at standard atmospheric pressure. It is dimensionless as it is a ratio of two magnitudes of the same kind. It is denoted by the symbol S.

The specific volume, denoted by v, is defined as the reciprocal of density and it is the volume occupied by a unit mass of substance.

$$v = \frac{1}{\rho}.$$

It has a dimension of L^3/M and it is expressed in m³/kg.

The specific weight, denoted by γ (gamma), is the ratio between the weight of a substance to the volume it occupies. γ is the weight per unit volume of a substance. The weight of a substance is the product of its mass and acceleration due to gravity at its location. As gravitational acceleration varies from place to place on earth the specific weight of a substance also varies depending on its location.

$$\gamma = \rho g.$$

It has a dimension of F/L^3 or $ML^{-2}T^{-2}$. As ρ is expressed in kg/m³ and g in m/sec², specific weight is expressed in newtons/m³ in SI units and in kgf/m³ in MKS units. Weight of fresh water at 4°C is 9·81 kN/m³ or 1000 kgf/m³.

Example 1·1 (MKS units). *One litre of petrol weighs 0·716 kgf. Calculate the specific weight, density, specific volume and relative density.*

$$\gamma = 0{\cdot}716 \times 1000 = 716 \text{ kgf/m}^3$$

$$\rho = \frac{\gamma}{g} = \frac{716}{9{\cdot}81} = 73{\cdot}03 \frac{\text{kgf-sec}^2}{\text{m}^4}$$

But $1 \dfrac{\text{kgf. sec}^2}{\text{m}} = 9{\cdot}81$ kgm (kilogram-mass)

Hence $\rho = 73{\cdot}03 \times 9{\cdot}81 = 716$ kgm/m³

$$v = \frac{1}{\rho} = \frac{1}{716} = 1{\cdot}395 \times 10^{-3} \text{ m}^3/\text{kgm}$$

$$S = \frac{716}{1000} = 0{\cdot}716.$$

Example 1·1 (SI units). *One litre of petrol weighs 7·02 N. Calculate the specific weight, density, specific volume and relative density.*

$$\gamma = 7{\cdot}02 \times 1000 = 7{\cdot}02 \text{ kN/m}^3$$

$$\rho = \frac{7{\cdot}02 \times 1000}{9{\cdot}81} = 716 \text{ kg/m}^3$$

$$v = \frac{1}{\rho} = \frac{1}{716} = 1{\cdot}395 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$S = \frac{7{\cdot}02}{9{\cdot}81} = 0{\cdot}716.$$

1·4·3·1. Viscosity. One of the main characteristics of a real fluid is its tendency to undergo angular deformation continuously as long as a shear force is applied. When the shear force is removed the fluid particles remain in their new positions. They do not go back to their original positions. The resistance to the rate of angular deformation, under steady condition, is equal to the external shear force. The internal resistance is caused due to the movement of one layer of fluid over the adjacent layer and it varies from fluid to fluid. This property of the fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid is called viscosity of the fluid.

Consider the velocity field between two parallel plates due to flow of a real fluid as shown in Fig. 1·2. It is taken that the velocity

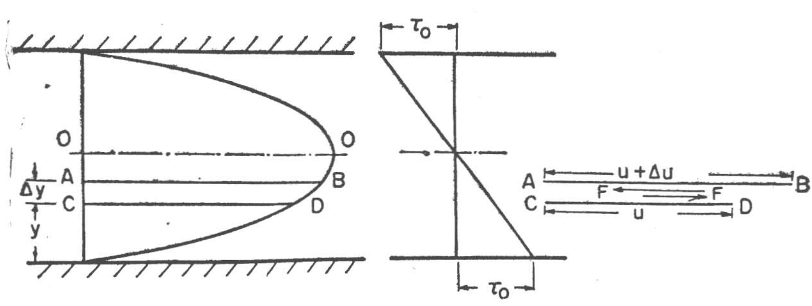

Fig. 1·2. Shear distribution in a velocity field.

distribution is parabolic. Let *AB*, *CD* be two adjacent layers separated by an elemental distance Δy, where velocities of flow per unit time are $(u+\triangle u)$ and u respectively. It can be seen from the figure that as the layer AB has higher velocity it moves past the layer CD by an additional distance per unit time. Because of this relative motion between two layers and as no slipping can occur along a surface within a fluid, a shear force F would be caused at the contact surface A causing rate of angular deformation of magnitude $\triangle u/\triangle y$. As $\triangle y\to 0$ the angular deformation becomes equal to the velocity gradient at D.

$$\lim_{\triangle y\to 0} \frac{\triangle u}{\triangle y}=\frac{\partial u}{\partial y}.$$

Partial derivative is used as velocity can also vary in other directions.

1·4·3·2. Dynamic Viscosity. Newton postulated that for a steady uniform flow the shear stress τ (tau) between two adjacent layers is proportional to the velocity gradient in the direction perpendicular to the layers. Hence

$$\tau=\frac{F}{A}\propto\frac{\partial u}{\partial y}$$

$$=\mu\frac{\partial u}{\partial y} \tag{1·1}$$

where μ (mu) the constant of proportionality in the above equation is called dynamic or absolute viscosity of the fluid. The above relationship is called Newton's law of viscosity. Any fluid which obeys the above law is called Newtonian fluid. The relationship between shear stress and rate of angular deformation of some non-Newtonian fluids are shown in Fig. 1·1.

Depending on the sign of velocity gradient, the direction of action of shear force changes. If the shear acts in the direction of velocity it is considered positive. The value of τ in the equation is with reference to the force on the layer CD.

It is evident from the equation that $\tau=0$ when $\frac{\partial u}{\partial y}=0$. Hence there would not be any shear force in uniform flow or at the symmetry of a flow (about axis OO in the figure). The velocity gradient cannot be infinite as it is not physically possible to have an infinite value for the shear stress. Hence the value of velocity gradient should change continuously without any jump throughout the flow region including the boundary. If the boundary is stationary then the fluid is very close to it should have zero velocity and if the boundary velocity is V, the fluid adjacent to it should also have a velocity V. This condition which must be satisfied in all real fluid flows is called '*no slip condition*'.

Viscosity of a fluid is manifested only when there is relative motion between adjacent layers of fluid. No shear force need to be considered for a fluid at rest or in irrotational flow. Hence study of hydrostatics and irrotational flow is greatly simplified as normal and gravity forces need only be considered.

The main cause of energy loss in real fluid flow is due to viscosity. Work has to be done against the force causing resistance to the movement of one layer of fluid over another layer during motion. This energy loss is ultimately dissipated as heat which will never be recoverable to the fluid. This loss of energy in fluid motion is usually called 'frictional loss' or 'skin friction loss'. The effect of viscosity can be conveniently ignored in many situations to arrive at quick mathematical solutions.

The cause of viscosity in fluids under all temperatures and pressures is not fully understood. It is believed that transfer of momentum and cohesion between fluid molecules are responsible for this fluid property. Except under very high and low temperatures, as the temperature increases, the viscosity of all gases increases while that of liquids decreases. This is because of the force of cohesion which is predominant in liquids diminishes with the rise in temperature while in gases the predominating factor is the interchange of molecules between layers of different velocities. The absolute viscosity of liquids and gases is independent of pressure for the range that is ordinarily encountered in engineering applications.

The dimension of dynamic or absolute viscosity on the basis of Eq. 1·1 is

$$\mu=\frac{\tau}{\frac{\partial u}{\partial y}}=\frac{F/L^2}{\frac{L}{T}\,\frac{1}{L}}=FL^{-2}\,T$$

Expressing in terms of M, L and T,

$$\mu=ML^{-1}\,T^{-1}$$

Since $$F=MLT^{-2}.$$

In the CGS (cm-gm-second) system as force is expressed in dynes, the unit of viscosity is dyne-sec/cm² which is called poise P after the French scientist Poiseuille. As this unit is large, one hundredth of poise or centipoise cP is frequently used in practice. Though newton-sec/m² would be the counterpart unit of measure of dynamic viscosity in SI units, the present practice of using poise will be continued. Newton-sec/m² is equal to ten times of poise. The relationships between various units of dynamic viscosity are shown below.

$$1\text{ Poise}=\frac{\text{dyne-sec}}{\text{cm}^2}$$

$$=\frac{1}{10}\,\frac{\text{newton-sec}}{\text{m}^2}$$

$$=\frac{1}{98{\cdot}1}\,\frac{\text{kgf-sec}}{\text{m}^2}$$

Specific viscosity is defined as a ratio between the dynamic viscosity of a fluid in centipoises and the dynamic viscosity of water at 20°C in centipoises. Water at 20°C has viscosity of 1·0 cP.

$$\text{Specific viscosity} = \frac{\text{Viscosity of a fluid in centipoises}}{\text{Viscosity of water at 20°C in centipoises}}$$

The specific viscosity of a fluid becomes numerically the viscosity of fluid itself. It is a pure number.

1·4·3·3. Kinematic Viscosity. The ratio between the dynamic viscosity and density is defined as kinematic viscosity of a fluid and it is denoted by ν (nu).

$$\nu = \frac{\mu}{\rho} \tag{1·2}$$

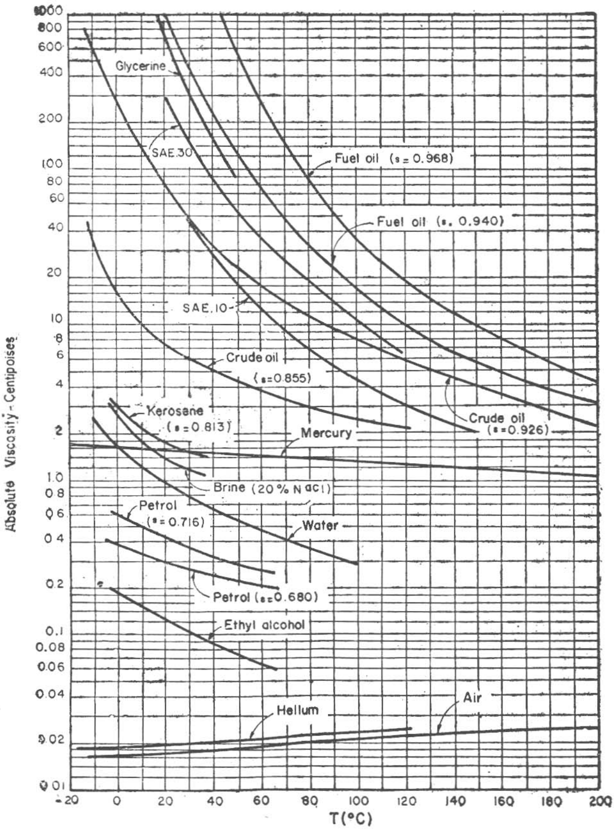

Fig. 1·3. Dynamic viscosity.

This ratio occurs often in engineering applications as a relation between viscous and inertial effects of fluid motion and hence it