

# **RF and Microwave**

## **Transistor Oscillator Design**

**ANDREI GREBENNIKOV**

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# **RF and Microwave Transistor Oscillator Design**

# About the Author

Dr Andrei Grebennikov, IEEE Senior Member, has obtained long-term academic and industrial experience. He worked with Moscow Technical University of Telecommunications and Informatics, Russia; Institute of Microelectronics, Singapore; M/A-COM, Ireland and Infineon Technologies, Germany, as an engineer, researcher, lecturer and educator.

Dr Grebennikov has lectured as a Guest Professor in University of Linz, Austria, and presented short courses as an Invited Speaker at the International Microwave Symposium, European Microwave Conference and Motorola Design Centre, Malaysia. He is also the author of more than 60 papers, 3 books and several US patents.

# Preface

The main objective of this book is to present all relevant information necessary for RF and microwave transistor oscillator design including well-known and new theoretical approaches and practical circuit schematics and designs, as well as to suggest optimum design approaches, which combine effectively analytic calculations and computer-aided design. This book can be useful for lecturing to promote the analytical way of thinking and combine effectively theory and practice of RF and microwave engineering. As often happens, a new result is a long-forgotten old one. Therefore, not only new results based on new technologies or circuit schematics are given, but some old ideas, schematics or approaches are also introduced, that could be very useful in modern practice or could contribute to the development of new ideas or techniques.

As a result, this book is intended for and can be recommended to:

- *university-level professors and researchers*, as possible reference and well-founded material for creative research and teaching activity which will contribute to strong background for graduates and postgraduates students;
- *R&D staff*, to combine the theoretical analysis and practical aspects, including computer-aided design (CAD) and to provide a sufficient basis for new ideas in theory and practical circuit techniques;
- *practising RF designers and engineers*, as an anthology of many well-known and new practical transistor oscillator circuits with detailed descriptions of their operational principles and applications and clear practical demonstration of theoretical results.

Chapter 1 presents the most commonly used design techniques for analysing nonlinear circuits, in particular, transistor oscillators. There are several approaches to analyse and design nonlinear circuits, depending on their main specifications. That means an analysis both in the time domain to determine transient circuit behaviour and in the frequency domain to improve power and spectral performances when parasitic effects such as instability and spurious emission must be eliminated or minimized. Using the time-domain technique, it is relatively easy to describe a nonlinear circuit with differential equations, which can be solved analytically in explicit form for only some simple cases. Under the assumption of slowly varying amplitude and phase, it is possible to obtain the separate truncated first-order differential equations for the amplitude and phase of the oscillation process from the original second-order nonlinear differential equation. However, generally it is necessary to use numerical methods. The time-domain analysis is limited to its inability to operate with the circuit immittance (impedance or

admittance) parameters as well as the fact that it can be practically applied only for circuits with lumped parameters or ideal transmission lines. The frequency-domain analysis is less ambiguous because a relatively complex circuit can often be reduced to one or more sets of immittances at each harmonic component. For example, using a quasilinear approach, the nonlinear circuit parameters averaged by the fundamental component allow one to apply a linear circuit analysis. Advanced modern CAD simulators incorporate both time-domain and frequency-domain methods as well as optimization techniques to provide all the necessary design cycles.

Chapter 2 introduces the principles of oscillator design, including start-up and steady-state operation conditions, basic oscillator configurations using lumped and transmission-line elements and simplified equation-based oscillator analysis and design techniques. An immittance design approach is introduced and applied to series and parallel feedback oscillators, including circuit design and simulation aspects. Numerous practical examples of RF and microwave oscillators using MOSFET, MESFET and bipolar devices, including the descriptions of their circuit realizations, are given.

Applying dc bias to the active device does not generally result in the negative resistance condition. This condition has to be induced in these devices and it is determined by the physical mechanism in the device and chosen circuit topology. The transistor in the oscillator circuits is mostly represented as the active two-port network, whose operation principle is reflected through its equivalent circuit. The influence of the circuit and transistor parameters can result in a hysteresis effect or oscillation instability in practical design. In high-frequency practical implementation, the presence of the parasitic device and circuit elements can contribute to the multi-resonant circuits. The possibility of an operation mode with different natural frequencies depends on the value of the coupling coefficient between resonant circuits. Therefore, the stability conditions for a steady-state single-frequency operation for a multi-resonant circuit, in general, and two coupled resonant circuits, in particular, are analytically derived. The several examples of stability criteria for different single-resonant and double-resonant oscillator circuits are described and analysed in Chapter 3. In addition, the phase plane method as a qualitative method of an analysis of the dynamics of the oscillation systems and a Nyquist stability criterion are shown and illustrated by several examples of the oscillator circuits described by second-order differential equations.

Generally, RF and microwave transistor oscillator design is a complex problem. Depending on the technical requirements, it is necessary to define the configuration of the oscillator circuit, choose a proper transistor type, evaluate and measure the parameters of the transistor nonlinear model under small- and large-signal conditions. Finally, an appropriate nonlinear simulator must be used to simulate the oscillator performance in time and frequency domains. An oscillator analysis can be based on the two-port network approach to describe the active device and feedback circuit. In this case, the basic parameters of the transistor equivalent circuit can be directly measured, or approximated on the basis of experimental data, with sufficient accuracy across a wide frequency range. However, the values of the external feedback circuit elements are initially unknown. The process of determining the optimum values of the feedback and load parameters can be time-consuming and, in a typical case, calls for much simulation. Consequently, it is convenient to use an analytic method of optimizing oscillator design. This method should incorporate the explicit expressions for feedback elements and load impedance in terms of the transistor equivalent circuit elements and its static volt-ampere and voltage-capacitance characteristics. Chapter 4 presents both the empirical and analytic optimum design approaches applied to series and parallel feedback oscillators, including circuit design and

simulation aspects, and high-efficiency design techniques as well. Typical practical examples of RF and microwave oscillators using MOSFET, MESFET, HEMT, and bipolar devices, including the descriptions of their circuit configurations, are given.

Chapter 5 describes different oscillator noise models to express a clear relationship between the resonant circuit and active device noise model parameters. The simple Leeson linear model for a feedback oscillator, which was derived empirically, is based on the expectations that the contribution to the real oscillator output spectrum is provided by two basic processes. The first process is a result of the phase fluctuations due to the additive white noise at frequency offsets close to the carrier. The second process is a result of the low-frequency fluctuations or flicker noise up-converted to the carrier region because of the active device nonlinear effects. The nonlinear Kurokawa analysis based on the sinusoidal representation of the current in the negative-resistance oscillator extends the oscillator noise model by introducing relationships between the noise power, stability conditions and amplitude-to-phase conversion. However, such a noise generation mechanism does not consider the mixing effect from the inherent nonlinear behaviour of the active device when the current at the output of the active device must be represented by a Fourier series expansion. Thus, the phase noise generated around the fundamental frequency of the oscillation generally is an equal contribution of two simultaneous and correlated phenomena: additive phase noise due to phase modulation process and converted phase noise due to conversion from one sideband to another.

Voltage-controlled oscillators are key components in many applications, especially in wireless communication systems, measurement equipment, or military applications. A growing market of wireless applications requires highly integrated circuit solutions, where both high-performance transistors and passive elements with high quality factors can be used. Chapter 6 discusses the varactor modelling issues, varactor nonlinearity and its effect to frequency modulation, and resonant circuit techniques to improve VCO tuning linearity using lumped and transmission-line elements. Various practical examples of VCO implementation techniques based on using different types of active devices, circuit schematic approaches and hybrid or monolithic integrated circuit technologies are shown and described.

The rapid growth of new-generation wireless communication systems has created a strong demand for designing single-chip radio transceivers in a fully monolithic CMOS process with extremely small size due to better integration, low cost and low operating voltage. To increase the integration level, all passive components must be integrated monolithically into a single chip. In this case, the elements of a resonant  $LC$  circuit of the voltage-controlled oscillator as a core part of the synthesizers should feature high quality factors over frequency tuning range. Chapter 7 discusses the technological aspects to realize MOS varactors and spiral inductors, basic concepts of circuit design and implementation issues, oscillator phase noise and the effect of low-frequency flicker noise. Also included are various practical examples of differential, complementary and quadrature CMOS VCOs using different process technologies.

Wideband voltage-controlled oscillators are used in a variety of RF and microwave systems, including broadband measurement equipment, wireless and TV applications and military electronic countermeasure systems. Among wideband tunable signal sources such as YIG-tuned oscillators, wideband VCOs are preferable because of their small size, low weight, high settling time speed and capability of fully monolithic integration. Therefore, modern radar and communication applications demand VCOs that are capable of being swept across a wide range of potential threat frequencies with a speed and settling time far beyond that of the YIG-tuned oscillators. This chapter discusses the basic concepts of wideband VCO circuit design and gives specific circuit solutions using lumped elements and transmission lines to improve



their frequency tuning characteristics. Various examples of the RF and microwave VCO circuit configurations using bipolar, MOSFET and MESFET devices are analysed, their circuit parameters are calculated or optimized to provide maximum tuning bandwidth or minimum tuning linearity. Also included are numerous practical examples of wideband VCOs for RF and microwave applications in radar or telecommunication systems.

Chapter 9 discusses phase noise reduction techniques and gives specific resonant circuit solutions using lumped and distributed parameters for frequency stabilization and phase noise reduction. Phase noise improvement can also be achieved by appropriate low-frequency loading and feedback circuitry optimization. The feedback system incorporated into the oscillator bias circuit can provide significant phase noise reduction over a wide frequency range from the high frequencies up to microwaves. Particular discrete implementations of a bipolar oscillator with collector and emitter noise feedback circuits are described. Also a filtering technique based on a passive  $LC$  filter to lower the phase noise in the differential oscillator is presented. Several topologies of fully integrated CMOS voltage-controlled oscillators using filtering techniques are shown and discussed. A novel noise-shifting differential VCO based on a single-ended classical three-point circuit configuration with common base can improve the phase noise performance by a proper circuit realization. An optimal design technique using an active element based on a tandem connection of a common source FET device and a common base bipolar transistor with optimum coupling of the active element to the resonant circuit is presented. The phase noise in microwave oscillators can also be reduced using negative resistance compensation increasing the loaded quality factor of the oscillator resonant circuit. Finally, a new approach utilizing a nonlinear feedback loop for phase noise suppression in microwave oscillators is discussed.

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# 1

## Nonlinear circuit design methods

This chapter presents the most commonly used design techniques for analysing nonlinear circuits, in particular, transistor oscillators. There are several approaches to analyse and design nonlinear circuits, depending on their main specifications. This means an analysis both in the time domain to determine transient circuit behaviour and in the frequency domain to improve power and spectral performances when parasitic effects such as instability and spurious emission must be eliminated or minimized. Using the time-domain technique, it is relatively easy to describe a nonlinear circuit with differential equations, which can be solved analytically in explicit form for only a few simple cases. Under an assumption of slowly varying amplitude and phase, it is possible to obtain separate truncated first-order differential equations for the amplitude and phase of the oscillation process from the original second-order nonlinear differential equation. However, generally it is required to use numerical methods. The time-domain analysis is limited to its inability to operate with the circuit immittance (impedance or admittance) parameters as well as the fact that it can be practically applied only for circuits with lumped parameters or ideal transmission lines. The frequency-domain analysis is less ambiguous because a relatively complex circuit can often be reduced to one or more sets of immittances at each harmonic component. For example, using a quasilinear approach, the nonlinear circuit parameters averaged by fundamental component allow one to apply a linear circuit analysis. Advanced modern CAD simulators incorporate both time-domain and frequency-domain methods as well as optimization techniques to provide all necessary design cycles.

This chapter also includes a brief introduction of simulator tools based on the Ansoft Serenade circuit simulator. In addition, some practical equations, such as the Taylor and Fourier series expansions, Bessel functions, trigonometric identities and the concept of the conduction angle, which simplify the circuit design procedure, are given.

### 1.1 SPECTRAL-DOMAIN ANALYSIS

The best way to understand the oscillator electrical behaviour and the fastest way to calculate its basic electrical characteristics such as output power, efficiency, phase noise, or harmonic suppression, is to use a spectral-domain analysis. Generally, such an analysis is based on the determination of the output response of the nonlinear active device when the multiharmonic

signal is applied to its input port, which analytically can be written in the form

$$i(t) = f[v(t)] \quad (1.1)$$

where  $i(t)$  is the output current,  $v(t)$  is the input voltage and  $f(v)$  is the nonlinear transfer function of the device. Unlike the spectral-domain analysis, time-domain analysis establishes the relationships between voltage and current in each circuit element in the time domain when a system of nonlinear integrodifferential equations is obtained applying Kirchhoff's law to the circuit to be analysed.

The voltage  $v(t)$  in frequency domain generally represents the multiple frequency signal at the device input in the form

$$v(t) = V_0 + \sum_{k=1}^N V_k \cos(\omega_k t + \phi_k) \quad (1.2)$$

where  $V_0$  is the constant voltage,  $V_k$  is the voltage amplitude and  $\phi_k$  is the phase of the  $k$ th-order harmonic component  $\omega_k$ ,  $k = 1, 2, \dots, N$ , and  $N$  is the number of harmonics.

The spectral domain analysis based on substituting Equation (1.2) in Equation (1.1) for a particular nonlinear transfer function of the active device determines an output spectrum as a sum of the fundamental-frequency and higher-order harmonic components, the amplitudes and phases of which will determine the output signal spectrum. Generally, this is a complicated procedure which requires a harmonic balance technique to numerically calculate an accurate nonlinear circuit response. However, the solution can be found analytically in a simple way when it is necessary to estimate only the basic performance of an oscillator in the form of the output power and efficiency. In this case, a technique based on a piecewise-linear approximation of the device transfer function can provide a clear insight into the basic oscillator behaviour and its operation modes. It can also serve as a good starting point for a final computer-aided design and optimization procedure.

The result of the spectral-domain analysis is shown as a summation of the harmonic components, the amplitudes and phases of which will determine the output signal spectrum. This problem can be solved analytically by using trigonometric identities, piecewise-linear approximation or Bessel functions.

### 1.1.1 Trigonometric identities

The use of trigonometric identities is very convenient when the transfer characteristic of the nonlinear element can be represented by the power series

$$i = a_0 + a_1 v + a_2 v^2 + \dots + a_n v^n \quad (1.3)$$

If the effect of the input signal represents a single harmonic oscillation in the form

$$v = V \cos(\omega t + \phi) \quad (1.4)$$

then, by substituting Equation (1.4) into Equation (1.3), the power series can be written as

$$i = a_0 + a_1 V \cos(\omega t + \phi) + a_2 V^2 \cos^2(\omega t + \phi) + \dots + a_n V^n \cos^n(\omega t + \phi) \quad (1.5)$$

To represent the right-hand side of Equation (1.5) as a sum of first-order cosine components, the following trigonometric identities, which replace the  $n$ th-order cosine components, can be



used:

$$\cos^2 \psi = \frac{1}{2}(1 + \cos 2\psi) \quad (1.6)$$

$$\cos^3 \psi = \frac{1}{4}(3 \cos \psi + \cos 3\psi) \quad (1.7)$$

$$\cos^4 \psi = \frac{1}{8}(3 + 4 \cos 2\psi + \cos 4\psi) \quad (1.8)$$

$$\cos^5 \psi = \frac{1}{16}(10 \cos \psi + 5 \cos 3\psi + \cos 5\psi) \quad (1.9)$$

where  $\psi = \omega t + \phi$ .

By using the appropriate substitutions from Equations (1.6–1.9) and equating the signal frequency component terms, Equation (1.5) can be rewritten as

$$i = I_0 + I_1 \cos(\omega t + \phi) + I_2 \cos 2(\omega t + \phi) + I_3 \cos 3(\omega t + \phi) + \dots + I_n \cos n(\omega t + \phi) \quad (1.10)$$

where

$$I_0 = a_0 + \frac{1}{2}a_2V^2 + \frac{3}{8}a_4V^4 + \dots$$

$$I_1 = a_1V + \frac{3}{4}a_3V^3 + \frac{5}{8}a_5V^5 + \dots$$

$$I_2 = \frac{1}{2}a_2V^2 + \frac{1}{2}a_4V^4 + \dots$$

$$I_3 = \frac{1}{4}a_3V^3 + \frac{5}{16}a_5V^5 + \dots$$

Comparing Equations (1.3) and (1.10), we find:

- For nonlinear elements, the output spectrum contains frequency components which are multiples of the input signal frequency. The number of the highest-frequency component is equal to the maximum degree of the power series. Therefore, if it is necessary to know the amplitude of  $n$ -harmonic response, the volt–ampere characteristic of nonlinear element should be approximated by not less than an  $n$ -order power series.
- The output dc and even-order harmonic components are determined only by the even voltage degrees in the device transfer characteristic given by Equation (1.3). The odd-order harmonic components are defined only by the odd voltage degrees for the single harmonic input signal given by Equation (1.4).
- The current phase  $\psi_k$  of the  $k$ th-order harmonic component  $\omega_k = k\omega$  is  $k$  times larger than the input signal current phase  $\psi$ :

$$\psi_k = \omega_k t + \phi_k = k(\omega t + \phi) \quad (1.11)$$

that is also applied to their initial phases defined as

$$\phi_k = k\phi \quad (1.12)$$