The Accuracy of Flow Measurements by Laser Doppler Methods

Proceedings of the LDA-Symposium Copenhagen 1975

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Contents

	rage
Preface Introduction	7 9
Signal properties W.K. George, Jr.: Limitations to measuring accuracy inherent in the laser Doppler	19 20
signal. L. Lading and R.V. Edwards: The effect of measurement volume on laser Doppler anemometer measurements as measured on simulated signals.	64
H.R.E. van Maanen, K. van der Molen and J. Blom: Reduction of ambiguity noise in laser-Doppler velocimetry by a cross correlation technique	81
J.M. Owen and R.H. Rogers: Velocity biasing in laser Doppler anemometers.	89
J.O. Asalor and J.H. Whitelaw: The influence of combustion-induced particle concentration variations on laser-Doppler anemometry.	115
D.F.G. Durão and J.H. Whitelaw: The influence of sampling procedures on the velocity bias in turbulent flows.	138
J.C.F. Wang: Measurement accuracy of flow velocity via a digital-frequency- counter laser velocimeter processor.	150
S. Hanson: Visualization of alignment errors and heterodyning constraints in laser Doppler velocimeters.	176
F. Durst and W.H. Stevenson: Moiré patterns to visually model laser-Doppler signals.	183
Electronics and signal processing F. Durst: Electronic processing of optical anemometer signals	207 208
W. Hösel and W. Rodi: Errors occuring in LDA-measurements with counter signal processing.	251
P. Buchhave: Biasing errors in individual particle measurements with the LDA-counter signal processor.	258

V. Bökemeier and N. Feige:	279
A correction method for removing Doppler-ambiguity broadening	
from analysed spectra.	
J. Adrian, J.A.C. Humphrey and J.H. Whitelaw:	287
Frequency measurement errors due to noise in LDV signals.	
J.C. Petersen and F. Maurer:	312
A method for the analysis of laser Doppler signals using a computer	
in connection with a fast A/D-converter.	
J.B. Abbiss, L.J.S. Bradbury and M.P. Wright:	319
Measurements in an axi-symmetric jet using a photon correlator.	
C. Fog:	336
A photon-statistical correlator for LDA-application.	
R.J. Baker and C. Wigley:	350
Design, evaluation and application of a filter bank signal processor.	
D.F.G. Durão, J. Laker and J.H. Whitelaw:	364
Digital processing of frequency-analysed Doppler signals.	504
Digital processing of frequency-aliasysed Doppler signals.	
Particles and particle measurements	381
A. Melling and J.H. Whitelaw:	382
Optical and flow aspects of particles.	302
F. Durst and M. Zaré:	403
	403
Laser Doppler measurements in two-phase flows.	420
F. Durst and H. Umhauer:	430
Local measurements of particle velocities, size distribution and	
concentration with a combined laser-Doppler particle sizing system.	465
F. Durst and B. Eliasson:	457
Properties of laser-Doppler signals and their exploitation for particle	
size measurements.	
	,
Contamo	479
Systems	480
R. Schodl:	400
On the extension of the range of applicability of LDA by means of	
the laser-dual-focus (L-2-F) technique.	400
F. Danel:	490
Fiber optic and endoscopic LDA systems.	400
N.A. Halliwell and J.E. Rizzo:	498
The application of laser Doppler anemometry in ship science.	
A. Melling:	505
A technique for simultaneous velocity and concentration measure-	
ments.	

Proceedings of the LDA-Symposium Copenhagen 1975	5
B. Lehmann:	510
Laser Doppler measurements of the probability density function of local velocities in two-phase droplet flows.	
W. Erdmann and W. Merzkirch:	519
Measurement of shock wave propagation in a non-uniform atmo-	
sphere by using a modified laser-Doppler interferometer.	1 12 2
P.W. Runstadler, Jr. and F.X. Dolan:	523
Design, development, and test of a laser velocimeter for high speed	
turbomachinery. J. Oldengarm:	553
Two-dimensional laser Doppler velocimetry.	333
I wo-dimensional laser Doppler velocificity.	
*/	- 4
Applications	565
J.M. Delhaye:	566
Two-phase flow instrumentation and laser beams.	44.0 F
	579
Comparison of accuracy of a laser Doppler velocimeter system with	
that of a Moiré fringe encoder.	2.14
Y.M. Trouilhet:	585
Laser Doppler anemometry by non-isothermal flow. G.C. Dubbeldam:	588
LDA measurements on man-made fibre processes.	
L. Masbernat, C. Suzanne and V. Referowski:	593
Turbulent velocity measurements in a stratified co-current two-phase	
flow. W.W. Fowlis and P.J. Martin:	599
Laser Doppler measurements of a low speed laminar flow.	377
G. Lörcher and G. Grötzbach:	603
LDA-measurements in turbulent incompressible flows used for verification of theoretical turbulence models.	
A. Müller:	608
Comparison of pitot-tube and LDA in a rotational flow.	
D.P. Ly and A. Bousquet:	612
Determination of velocity profiles, by laser Doppler anemometry in pulsating laminar flow of non-Newtonian fluids.	
O.R. Ferreira, R.E.W. Jansson and J.E. Rizzo:	618
Correlation of laser anemometer information with measurements of	
mass transfer.	
K.L. Orloff, V.R. Corsiglia, J.C. Biggers and	*
T.W. Ekstedt:	624
Investigating complex aerodynamic flows with a laser velocimeter.	

A. Boutier and J. Lefevre:	644
Some applications of laser anemometry in wind tunnels.	
P. Scheel Larsen, H. Saustrup Kristensen and	
C. Borgnakke:	663
Bunsen flame velocity distributions by LDA.	
A. Ballantyne and K.N.C. Bray:	674
Velocity power spectral measurements in jet diffusion flames using	
laser anemometry.	
N.E. Kotsovinos and E.J. List:	684
Turbulence measurements in a two-dimensional bouyant jet using	
laser Doppler velocimetry.	
A.J. Loader and M.T. Thew:	690
Analysis and characteristics of turbulence in confined swirling flow	
studied with an LDA.	
A.R. Freeman:	704
Laser anemometer measurements in the recirculating region down-	
stream of a sudden pipe expansion.	
K.V. Rao:	710
Secondary flow in a curved channel as revealed by laser Doppler	
anemometer.	
L.E. Drain, M.L. Yeoman and W. Dalzell:	719
Measurements of spatial velocity correlation for turbulent flow in a	
pipe.	
R. Teitgen:	725
Laminar turbulent transition in pipe flow measurements made with	
pitot probe, hot wire sensor and LDA.	
J.R. Clermont, J.M. Piarrard and O. Scrivener:	731
Evolution of the velocity profiles in the outlet of a tube and in the	
jet polymer solutions.	

Preface

It is well known that laser-Doppler anemometry can be used to effect local velocity and velocity-correlation measurements in fluids which are transparent and which contain particles suitable for light scattering. The precision of measurement depends substantially on the particular instrumentation and the flow and is often difficult to quantify. The contributions contained in this volume were presented at an International Symposium which had the following purposes:

 To establish the value of measurements by laser-Doppler anemometry by discussing possible sources of error and the consequent precision.

 To demonstrate the range of applicability of laser-Doppler anemometry and the precision of measurements achieved.

It was held at the Technical University of Denmark from 25th - 28th August, 1975.

This volume of proceedings makes available papers which were presented at the Symposium and, thereby, makes the results of the deliberations at the Symposium available to the scientific community at large.

The Symposium is clearly reflected in these proceedings. It contains papers which are presented in five sections and which relate respectively to the characteristics of the Doppler signal, signal processing techniques, lightscattering particles, systems and applications. They are proceeded by a general introduction, prepared by the Editorial Board, which provides a foundation for the subsequent sections by describing the basic principle of laser-Doppler anemometry, introducing terminology and making reference to the more important past contributions. The sections dealing with the Doppler signal and signal processing provide papers which are particularly concerned with the quantification of obtainable precision. The three papers which discuss particles describe methods of measuring particle size by optical techniques which are complementary to laser-Doppler anemometry; they are also concerned with the precision of measurement. The various complete laser-Doppler anemometers presented at the Symposium are described under Systems and the Application Section discusses, often in a few pages, a wide range of flow configurations which have been investigated by laser-Doppler anemometry.

The volume cannot completely communicate the stimulating atmosphere of the Symposium, but we hope that it will assist experimenters to appreciate current views of leading researchers and that, as a result, they will be able to assess the value of laser-Doppler anemometry for their application and the likely precision of measurement.

The Symposium was arranged at the initiative of the Technical University of Denmark and DISA ELEKTRONIK A/S; we are glad to acknowledge their contribution. In particular, we would like to thank Mr. N.J. Madsen and Mrs. B. Christensen for their advice and the time which they generously gave to the organization of the Symposium.

64

December, 1975

The Editorial Board

Introduction

In many areas of science and technology it is of vital importance to be able to measure the local velocity of various types of liquid or gaseous flows. Design problems or experimental verification of theories within scientific disciplines ranging from mechanical and aeronautical engineering to such diverse field as biology, medicine, meteorology and oceanography, to name a few, require detailed knowledge of three-dimensional flow fields. The flow velocities of interest range from the extremely slow motion of biological fluids in capillary tubes or oil films to the hypersonic speeds in wind tunnel tests.

The laser Doppler method with which we are concerned in this Proceedings volume has already been successfully applied to many of these measurement problems and the Laser Doppler Anemometer (LDA) has developed into a research tool of unique properties. It is potentially able to solve many so far inaccessible flow problems and often allows measurements to be carried out with greater precision and convenience than previous methods. However, it is a technique which is not always straight forward to use and the measured velocity values may need to be corrected for certain effects having to do with the fundamentals of the laser Doppler measurement.

In the preceding years much effort has been invested to achieve a better understanding of the fundamental principles of the laser Doppler method and to develop and perfect the optical and electronic hardware. The intention with the LDA-SYMPOSIUM in Copenhagen 1975, which is also reflected in the selection of papers presented in this Proceedings volume, was to focus the attention on a particular aspect of laser Doppler anemometry, namely the accuracy attainable with the various optical methods and types of signal processors. The aim was to help in establishing the theoretical boundaries for the LDA-method in flow measurements and the extent to which present day LDA-technology is able to approach these boundaries in practical measurement situations. By bringing together experts in LDA-theory and the fluid mechanics researcher, who uses the technique for a specific purpose, it was hoped to bring about new insight into the fundamental problems of LDA and to learn to extend the method to still further applications.

Optical methods for local flow measurement using incoherent light sources are possible, but not very practical, and pre-laser flow measurements were nearly exclusively carried out by means of mechanical probes (pitot tubes, hot-wire or hot-film probes) supported at the point of measurement by mechanical means. Such methods obviously are likely to exert uncontrollable influence on the flow phenomenon under study. In cases, where the flow region is accessible for light beams either through windows or through transparent sections, optical methods have the great advantage of introducing negligible

distortions into the flow field. The laser Doppler method, which is based on the detection of the Doppler shift of laser light scattered from small particles moving with the medium, furthermore has the potential of complete linearity between transducer response (Doppler frequency shift) and velocity. In contrast, other commonly used transducers such as the pitot tube or the hot wire are only approximately linear within limited ranges. Thus the advent of the gas laser as a source of coherent, highly collimated light presented a welcome opportunity to improve and extend available flow measurement techniques.

Historically LDA grew out of work at Columbia University by Cummins et al on light beating spectroscopy in the early sixties 1,2,3). By analysis of the spectrum of the photodetector current, caused by photo beats between light scattered from small particles moving with the medium through a focused laser beam and a reference beam derived directly from the laser, it was possible to determine the local mean velocity of the flow and also to obtain an estimate of the turbulence intensity. In the following years various types of optical configurations were analyzed 4,5,6,7,8), and the properties of the detector current (usually known as the Doppler signal) from the point of view of various types of electronic signal processors were analyzed 9,10,11,12). In recent years, the properties of LDA-systems based on the detection of single particle traversals have received a great deal of attention. It has turned out that neither signal processing nor the subsequent data processing of the measured velocity data (e.g. the calculation of correlation functions or velocity power spectra in the case of single particle detection) were as simple as initially assumed. Much of the progress in recent years has been reported (and initiated) at a series of LDA-meetings and workshops around the world, (for example, refs. 13,14,15) and even now the amount of literature on LDA and related techniques shows that the field is still developing and that further progress can be expected in coming years.

As previously implied, the main ingredients of any LDA-arrangement are the laser source, the optical transmitter/receiver and the electronic signal processor. These elements are basic for the discussions in the papers in the following part of the book. We do not here pretend to be able to bring a thorough introduction to LDA in a few pages (see, for example ref. 16), but merely wish to name the main terms used in the general LDA-measurement and perhaps clarify the definitions of a few concepts frequently used in the following pages.

Various physical models have been used to describe the laser Doppler principle. Most often it is described as a beat or heterodyning on a square law optical detector of two optical fields having slightly different optical frequencies as a result of one or both fields having been scattered from small particles moving with the medium through the point of measurement. Another explanation of the Doppler phenomenon describes the Doppler frequency as resulting from the detection of light scattered from particles passing through a real or

virtual set of "intensity" fringe planes which are themselves a result of the interference of two optical fields in the measuring volume. However, this so-called fringe model should not be used too rigorously. The fact that the non-linear process, which causes the difference frequency to appear in the detector current, is actually taking place at the square law detector (after the scattering process) and not in the intersection volume of the laser beams (before or independent of the scattering process) explains why the fringe model fails to describe the concept of coherent detection, when scattered light from many particles reaches the detector ^{17,18,19}). Still other descriptions of the Doppler method visualize the LDA as a special form of a Mach-Zender interferometer, in which the optical path length is modified by scattering from the moving particles, or use the language of quantum mechanics to explain the Doppler frequency shift.

Properly used, however, all models result in the well known formula for the Doppler shift, f_D :

$$f_D = 2\pi \underline{V} \cdot (\underline{k}_s - \underline{k}_i)$$

where \underline{V} is the particle velocity, and \underline{k}_s and \underline{k}_i are the wave vectors of the scattered and the incident light waves respectively. In a coordinate system with z-axis along the bisector of \underline{k}_s and \underline{k}_i and x-axis in the plane of \underline{k}_s and \underline{k}_i the expression becomes

$$f_D = (2V_x/\lambda)\sin(\theta/2)$$

where θ is the angle between the two wave vectors, λ the wavelength of the laser light in the medium and V_x the projection of the velocity vector on the x-axis. In laser Doppler measurements, the light field may be considered a superposition of elementary plane waves, and the resulting Doppler current consists of a time varying d.c. part $i_{dc}(t)$ of relatively low frequency and a Doppler frequency modulated a.c. term of time varying amplitude $i_{ac}(t)$ and phase $\varphi(t)$:

$$i = i_{dc}(t) + i_{ac}(t)\cos(2\pi f_D t + \varphi(t)).$$

The function of the LDA optics is partly to transmit and direct the laser light into a small, well defined volume at the desired point of measurement, and partly to receive the scattered light and direct it to the photo detector. The optics selects two light beams originating from the same laser of which at least one consists of scattered light from the measuring volume. These two beams must be combined such that their wavefronts are parallel and overlapping at the surface of the photodetector. The splitting and recombination of the beams

can be realized in a great number of ways as is evident from the early LDA-literature. It has been established that the most stable systems are those in which the recombination of the two beams takes place within the measuring volume by means of the scattering particles themselves: by causing two beams to intersect in a common focal volume light scattered from one beam is automatically injected into the other.

There are different criteria by which to define the various LDA-modes. By noting how the two recombined beams are selected one may distinguish between three modes to which all LDA-systems may be referred: (1) the reference beam mode in which one beam, the reference beam, is derived directly from the laser by means of beam splitters and mirrors and the other beam consists of scattered light from the measuring volume; (2) the dual beam mode (also known as the differential or fringe system mode) in which the two recombined beams are composed of scattered light from the same measuring volume, but scattered from two different incident beams intersecting in the measuring volume; and (3) the dual scattered beam mode in which, by means of beamsplitter and mirrors, two beams of light scattered in two different directions from one incident beam are recombined on the surface of the detector. Today most LDA measurements are carried out by means of integrated optical units capable of operating in two or all three of these modes ²⁰). Systems employing backscattered light do not differ from forward scatter systems in principle, but the low intensity of backscattered light from small particles necessitates special care in the design of backscatter systems.

Another concept often used in describing LDA-systems is that of coherent versus incoherent detection. In coherent detection, which is of special importance when many particles are present in the measuring volume of one time, the receiver aperture A and the solid angle Ω subtended by the measuring volume as seen from the detector satisfy the antenna condition A $\Omega \lesssim \lambda^{2}$ 21). In this case the detector current is of the form $(\Sigma_n E_n)^2$, where E_n is the contribution to the field at the detector from the n'th particle, and the signal-to-noise ratio of the detector current is proportional to the number density of the scattering particles. As the receiver aperture is increased beyond the size given by the antenna condition we reach the realm of incoherent detection, where ultimately the current reduces to the form $\Sigma_n E_n^2$. Although more light reaches the detector, the coherence is reduced resulting in a reduction of the contributions from beats between different particles. In the many particle case this results in a signal-to-noise ratio largely independent of A, when A is larger than the size given by the antenna condition. The signal-to-noise ratio of the current resulting from beats between light scattered from the same particle increases, thus explaining the superiority of the fringe system in cases where on the average less than one particle is present in the measuring volume at a time.

The electronic signal processing is decisive in determining the over-all char-

acteristics of an LDA system, especially with regard to the magnitude and type of errors in the measurement. Already a surprising variety of signal processing methods have been applied in LDA measurements.

One of the first signal processors used in LDA-work, and one which even today is a useful tool, is the conventional spectrum analyzer. The measured probability density distribution of frequency allows the mean velocity and correlations of the fluctuating velocity to be determined. The operation of the spectrum analyzer as an LDA-processor and the associated measurement errors, which result in both a skewness and a broadening of the spectrum, are now largely understood 10). The multi-channel spectrum analyzer or filter bank operating in a time averaging mode has the same characteristics as the spectrum analyzer, but improves one of the basic weaknesses of the spectrum analyzer, the inefficient utilization of the signal caused by the necessity of sweeping the filter across the whole frequency range of interest 22). Another averaging system is the photon correlator which directly constructs the time averaged autocorrelation function of the detector signal 12). Correlators are multichannel instruments and in principle as efficient in utilizing the available signal information as the filter bank. For economical reasons however, present correlators work with an internal one bit quantization of the signal (hard limiter) thus losing some signal information on that account. Only in the limit of very weak scattered light, where the detector signal approaches a one-bit form anyhow (photon - no photon), does the photon correlator come into its own as one of the most efficient LDA-signal processors. All time averaging systems are of course unable to provide information relating to the short time history of the flow fluctuations thus preventing the formation of turbulence spectra and related information. The first system capable of real-time information was the frequency tracking filter or "tracker" based on a phase or frequency locked loop 23). In order to maintain lock this processor most naturally works with continuous or quasi-continuous signals. Although present trackers have built-in "drop out detection" and "hold" features, the interpretation of tracker outputs from signals with a large percentage drop out time still presents a problem. The "burst counter" or LDA counter was developed as an alternative to the tracker for applications in which appreciable signal drop out occurs, 24,25). It is basically a timing device which measures the time for a certain number of zero crossings of the high pass filtered Doppler signal. The measured time is the time of flight of a particle through a corresponding number of interference fringe planes in a fringe mode LDA. Fast digital electronics calculates the velocity by inversion and multiplication by the appropriate scale factor and presents a digital output immediately following each measurement. In most cases, counters are used in situations where the data rate is far below the data rate required for a real-time analog output, and the required flow parameters must be found by statistical analysis of the counter output data. The errors associated with the

non-uniform sampling provided by the particle arrival rate in various optical configurations and flow situations are presently a matter of high priority and are also the subject of a number of papers in this publication.

The counter and the tracker, at least in their conventional forms, are in a certain sense complementary: the counter must be ready to accept bursts within the whole frequency range of interest with equal probability, and must therefore have wide band characteristics, whereas the essential property of the tracker is its narrow band character. Conversely, the maximum slew-rate, i.e. maximum acceptable rate of change of Doppler frequency, which limits the frequency response of the tracker, is a problem which in principle does not exist in counter measurements (in practise, signal validation circuits introduce slew-rate limits even for the counter). The wide band character means that the counter normally requires a better signal-to-noise ratio, and thus higher laser power than tracker based systems, for their proper operation.

Other methods of signal processing have been used or proposed, and the more conventional methods have been combined to obtain special advantages. As an example it has been proposed to combine the LDA-counter with a fast automatic filter bank at the input to improve signal-to-noise ratio ¹⁶) or to combine a fast and a slow loop in a tracker to improve speed and lock range. Recently both the filter bank and the photon correlator have been constructed for operation on single Doppler bursts with the ultimate goal of providing real-time output. Also along these lines are the attempts to use fast A/D converters to digitize the LDA-signal for on-line computer analysis. Reports on advances in many of these projects are to be found in this publication. Finally in this summary of signal processing methods it must be mentioned that the Fabry-Perot etalon has been successfully used as a purely optical signal processor for high frequency continuous signals, and that also this method recently has been extended to real-time analysis ²⁶.

An important concept in LDA-measurements is that of optical frequency preshift. Frequency shift by means of Bragg cells was used early on in the study of diffusion broadened optical spectra ²⁷). Since then a number of practical methods have been developed, and the use of frequency shift has become a factor of great significance ²⁸). An optical frequency shift not only allows the direction of the flow velocity to be determined, but also improves the performance of electronic signal processors in the case of highly fluctuating flow velocity, ²⁹, ³⁰).

The origins of noise and measurement error in an LDA-system may be classified into two groups: The first comprises purely electronic noise from the optical detection process (shot noise) and from electronic circuitry. The other causes noise which is present even in cases of good electrical signal-to-noise ratio and is a consequence of the random distribution of scattering particles. A third possible type of measurement errors in the determination of the flow

velocity is related to the fact that the LDA-method measures the velocity of scattering particles inbedded in the medium, not of the medium per se. Thus it must be ascertained that the size of particles active in the measurement are actually following the motion of the fluid with negligible drag error.

The LDA distinguishes itself in an important way from many other flow velocity transducers: the input/output conversion factor (velocity vs Doppler frequency) for the laser anemometer is independent of the material parameters of the medium. The only property, which affects the beam path in a transparent medium, is the index of refraction n, but as one can readily see from simple ray tracing the Doppler formula is independent of the index of refraction: λ and $\sin(\theta/2)$ both scale with n. Index gradients in two phase flows or in flows containing temperature, pressure or density fluctuations do not influence the relation between the Doppler shift and the particle velocity directly. Indirectly the accuracy of the measurement is influenced by the uncertainty as to the actual location of the measuring volume or by lens effects causing changes in the intersection angle.

Even when no index gradients are present, the random distribution of the particles introduces uncertainties into the measurement. If spatial or temporal velocity gradients are present, the finite size of the measuring volume will cause a fluctuation of the output as particles of random position pass through the volume. These measurement errors are common to all types of LDA-systems, but the way in which they materialize depends on the particular signal processor. In spectral measurement the term broadening is used; in tracking systems "ambiguity" noise; in counters r.m.s. noise and biasing. These phenomena are treated in detail in several of the following papers. Also the matter of electronic noise and its consequences for the measurement error and the question of particle drag errors are considered in later sections of this publication.

At the present stage the value of the LDA as a useful measuring technique is well established. The range of applications is evident from the many important contributions assembled in the last section of this Proceedings. These applications illustrate the applicability of the LDA technique to widely different flow problems. A still wider acceptance of the technique will doubtless follow with future improvements in the basic technique and with a further simplification of the operation of LDA-instruments. Also needed are correction procedures or algorithms for error and bias compensation and possibly the development of LDA-systems containing automatic correction procedures as an integrated part. However, the great variety of flow situations and measuring situations are unlikely to ever replace sound judgement by the user on the possibility of applying the LDA-technique to his particular problem.

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