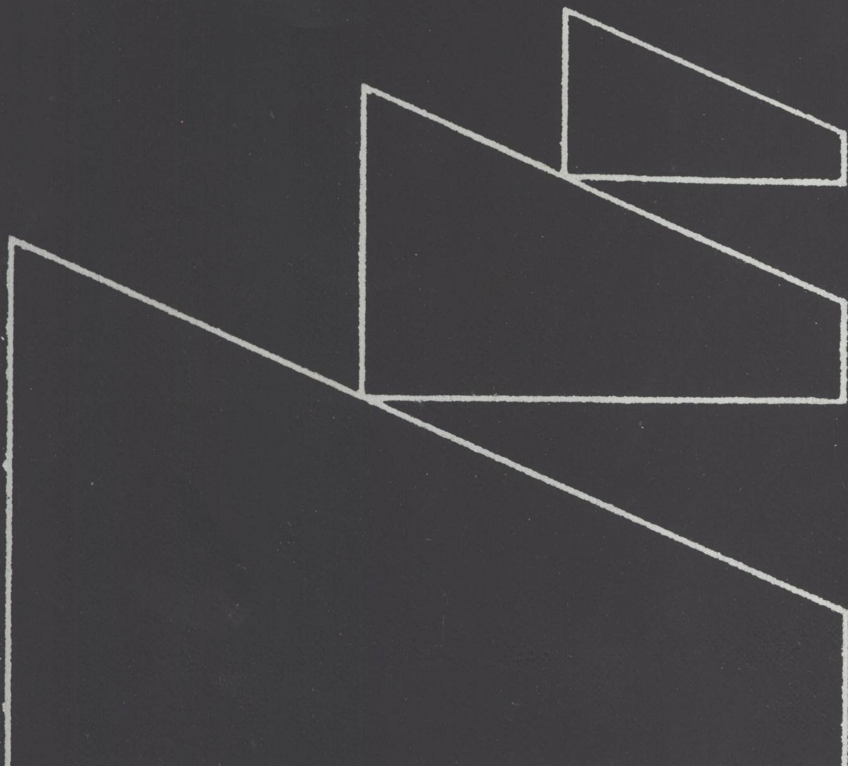
SIMILARITY AND DIMENSIONAL METHODS IN MECHANICS

L. I. Sedov



ИЗДАТЕЛЬСТВО · МИР ·

Л. И. Седов

МЕТОДЫ ПОДОБИЯ И РАЗМЕРНОСТИ В МЕХАНИКЕ

Издательство «Наука»
Главная редакция физико-математической литературы
МОСКВА

L. I. Sedov

SIMILARITY AND DIMENSIONAL METHODS IN MECHANICS

Translated from the Russian
by
V. I. Kisin

MIR PUBLISHERS MOSCOW

First published 1982
Revised from the 1981 Russian edition

На английском языке

Printed in the Union of Soviet Socialist Republics

CONTENTS

FOREWORD TO THE FIRST RUSSIAN EDITION

Physical similarity and dimensional properties play a very important part in experiments and calculations in physics and engineering. The construction of airplanes, ships, dams, and other complicated engineering structures is based on preliminary, broad investigations, including the testing of models. Dimensional analysis and similarity theory determine the conditions under which the model experiments are to be carried out and the key parameters representing fundamental effects and modes of operation. In addition, dimensional analysis and similarity theory when combined with the usual qualitative analysis of a physical phenomenon can be a fruitful means of investigation in a number of cases.

Dimensional analysis and the use of models are encountered in the earliest study of physics in schools and in the initial stages of formulating new problems in research work. Moreover, these theories are of an extremely simple and elementary character. In spite of this it is only in recent years that the reasonings of similarity theory have been widely and consciously used; in hydromechanics, for example, in the past thirty to forty years.

It is generally acknowledged that the explanation of these theories in textbooks and in actual teaching practice in colleges and universities is usually very inadequate; as a rule, these questions are only treated superficially or in passing. The fundamental concepts, such as those of dimensional and dimensionless quantities, the question of the number of basic units of measurement, etc., are not clearly explained. However, such confused and intuitive representations of the substance of the dimensions concept are often the origin of mystical or arcane physical imports attributed to dimensional formulas. In some cases, this vagueness has led to paradoxes which were a source of confusion. We shall examine in detail one example of such a misunderstanding in connection with Rayleigh's conclusions on heat emission from a body in fluid flow. Often, relations and mathematical techniques not related to the substance of the theory are used to explain similarity theory. As in every general theory, it is desirable to construct the dimensional analysis and similarity theory by using methods and basic hypotheses which are adequate to the substance of the theory. Such a construction permits the limitations and possibilities of the theory to be clearly traced. This is necessary especially in dimensional analysis and similarity theory since they are regarded from widely different points of view: at one extreme they are considered to be all-powerful while at the other they are only expected to give trivial results. Both of these extreme opinions are incorrect.

However, it should be noted that similarity theory gives the most useful results when used in combination with general physical assumptions which do not in themselves yield interesting conclusions. Consequently, to show the range of application more completely, we consider a whole series of mechanical problems and examples in which we combine dimensional methods with other reasonings of a mechanical and a mathematical nature.

With this in mind, special attention is paid to the problems of turbulent fluid motion. Similarity methods are the basic techniques used in turbulence theory, since we still do not have a closed system of equations in this field which would permit the mechanical problem to be reduced to a mathematical one. New results are contained in the section on turbulent fluid motion which supplement and explain some aspects of turbulence theory. In addition to examples illustrating the use of methods of similarity and dimensional analysis, we discuss the formulation of a number of important mechanical problems some of which are new and hardly worked out.

We dwell in some detail on the analysis of the fundamental equation of mechanics derived from Newton's second law. This is of interest on its own account and also helps to illuminate the usual reasoning about basic mechanical properties. Our viewpoint on this matter is not new; however, it differs radically from the treatment given in certain widely used textbooks on theoretical mechanics.

The number of familiar applications of dimensional analysis and similarity theory in mechanics is very large; many of them are not touched upon here. The author hopes that the present book will give the reader an idea of standard methods and of their possibilities, which will be of assistance in the selection of new problems and in the formulation and treatment of new experiments.

A large part of the book does not require any special preparation by the reader. But in order to understand the material in the second half of the book, some general knowledge of hydromechanics is necessary.

Moscow, 1943 *L. I. Sedov*

FOREWORD TO THE THIRD RUSSIAN EDITION

In recent years, scientific investigations of physical phenomena have relied more and more on the invariant character of the governing mathematical and physical laws relative to the choice of units for measuring the physical variables and scales.

The practical and theoretical power of these methods has been recognized more and more by scientists contrary to the recently held opinion that the methods of similarity and dimensional analysis are of only secondary value.

A certain analogy exists between dimensional analysis and similarity theory and the geometric theory of invariants relative to coordinate transformation, a fundamental theory in modern mathematics and physics.

Since the first edition of this book appeared, many new applications of dimensional analysis and similarity theory have been made to widely different problems in physics and continuum mechanics, to certain mathematical problems related to the use of group theory in solving differential equations [1] and to statistical problems of sampling and inspection of goods and finished products [2].

Some corrections and additions to emphasize better the basic ideas of the theory of similarity and dimensional analysis are introduced in this edition. One example of this is the discussion of the proof of the Π-theorem. Furthermore, the definition of dynamic or physical similarity of phenomena has been given in more detail. This new definition is still not in general use in the similarity literature; however, from the practical viewpoint, it includes the essential peculiarities of physically similar processes; moreover, it is convenient for direct use and, apparently, satisfies all the needs of different applications.

Beyond this, §§ 8-12 in Chapter IV and an entirely new chapter have been added. The additions to Chapter IV are devoted to certain problems of explosions and the attenuation of shock waves, besides a discussion of the general theory of one-dimensional gas motion. Applications of the theory of one-dimensional unsteady gas motion and the methods of dimensional analysis to certain astrophysical problems are considered in new Chapter V[1]).

The theory developed in the additions to Chapter IV and in Chapter V[1]) is completely new in its basic approach. The proposed formulation and solution of the gas dynamics problems illustrate the applications of dimensional analysis methods to astronomy and provide a stock of simple simulating ideal motions which can be used to investigate problems of cosmogony. Many of these results I obtained in collaboration with my young pupils in the course of the work of the hydromechanics seminar in Moscow University during the 1952-3 school year.

N.S. Mel'nikova and S.I. Sidorkina contributed to the preparation of § 14 of Chapter IV, V.A. Vasil'ev and M.L. Lidov to § 16, item 1° of Chapter IV, and I.M. Yavorskaya to § 6 of Chapter V[1]).

I express my deepest gratitude to them all.

Moscow, March 1954

L. I. Sedov

REFERENCES

[1]. Birkhoff, G., *Hydrodynamics. A Study in Logic, Fact, and Similitude*, Princeton University Press, 1950.

[2]. Drobot, S., Warmus, M., "Dimensional analysis in sampling inspection of merchandise", *Rozprawy Matematyczne*, V, Warszawa (1954).

FOREWORD TO THE SIXTH RUSSIAN EDITION

The sixth edition is supplemented with Chapter V, i.e. Introduction to the Theory of Gas Engines; some additions were made to Chapters I, IV, and VI. The texts of the earlier editions were checked out and a number of shortcomings was corrected.

References to recently published papers were enlarged. I shall specially single out a monograph *The Theory of Point Explosion* by V.P. Korobeinikov, N.S. Mel'nikova, and E.V. Ryazanov, which

---

[1]) Chapter VI in the sixth and subsequent editions.

is a conceptual relative of the present book and gives a further elaboration of the theory of explosion.

I am very much indebted and deeply grateful to N.S. Mel'nikova who undertook the general editing and refinement of the text, as well as supervised additional computational work and preparation of new diagrams.

Moscow, February 1967 *L. I. Sedov*

FOREWORD TO THE EIGHTH RUSSIAN EDITION

As compared to the seven earlier editions, this one has a number of additions, comments, and improvements.

The topic that has been added to significantly is a comparison of the theory of isotropic turbulent flow (included in the first edition, i.e. in 1944) with the latest experimental results. Now that about forty years have elapsed, we have ample evidence that this theory, based on the dimensional analysis and similarity theory, is in good agreement with the experiments carried out within this period.

It can also be noted that the application-oriented aspects of the gas dynamics and dimensions theory, developed in Chapters IV and VI, gradually penetrate the realms of modern astrophysics and numerous other fields of science.

The arguments based on dimensions of various variable and constant quantities and on physical similarity (scaling) are widely used nowadays to formulate cognition problems as well as those in diverse fields of science and technology.

Moscow, May 1976 *L. I. Sedov*

FOREWORD TO THE NINTH RUSSIAN EDITION

The present, ninth, edition of the monograph has been purged of misprints that crept into the earlier ones, and supplemented with a considerable number of briefly annotated references to recent research results relevant to the body of the text on the theory of unsteady flow in continuous compressible media.

Apparently, not all significant contributions, especially those published outside of the USSR, are covered in the lists of References. I mostly cite the results connected with my line of work and contributing to the progress of the theories exposed in this monograph.

I hope that future will present us with history-oriented reviews which will fill this gap.

I wish to express my gratitude to V.V. Rozantseva who has prepared this edition for the publication and N.V. Morugina who did all the technical work with the manuscript.

Moscow October 1980 *L. I. Sedov*

CHAPTER I

GENERAL DIMENSIONS THEORY

§ 1. Introduction

Every phenomenon in mechanics is determined by a series of quantities, such as energy, velocity, and stress, which take on definite numerical values in specific cases.

Problems in dynamics or statics reduce to the determination of certain functions and characteristic parameters. The relevant laws of nature and geometrical relations are represented as functional equations, usually differential equations.

In purely theoretical investigations, we use these equations to establish the general qualitative properties of motion and to calculate unknown physical quantities using mathematical techniques. However, it is not always possible to solve a mechanical problem solely by the processes of analysis and calculation; sometimes the mathematical difficulties are too great. Very often the problem cannot be formulated mathematically because the mechanical phenomenon to be investigated is too complex to be described by a satisfactory model and the equations of motion are unknown. This situation arises in many important problems in aeromechanics, hydromechanics, and the theory of structures; in these cases, we have to rely mainly on experimental methods of investigation to establish the essential physical features of the problem. In general, we begin every investigation of a natural phenomenon by finding out which physical properties are important and by looking for mathematical relations between them which govern the phenomenon.

Many phenomena cannot be investigated directly, and to determine the laws governing them we must perform experiments on similar phenomena which are easier to handle. To set up the most suitable experiments we must make a general qualitative analysis and bring out the essentials of the phenomenon in question. Moreover, theoretical analysis is needed when formulating experiments to determine the values of particular parameters of the phenomenon. In general, and particularly in designing experiments, it is very important to select the dimensionless parameters correctly; there should be as few parameters as possible and they must reflect the fundamental effects in the most convenient way.

This preliminary analysis of a phenomenon and the choice of a system of principal dimensionless parameters are made possible by dimensional analysis and similarity theory: it can be used to analyse very complex phenomena and is of considerable help in processing experimental data. In fact it is out of the question to formulate and carry out experiments nowadays without making use of similarity and dimensions concepts. Sometimes dimensional analysis is the only theoretical means available at the beginning of an investigation of some phenomenon. However, the potentialities of the method should not be overestimated. In many cases, only very limited or trivial results are obtained from dimensional analysis. On the other hand, the widely held opinion that dimensional analysis rarely yields results of any importance is completely unjustified: quite significant results can be obtained by combining similarity theory with the data obtained from experiment or from the mathematical equations of motion. In general, dimensional analysis and similarity theory are very useful both in theory and practice. All results derived from this theory are obtained in a simple and elementary manner. Nevertheless, in spite of their simple and elementary character, the methods of dimensional analysis and similarity theory require considerable experience and ingenuity on the part of an investigator when probing into the properties of some new phenomenon [1].

In the study of phenomena which depend on a large number of parameters, dimensional analysis is especially valuable in determining which parameters are irrelevant and which are significant. We shall illustrate this point later by examples. The methods of dimensional analysis and similarity theory play an especially large part in simulating various phenomena.

§ 2. Dimensional and Dimensionless Quantities

Quantities are called dimensional or concrete if their numerical values depend on the scale used, that is, on the system of the units of measurement. Quantities are called dimensionless or abstract if their values are independent of the system of units. Typical dimensional quantities are length, time, force, energy, and moment. Angles, the ratio of two lengths, the ratio of the square of a length to an area, the ratio of energy to moment, etc. are examples of dimensionless quantities.

However, the subdivision of quantities into dimensional and dimensionless is to a certain extent a matter of convention. For example, we have just called an angle dimensionless. It is known that angles can be measured in various units, such as radians, degrees, or fractions of a right angle. Therefore, the number defining an angle depends on the choice of units; consequently, an angle can be considered a dimensional quantity. Suppose we define an angle as the ratio of the subtended arc of a circle to its radius; the radian—the angular unit—will then be defined uniquely. Now, if an angle is measured only in radians in all systems of units, then it can be considered a dimensionless quantity. Exactly the same argument applies if a single fixed unit of measurement is introduced for length in all systems of units. In these circumstances length can be considered dimensionless. But it is convenient to fix the unit for angle and inconvenient for length: this is explained by the fact that corresponding angles of geometrically similar figures are identical while corresponding lengths are not and, consequently, it is convenient to use different basic lengths in different problems.

Acceleration is usually considered a dimensional quantity with the dimensions of length divided by time squared. In many problems, the acceleration due to gravity g, equal to the acceleration of a body falling in a vacuum, can be considered constant (9.81 m/s^2). This constant acceleration g can be selected as a fixed unit of measurement for acceleration in all systems of units. Then any acceleration will be measured by the ratio of its magnitude to the magnitude of the acceleration due to gravity. This ratio is called the load factor, a numerical value of which will not vary when converting one unit to another. Therefore, the load factor is a dimensionless quantity. But the load factor can be considered a dimensional quantity at the same time, namely, as acceleration when the acceleration due to gravity is taken as the unit. In this latter case, we assume that the load factor—the acceleration—can be taken as a unit which is not equal to the acceleration due to gravity.

On the other hand, abstract (dimensionless) quantities can be expressed in various numerical forms. In fact, the ratio of two lengths can be expressed as an arithmetic quotient, as a percentage, or by other means.

The concepts of dimensional and dimensionless quantities are therefore relative. A certain excess of units is employed. When these units are identical in all systems, the corresponding quantities are called dimensionless. Dimensional quantities are defined as those for which the units can vary in experimental or in theoretical investigations. Here it is irrelevant whether or not the investigations are actually carried out. It follows from this definition that certain quantities can be considered dimensional in some cases and dimensionless in others. We gave examples of these above and later we shall encounter a number of others.

§ 3. Fundamental and Derived Units of Measurement

Different physical quantities are interrelated via a number of relationships. Therefore, if certain physical quantities are taken as basic with assigned units, then the units of measurement of all the remaining quantities can be expressed in a definite manner in terms of those of the fundamental quantities. The units taken for the fundamental quantities will be called fundamental or primary, and all the rest will be derived or secondary.

In practice, it is sufficient to establish the units for three quantities; precisely which three depends on the particular conditions of a problem. Thus, in physical investigations it is convenient to take the units of length, time, and mass as the fundamental units, and in engineering investigations to take the units of length, time, and force. But the units of velocity, viscosity, and density, etc. could also be taken as the fundamental units.

At the present time, the physical and absolute mechanical systems of units have become most widespread. The centimetre, gram, and second have been adopted as the fundamental units in the physical system (hence the abbreviation—cgs system of units). The metre, kilogram-force, and second have been adopted as the fundamental units in the absolute mechanical system (hence the abbreviation—mks system of units).

The units of length, the metre (equal to 100 cm), of mass, the kilogram (equal to 1000 g), and of time, the second, have been established experimentally by definite agreement. Until 1960 the length of a bar of platinum-iridium alloy, stored in the French Bureau of Weights and Measures, was taken as the metre; the mass of another bar of platinum-iridium alloy, stored in the same Bureau, was taken as the kilogram. The second was assumed to be $1/(24 \times 3600)$ part of a mean solar day [2].

A system of units which is becoming more and more widespread is the unified International System of Units, SI (from the French, Système International d'Unités). It was enacted as a mandatory system in the USSR in 1963, and in the COMECON as a whole in 1979.

The fundamental mechanical units in the SI are: the metre, kilogram of mass, and second; the unit of current is the ampere, that of the thermodynamic temperature is the kelvin, the unit of the luminous intensity is the candela, and that of the amount of substance is the mole [3].

Once the fundamental units have been established, the units for the other mechanical quantities, such as force, energy, velocity, and acceleration, are obtained automatically from their definitions.

The expression of the derived units in terms of the fundamental units is called their dimensions. The dimensions are written as a formula in which the symbol for the dimensions of length, mass, and time is denoted by L, M, and T, respectively (in the absolute mechanical system, the unit of force is denoted by K). When discussing dimensions, we must use a fixed system of units. For example, the dimensions of area are L^2; the dimensions of velocity are L/T or LT^{-1}, the