FUZZY SYSTEMS

C.V. NEGOITA

FUZZY SYSTEMS

C.V. NEGOITA
Faculty of Cybernetics and the Institute of Management, Bucharest, Romania

ABACUS
PRESS

SERIES PREFACE

The inter- and trans-disciplinary sciences of cybernetics and systems have made tremendous advances in the last two decades. Hundreds of books have been published dealing with various aspects of these sciences. In addition, a variety of specialist journals and voluminous post-conference reports have appeared, and learned societies, national and international, established. These substantial advances reflect the course of the Second Industrial Revolution, otherwise known as the Cybernetic Revolution.

In order to extend the readership from experts to the public at large and acquaint readers with up-to-date advances in these sciences which are rapidly achieving tremendous importance and which impinge upon many aspects of our life and society, it was considered essential to produce a series of concise and readable monographs, each concerned with one particular aspect. The twelve topics constituting the first series are as follows (in alphabetical order).

Artificial Intelligence, Automation and Cybernetics, Computers and Cybernetics, Cybernetics and Society, Economic Cybernetics, Fuzzy Systems, General System Theory, Management Cybernetics, Medical Cybernetics, Models and Modelling of Systems, Neurocybernetics, Philosophical Foundations of Cybernetics.

The authors are experts in their particular field and of great repute. The emphasis is on intelligible presentation without *excess* mathematics and abstract matter. It is hoped each monograph will become standard reading matter at academic institutions and also be of interest to the general public. In this age of enormous scientific advances and of uncertainty concerning the welfare of societies and the very future of mankind, it is vital to obtain a sound insight into the issues involved, to help us to understand the present and face the future with greater confidence.

J. Rose
Blackburn

PREFACE

Trying to apply mathematics to soft sciences, researchers are susceptible to trends of fashion. The system approach has shown their readiness to accept a new methodology. It seems to me that not only does an examination of this methodology cast light on the challenges currently facing the human systems, but it is also a case study of a topic that has come of age.

There can be little argument that the expertise of System Theory is potentially relevant to soft sciences. But the way in which this potential can be fruitfully realized is less obvious. This book contends that the first step in the process of applying System Theory to human systems must be the development of a paradigm. The basic assumption is that synthesis means conflict resolution. Structural stability questions motivate the introduction of new mathematical tools, and fuzzy systems theory shows in a very precise way how modern algebra can be used in the understanding of decision processes.

The book consists of a series of essays. It is consistently characterized by the experience of the author as head of research staff at the Institute of Management in Bucharest, and owes much of its present form to the comments and contributions of the participants in seminars, in which the first concern was to formulate the questions before obtaining the answers.

This book owes its existence in large measure to the editor, Dr J. Rose, who convinced me that whatever work has been done by us in this field should be published in the form of an unified and readable presentation.

C.V. Negoita
Bucharest

CONTENTS

British Library Cataloguing in Publication Data
Negoita, C.V.
Fuzzy systems.—(Cybernetics and systems series: 2).
1. Decision-making—Mathematical models
2. Set theory
I. Title II. Series
658.4′033 HD30.23
ISBN 0-85626-164-5

Typeset by Pearl Island Filmsetters in Hong Kong
and printed and bound by
Tien Wah Press (Pte.) Ltd in Singapore

Chapter 1

INTRODUCTION

This book represents an attempt to explore the relationship between fuzzy systems theory and homeostasis. It is hoped that it will provide the conceptual framework, the basic methods and the perspective essential for a better understanding of a highly important subject. It hardly needs to be said that in a relatively small book such as this a big subject cannot be treated in a fully authoritative way. But the fundamental difficulty is not really one of space. Rather, it is our still limited and imperfect understanding of the causes and consequences of the process that we call 'qualitative change'.

The central goal of General Systems Research seems to be an orderly way of handling the shifting, interacting individual events of existence. Therefore, truth is relative because empirical existence itself is in a process of change, and stability is to be found only in the process of interaction. Some systems show a phenomenon of robustness in that they are stable despite substantial alteration of their structure. That is, some aspects of the balance of the system are insensitive to quite radical changes in seemingly importance aspects. This is precisely the case of cognition and decision-making processes.

It is quite natural to think of change as a succession of states in which each holds the stage for a while before being following by its successor. Perhaps for this reason, methods from mechanics have been tried to explain the change. However, if we accept the dynamics as conflict resolution, then homeostasis also means movement toward equilibrium, but the theory of homeostasis cannot be more satisfactory than that offered by the framework of mechanics. *Autopoietic processes*, i.e. organic growth, require a better understanding of the concept called 'synthesis'. It seems to me that a good model for this concept could be formulated using the notion of 'pullback' introduced by the category theorists. Initial contradictions are reconciled in higher synthesis, for to remove differences is to achieve the purpose of equilibrium, and this process can be formalized, represented and modelled.

The development of General Systems Research at the moment is characterized by its own mathematical concepts, the penetration of mathematical methods into traditionally soft sciences now being generally acceptable. Along with the use of mathematics, there is taking place a saturation of mathematics itself with set-theoretical concepts, use of mathematics having become almost equivalent to using set theory. But the language of set theory is not always adequate for expressing the purposes of soft sciences, and applied mathematics is not merely an application of set theory to practical problems but something else. Fuzzy set theory is an example of mathematics based on a different concept.

One aspect of General Systems Research is its development in a direction that will enable more and more complex systems to be analysed or designed. The trend of fuzzy systems reflects the desire to represent human systems. The major unsolved problem in the field of General Systems Research seems to be that of transcendence. If we explore the mode of cognition as a procedure for describing complex systems and the way in which natural language is involved in this process, we see that mankind copes with complexity by assembling partial representations, and that learning means conceptualization, which is concerned with getting a feel for the whole. The unfolding of multi-criteria decisions follows the same procedure. Transcendence signifies a trend towards totality, completeness and wholeness. As such we are speaking of timeless and universal concepts or criteria that clarify everything and establish relationships. Transcendence seeks a unification of particular existence in such a way that every detail would be a unit of the totality. There is, indeed, a sense in which transcendence expresses the synthesis of empirical existence. If we accept that transcendence discloses a constant openness in which man grasps freedom, then it is high time to obtain a deep insight into this natural phenomenon.

1.1. FUZZY SYSTEMS

The expression 'fuzzy system' was coined by Lotfi Zadeh in 1965. It is generally taken to embrace the whole field of imprecisely described systems, but though this implies the use of verbal descriptions, there is a tendency to regard the subject as providing a substitute for methods such as this.

There is, however, no recognizable or accepted body of theory that can be considered as an established discipline. For this reason, those writers who have attempted to survey the field have studiously avoided the use of a title that would suggests any claim that they were dealing with a subject which has recognizable boundaries.

The problem of finding a way to handle inexact descriptions has created a challenge that is attracting the attention of workers from an increasing number of disciplines. The proliferation of different kinds of approach has tended to cloud the issue, and this book was born of a conviction that, within the limits of our present knowledge, we can isolate and exploit some principles.

Chapter 2 is an attempt to explain the need for a study of fuzzy systems and provides some bibliographical hints. The principles, which to my mind are fundamental to the problem, form the subject of Chapter 3. Chapter 4 and 5 give sketch the mathematical background, Chapter 6 introduces the concept of pullback as a model of synthesizing processes, and Chapter 7 illustrates the theory.

The principal theme of the book forms a continuing argument through Chapters 3 to 7 and comprises an integrated study of fuzzy relations viewed as states of dynamic systems.

I shall define fuzzy systems more comprehensively later, but I think that for the purpose of this introduction and for drawing the interested reader further into the book, I would describe them as systems of systems, or systems of logical objects. The object of studying such systems is to gain understanding so that they may be more readily controlled, just as systems in the natural sciences are.

Most of the techniques described will be illustrated by examples from my own work and that of my colleagues. This is not merely conceit, but based upon a belief that first-hand knowledge is best. The views presented here might have been elaborated with greater mathematical refinement. But there may be some virtue in articulating new ideas briefly and simply to an intelligent unprofessional audience whose response can give the enterprise an authentic air of intellectual adventure. The book will also provide the specialist with some original but worthwhile concepts and some new ways of looking at well-known problems.

In the course of the book, there will be found references to the writings of other workers in the field, when it has been found possible to interpret their work in relation to our model. These are scattered throughout the book with appropriate citations when they occur.

This book is not intended to be a treatise on the field, nor is it intended to be a critique of the work of other schools, nor a comparative evaluation of the success and failure of different approaches to the subject. It is intended to set forth in an orderly manner the motivations and viewpoints of a school of thought that cherishes the belief that the field of fuzzy systems can and should be discussed with mathematical skill.

1.2. SYMBOLS AND TERMS

Now, a short explanation of mathematical symbols and terms used in the book. Elements are indicated by lower case letters x, y, z. Sets are designated by capital letters A, B, C, etc. Set membership is indicated in symbols as $x \in A$ which reads 'x is a member of A' or 'x belongs to A'. Given two sets A and B the Cartesian product of A and B is the set of ordered pairs in which the first element belongs to A and the second to B, denoted as $A \times B = \{(x, y) | x \in A \text{ and } y \in B\}$. If A and B are two sets we write $A \subset B$ to indicate that A is a subset of B, i.e. that every element of A is also an element of B. $A \cap B$, called the intersection of A and B, is the set $\{x | x \in A \text{ and } x \in B\}$. $A \cup B$, called the union of A and B, is the set $\{x | x \in A \text{ or } x \in B\}$.

The set of all corresponding pairs (x, y) such that y is the single value associated with the element $x \in A$ constitutes a function f on A. It is not necessary, however, that to each y there corresponds only one x. The inverse function f^{-1} is the set of correspondences between each $y \in B$ and the subset of A which is mapped into y by f. We use the notation $f: A \to B$ for 'f is a function from the set A to the set B', i.e. 'to each $x \in A$, f assigns an element $f(x) \in B$. The composition of two functions f, g—denoted by a small circle, $g \circ f$, means a succession of two arrows $A \xrightarrow{f} B \xrightarrow{g} C$.

A *lattice* is a partially ordered set such that all two elements have a greatest lower bound and a least upper bound. The existence of these unique bounds for each pair of elements in a set A implies the existence of two functions $A \times A \to A$, called *supremum* and denoted as $\vee$ and called *infimum* and denoted as $\wedge$.

A *category* is a formal system. Formal means explicit, not merely understood. A formal system has a collection of axioms from which we obtain theorems by the repeated application of a number of rules. Formal means also 'of the essence of the thing' rather than its matter. The Aristotelian logic uses categories as fundamental classes among which all things can be distributed. Categories viewed as formal systems are also fundamental classes among which mathematical things can be distributed.

At many stages of the development the discussion will tend to be somewhat abstract. The reader is invited to relax and enjoy the trip. The ideas are, in fact, simple.

Chapter 2

AN APPROACH TO COMPLEXITY

2.1. WHY FUZZY SYSTEMS?

The systems concepts have been under-utilized in relation to the obvious and urgent need for their employment in dealing with complex problems. This book is an attempt to answer the question: what can be done? The main theme is that although many scientists boldly apply existing models of System Theory to complex problems, little research is devoted to the foundations of this model building.

System Theory is defined as a body of concepts and techniques used to analyse and design systems regardless of their nature. The important aspect of systems analysis and design is the development of a model that describes the 'cause and effect' relationships between the system variables. But an exact description of any real system is virtually impossible. Our inability to make precise statements about complex behaviours is a fact we have to accept and adjust to. Complexity is associated with description rather than being thought of as an intrinsic property of objects. Hence, we may well consider reducing the complexity of an object, not by changing that object, but by changing our views about it.

People who have attempted to analyse complex problems from a systems point of view have frequently remarked that symbolic language describing a model is too poor to convey all the nuances a person can carry in the mind, which is why social scientists require the medium of ordinary, natural language and its richer content. In some ways it is unfortunate, though almost inevitable, that the modern concept of the systems approach has evolved out of technical problems burdened with the artificial language of pure mathematics. Many system theorists have failed in what they tried to do because they never looked at the language they used. If they had, they might have begun to question some of the bases of what they were doing.

Conventional systems approach has tended to assume that what needs to be done is to survey the whole system. The fuzzy systems approach challenges this assumption and proposes to construct a global representation on the basis of assumptions about partial representation. In this way we are able to hold different imperfect representations of reality in hand at the same time. A theory based on consideration of individual representations, such as the one proposed by fuzzy systems theory, is inherently richer in its consequences than a theory that treats a system as if it were an individual representation. A description depends not only on each partial representation but also on interaction among representations.

Humans obtain knowledge of their environment by perception, the link between the outer world and the inner world, between what is commonly called 'material' and what has been named 'mind'. We see, hear, smell, taste, feel heat and cold, and form size and texture. Finally, all the perceptions merge into a complex background and are no longer isolated. A complex system is perceived as a single totality. Abstraction is viewed as a set of interrelated mental images. Words are mere labels for general functions. The interesting question is not whether such functions exist, for they are clearly implied in the phenomenon we call mental experience, but how do they develop? How do they work?[1]

Each word of natural language is inherently vague. It is an essential characteristic of a vague concept that the boundaries of the domain of its applicability are not fixed and, therefore, we do not know precisely where this domain ends and some other begins; that vagueness is not thought of as inherent in the real world, but rather in our use of words. Put in another way, inexact concepts are associated with the inability to decide, which is generated by conflict. The principle of the excluded middle is regarded as inapplicable in these situations because the notion of contradiction cannot be directly applied there. People understand vague concepts as if these were represented by fuzzy sets. The assumption underlying fuzzy sets is that the transition from membership to non-membership is a gradual specifiable change. Membership in a fuzzy set is specified by a mapping. This mapping can be further used in fuzzy inferences. People manipulate vague concepts as if they are processed according to the rules of fuzzy logic; several studies appear to support this theoretical position.[2]

People face complexity because they have the ability to see essential features by a process of abstraction. Abstract concepts are amalgamations of subconcepts, each of which has different conflicting meanings. This is, indeed, a powerful model with many potentially fruitful applications and it deserves serious attention. Abstraction is the way people cope with complexity.

The moral is clear: fuzziness far from being a difficulty is often a convenience. The theory of fuzzy systems may well have a substantial impact on scientific methodology, particularly in the realm of soft sciences. Many people suppose that once you say a process is subjective, that it depends on human judgement and beliefs, analysis ends there. I disagree. Subjective processes are fully as amenable to scientific understanding as any other process. Some results in this field are encouraging, for these results tend to show a surprising and beautiful unity hidden in the apparent diversity of approaches. The key to the unification lies in conflict resolution. As I have already said, inexact concepts are associated with the inability to decide, which is generated by conflict. Domains of applicability are always overlapping. Our work makes the fuzzy set approach to hierarchical systems completely explicit and extends their applicability by demonstrating that the ability to coordinate means transformation of fuzzy sets. The existence of a conflict at one level generates the next level. This movement from one level to the other is precisely the dynamics of a fuzzy system. The result of this movement is compact representation, i.e. abstraction.

This is exactly the sense in which one distinguishes between an organic and a mechanical theory of movement. The growth of a fuzzy system is an autopoietic model; a spontaneous order, corresponding to an organism, is achieved. The structure is not designed from without, the forces that determine such an order are inner conflicts.

Conflict resolution at a higher level means composition of fuzzy sets. Under such a composition, a new fuzzy set is generated. Fuzzy sets are associated with ordering in a particular aspect. I define it by referring to an ideal point. Therefore, by composing fuzzy sets, the autopoietic process moves the ideal point. In other words, the dynamics of fuzzy sets means dynamics of ideal points. Self-coordination induces a displacement of ideal and can model the process of dynamic decision-making. In technical language, the absolute ideal corresponds to the last level in the hierarchy of fuzzy sets, and, in the jargon of category theory this is a terminal object in the category of fuzzy sets. Moving up in the hierarchy means to become aware of new criteria or points of view. Scientific knowledge represents partial truth and there may be material changes as new knowledge accumulates. Clearly, abstraction means unification.

There are many problems in management science in which hierarchically organized but incompletely specified information is important. Planning is notorious for its often ambiguous underlying assumption and its nearly always, vague, multilevel approach. Take, for instance, the linear programming problem. When speaking of objectives or constraints it is most often understood that these are well-known. Real life problems frequently lack this property. In fact, management decision-making involves the choice of an action from alternative courses of action to achieve fuzzy objectives. The significance of studying optimization problems in fuzzy environment has been recognized and some problems have been solved.

A fuzzy programming problem is one of multicriteria optimization. Constraints and objectives are fuzzy sets, i.e. functions; there is no distinction between means and ends. They are functions defined on the same carrier and we are speaking about their confluence. To solve a fuzzy programming problem means to move the decision to a higher level, to consider all the points of view as a whole. In the language of hierarchical systems theory this is coordination. To coordinate is to find a special kind of function that describes the relationship between the global objective and the local ones. This function must preserve order, a condition named *monotonicity*, and the intersection of fuzzy sets is clearly a monotone function.

Critiques of the systems approach remark that in all systems analysis, there is a moral overtone which can also be seen in planning and other activities where intellectuals become involved in trying to improve society; it addresses itself to collective properties and is incapable of considering people as unique individuals. If one looks at the linear programming model, one will see that, say the critics, the model for a machine would be the same as a model for a social system. Systems analysts are do-gooders; they are trying to help to improve social systems. But in all their studies they treat individuals as means rather than as ends. Hence, in some sense, they fail to deal with the moral dimension. We need a different approach if we are going to be honest. Essentially, what must change is the level. If different individuals prefer different alternatives there is a conflict again. As in any conflict situation, it can be solved by considering a move towards a higher level. We can bring the merits of judgement and beliefs to the linear programming model in a quantitative manner by using fuzzy sets instead of numbers as parameters; this is a compact representation.

Difficulties with the present-day soft sciences do not include lack of good results, but *do* include lack of theoretical foundation. It is hoped that the fuzzy systems theory will put us in a better position to tackle some of the questions concerning organic growth by understanding the rules of autopoietic processes.

Modelling in an abstract setting requires, and when done automatically provides, a refined analysis of the fundamental concepts and constructions involved. Thus, a foundation study of organisms is an inevitable and welcome byproduct. For example, what is called displacement of preferences in decision-making, and what is called displacement of possibilities in economics, is more exactly analysed as dynamics of fuzzy states, described here as transformation of fuzzy sets, i.e. arrows in the category of fuzzy sets.

The dynamics of fuzzy systems is a deterministic movement of imprecise evaluations. The idea of using possibility as an uncertainty variable was considered in order to lay down precise lines of action, and to count upon their being followed. Since we need at least to know the circumstances that will prevail, including the actions that at future times will be taken by other people not subjected to the planned instructions, policy must legislate for tolerances. In order that policy may command loyal and resolute endeavour it must offer some assumptions of being able to cope with situations that cannot be foreseen in detail; this is robustness.

The mathematical System Theory extensively promoted during the 1960s has come under a barrage of criticism, and a new movement—back to the basics—has come to the fore. There are complaints that mathematical System Theory produced computational cripples who are seriously hindered in their attempts to use mathematics in soft sciences.[3] The past decades have served to demonstrate that System Theory and soft sciences move in separate orbits. Those orbits will have to coalesce if major reforms are ever to be successfully implemented. The development of a fuzzy systems theory is posed as a challenge for both applied mathematics and soft sciences.

The struggle to reconcile mathematics and the soft sciences has its parallel in automatic control, also dominated by pure mathematics. Among control theorists there have emerged, also, advocates of the need to develop a logic of inexact concepts.[4] Taking a retrospective view of the development of the theory of automatic control from its beginning to the time when interest in variational techniques reached its peak, they observed that this theory is permeated with unfulfilled hopes. Many problems remain unsolved to this day. The problems in question reduce, in the main, to the analysis of an equation which depends on parameters. In general, simple solution properties abide in complex, difficult to describe and highly irregular, parameter regions. By questioning the language used to describe these regions, they observed that applied mathematics is not a second-class kind of mathematics, nor is it merely an application of pure mathematics to practical problems, but a different kind of discipline which should be based on a different concept of precision.

Fuzzy set technology has already been born and is growing. It is based upon subjective judgements and objective aggregation rules are required to put them together. Treating autopoietic processes as fuzzy systems has made possible recognition of equal structures from which can be developed fundamental laws and principles that operate commonly at different levels of organization. Its adoption as a scientific approach does much to mitigate the mathematics-nonmathematics dichotomy and improve communication across scientific disciplines.

2.2. THE MILESTONES

A few key references make it fairly easy to penetrate the literature of fuzzy sets and systems rapidly and to a reasonable depth. Almost all the papers in this field are listed in the huge bibliography, published in 1977 by Gaines and Kohout,[5] which consists of some 1150 items, each keyword-indexed, about 750 of them classified as being related to fuzzy systems theory and its applications. The remaining items deal with closely related topics in many-valued logic, linguistics, the philosophy of vagueness, and similar topics. These background references are annotated in an initial section that outlines the relationship of fuzzy systems theory to other developments and provides pointers to various possible, fruitful interrelationships. As stated by the authors, the organization of this preamble—the best study I know—is aimed primarily at helping those interested to find their way around the bibliography, and secondly as a tribute to Lotfi Zadeh, who opened this area of study and has been a consistent driving force behind its development.

The basic aspects of the theory can be rapidly gleaned from two papers of Zadeh,[6,7] entitled *Fuzzy Sets*, the second one being written after an interval of ten years. An extensive presentation of the basic concepts is given by Kaufmann.[8] His textbook provides a concise, rigorous and up-to-date introduction to the theory of fuzzy sets and is intended to prepare those with little or no previous knowledge of this field for work with advanced and specialized texts. Although written in an applied mathematical style, it does not demand an advanced mathematical background and is illustrated by many examples and exercises. A broad spectrum of applications of the theory of fuzzy sets, ranging from its mathematical aspects to applications in human cognition, communication, decision-making and engineering system analysis are presented in a volume containing the papers contributed by the participants in the US-Japan Seminar on 'Fuzzy Sets and their Applications', held at the University of California, Berkeley, July 1974.[9]

A compact presentation of the algebraic theory of fuzzy sets can be found in a book written by Ralescu and me and published in 1975.[10] Here we have introduced the so-called theory of representation (the arrow $X \to (0, 1)$ was reversed as $(0, 1) \to p(X)$, whose significance was not seen until recently. I refer to the attempt by Boyd[11] to present topoi as models of fuzzy sets.

In a long and serious essay, Arbib[12] reviews these three books and notices that the most disturbing feature of the literature collected within these volumes is its insularity. The applied papers, he says, use the language of fuzzy sets to tackle some problems, but never provide a hard-headed comparative study to show that the method is better than any other. However, very respectable international journals like *Man-Machine Studies* or *Kybernetes* devoted special issues to fuzzy topics, and meetings such as I.E.E.E., ORSA/TIMS, and S.G.S.R. allocated special sessions to the topic.[13,14,15] The international journal *Fuzzy Sets and Systems* published by North-Holland, Amsterdam, seeks to present papers of real significance and broad interest (1978). Its history can be traced to the discussions of the second meeting of the European Working Group on Fuzzy Sets.[16]

I shall now mention a few references that mark important milestones in the evolution of the theory of fuzzy systems. One, due to Zadeh,[17] is an outline of a new approach to the analysis of complex systems and decision processes, a substantial departure from the quantitative techniques of systems analysis. The linguistic variable is suggested in place of, or in addition to, numerical variables. Simple relations between variables are characterized by fuzzy conditional statements. By relying