BASIC ELECTRIC CIRCUITS

2nd Edition

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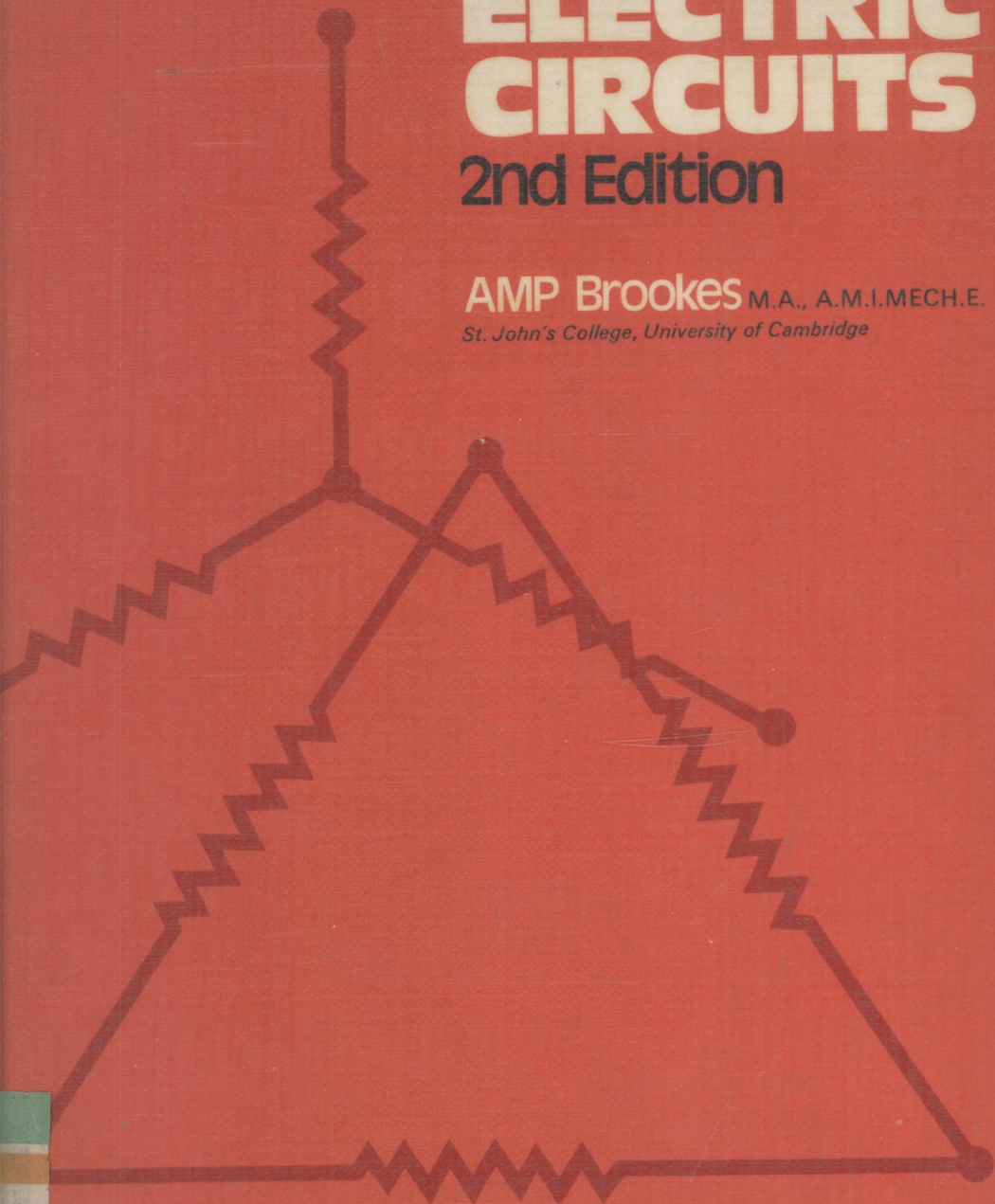
Basic Electric Circuits

SECOND EDITION

by

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Preface

THERE are many excellent books on advanced circuit analysis, but singularly few on the elements of this branch of electrical engineering, the topic usually being treated in one or two chapters in comprehensive electrical textbooks.

Familiarity with the elements of electric-circuit theory contributes greatly towards the full understanding of many branches of electrical engineering, both light and heavy and, by the use of the technique of equivalent circuits, a very large range of problems may be transformed into circuits capable of quick and straightforward solution.

To a large extent this book is based on lectures given to undergraduates in their first or second year of a University engineering course, and it is hoped that it will produce a solid grounding in the elements of circuit analysis both for university students and for those students reading engineering at technical colleges and other similar institutions. The earlier chapters should not be beyond the scope of sixth-form secondary school students.

Typical examination questions have been given at the end of each chapter and there are a number of worked examples in the text. The examples and questions have been widely chosen so as to illustrate various applications of the ideas discussed in any particular chapter, and duplication of an example on any specific aspect has been avoided as far as possible as it is felt that by far the best source of examples for practice is a collection of past examination papers for the examination for which any

particular student is preparing. In general the M.K.S. system of units has been used.

The physical appearance and construction of common circuit elements has been included, together with some mention of stability and accuracy, since experience shows that many students, when they come to do their practical work, have very little idea of the size, shape or suitability of the various components used in their experiments. An engineer is essentially a practical man and the sooner he gets the " feel " of his components and apparatus the better.

During the few years since the original publication of *Basic Electric Circuits* considerable advances have been made in the semiconductor field, particularly in respect of integrated circuit techniques. This second edition of the book includes considerable new material on transistors and semiconductor devices, and its general coverage has been considerably widened.

Cambridge   A. M. P. BROOKES

Acknowledgement

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Contents

Conducting and Insulating Materials

IF A MATERIAL will allow an electric current to flow through it easily it is known as a *conductor* and if it effectively prevents the flow of current it is known as an *insulator*. The majority of common materials fall definitely into one class or the other and the difference between the two classes is very considerable. If for instance two rods of the same size were made, one from a typical conducting material and the other from a typical insulating one, the ratio of the ability to conduct electricity would be of the order of 10^{20}. This enormous difference is due to the way in which an electric current is carried through a material. All materials are made up of atoms which themselves consist of a dense central part called a nucleus surrounded by a cloud of electrons. The nucleus has a positive charge and the electrons have a negative charge. The electrons are held in the atom by electric forces but the number and arrangement of electrons is different for different elements with the result that certain materials have electrons very closely held by these forces. In others some of the electrons on the surface of the atoms can be fairly readily detached so that they can wander about in the material from atom to atom. The closely bound electron system is a characteristic of insulators and the free outer electron system is characteristic of conductors.

Figure 1.1 shows in simplified form the arrangement of electrons around different nuclei. It will be seen that the nucleus can contain one or several positive particles or protons and similarly that the number of electrons can be one or more. There are, however, always the *same* number of protons as electrons in the

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atom in its normal state because each proton carries one positive charge and each electron one negative charge and these must balance if the atom as a whole is to be neutral. It can also be seen that the electrons are arranged in circles in the diagram which correspond to spherical shells designating the orbits of the electrons. There is a maximum number of electrons which will remain in any one shell and hence when one shell is full another at a greater radius is started. The number of protons and electrons that an atom contains differs for every element and, due to the shell arrangement and the finite number of electrons which each shell can contain, it will be clear that some elements will have outermost
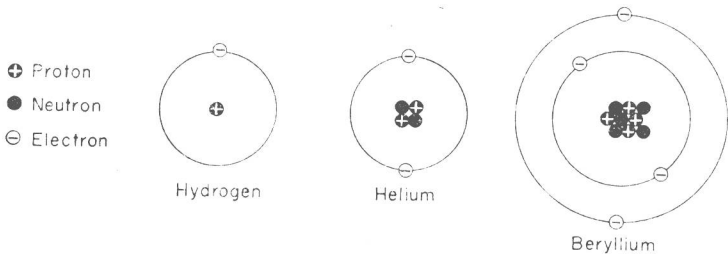
Fig. 1.1

shells which are full whilst others will have several places unoccupied. It is this difference which accounts for the ease or otherwise with which an electric current can pass through a material because a material with its outer shell full does not easily lose or take up an electron whilst, if there is an unoccupied place in an outer shell, an electron can slip into it and another electron leave the shell much more readily.

In metals the outer shells have not a full complement of electrons and also the atoms are closely packed together so that electrons can wander from atom to atom readily. In the absence of any unidirectional attractions the electrons will wander at random and on the average no net charge will be carried from one part of the metal to another as shown in Fig. 1.2.

However if at one point in the metal, at the end of a wire for instance, electrons are being attracted away by any means then the motion of electrons in the vicinity will no longer be completely random and there will be a drift towards the place from which they are being removed, as shown in Fig. 1.3, this drift constituting an electric current.
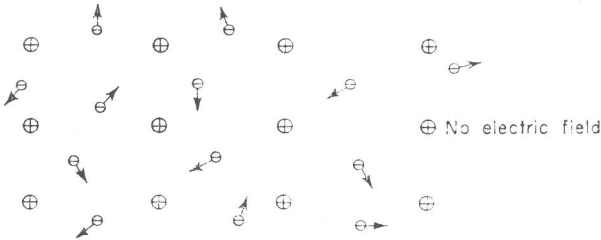
FIG. 1.2
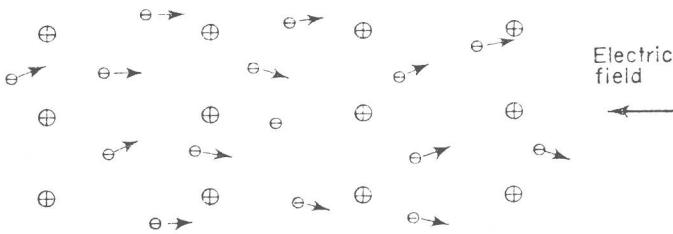
FIG. 1.3

It is important to distinguish between the velocity of this drift and the velocity of propagation of the change from random motion. If an electron drift is established at one end of a wire this drift will propagate through the wire to the other end in a time very much shorter than the time it would take an electron travelling at the mean drift speed to travel the same distance. This is analagous to a transverse vibration being transmitted down a taut string where the transverse velocity of the parts of the string is quite unconnected with the rate at which the disturbance is propagated along the string.

Not only is the drift velocity of the electron small compared with the velocity of propagation, it is also much smaller than the random motion velocity. Suppose that a conductor has a cross-sectional area A and that there are n free electrons, of charge e, per cubic metre moving with drift velocity v. The current I flowing will be given by

$$I = A.n.e.v. \text{ amps}$$

$$\therefore \quad v = \frac{I}{A.n.e}$$

For a copper wire of square section of side 1 mm and a current of 1 amp we have:

$$A = (10^{-3})^2 = 10^{-6}$$
$$I = 1$$
$$n = 8\cdot3 \times 10^{28}$$
$$e = 1\cdot6 \times 10^{-19}$$

which gives $v = 7\cdot5 \times 10^{-5}$ m/sec.

As would be expected from the above description of the mechanism of conduction the ability to conduct an electric current is not the same for all conducting materials nor is it the same at all temperatures. Changes in temperature imply changes in the energy of the electrons in the atoms of a substance and, since the mobility of these electrons governs the ease with which the electricity can be conveyed, it is natural that temperature and conduction should be interrelated. The property of opposing conduction is known as the *resistivity* of a material, symbol ρ, and the reciprocal $1/\rho$ is known as the *conductance*. It is more usual to specify materials by their resistivities and to give these a unit *ohm-metres* or *ohm-centimetres*. Numerically the resistivity of a homogeneous material at a given temperature could be obtained by the measurement of the current I through a cube of edge one metre when a potential difference V volts was applied to one pair of opposed faces of the cube. If the experiment were performed on a cube of edge one centimetre the result would, of course, then be in ohm-centimetres.

Table 1 gives the resistivities of several common conducting materials in order of ability to conduct.

The last three items in the table are alloys especially developed to have fairly high values for their resistivities so that resistors may be manufactured without requiring large quantities of wire. Brass is also an alloy and has different values of resistivity for different proportions of its constituent elements.

TABLE 1

Resistivities of Metals at 20°C

	ohm-metre
Silver	$1\cdot62 \times 10^{-8}$
Copper	$1\cdot76 \times 10^{-8}$
Gold	$2\cdot40 \times 10^{-8}$
Aluminium	$2\cdot83 \times 10^{-8}$
Tungsten	$5\cdot48 \times 10^{-8}$
Nickel	$7\cdot24 \times 10^{-8}$
Brass	$8 \quad \times 10^{-8}$
Iron	$9\cdot4 \quad \times 10^{-8}$
Platinum	$10 \quad \times 10^{-8}$
Lead	$20 \quad \times 10^{-8}$
Manganin	$45 \quad \times 10^{-8}$
Eureka	$49 \quad \times 10^{-8}$
Nichrome	$108 \quad \times 10^{-8}$

Since the variation of resistivity with change in temperature can be very large for some materials and is quite considerable for many, it is as necessary to have a method of estimating this change as it is to know the resistivity itself. The relation between resistivity and temperature is not a linear one as can be seen from Fig. 1.4 which is for lead. It will also be noted that there is a sudden change at the temperature at which lead melts. Thus any accurate figure for resistivity at a given temperature is best taken from an experimentally obtained curve for the material. However, in many practical circumstances the variation in temperature

from one at which the resistivity is known is not too great and a linear relation can be used. This relation is usually of the form

$$R_T = R_1[1 + \alpha_1(T - T_1)]$$

where R_T is required resistivity at temperature T

$\quad$ R_1 is the known resistivity at temperature T_1

$\quad$ α_1 is the *temperature coefficient of resistance* for the material appropriate to T_1.

It is important to note that α_1 will be different for different values of T_1 because the assumption we are making in assuming a linear relation is that we may extrapolate linearly from any point on
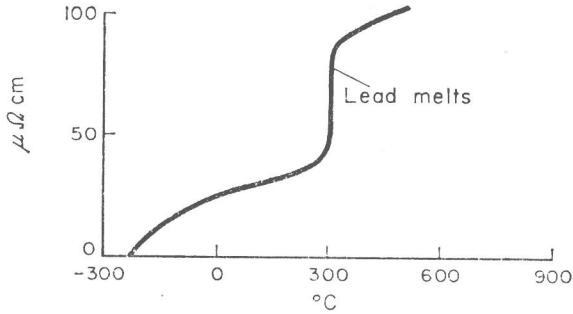

FIG. 1.4

our actual true curve for the material. This means that at any given point corresponding to T_1 on the curve we must draw a tangent and work along it. Obviously the slope of the tangent will vary with the value of T_1 and hence the value of α_1 will vary.

It is not usual, however, to give extensive tables of α for large numbers of temperature values and so Table 2 gives values of α_0 which assumes the resistivity T_0 is known at 0°C.

It will be noticed that Eureka may have a small *negative* temperature coefficient. Carbon is another substance which has a negative coefficient which is about -0.0005. Some other materials

consisting for example of oxides of manganese, nickel, iron, zinc, titanium and magnesium have very large negative temperature coefficients and also have high resistivities compared with the pure metals and alloys we have been considering, although not nearly so high as for insulating materials. These materials are known as *semi-conductors* and are extremely important as the

TABLE 2

Temperature Coefficients of Resistance

	°C units
Manganin	0·000002
Eureka	−0·00007 to +0·00004
Nichrome	0·0001
Brass	0·001
Gold	0·0034
Platinum	0·0037
Silver	0·0038
Copper	0·0043
Lead	0·0043
Aluminium	0·0043
Tungsten	0·0045
Iron	0·0055
Nickel	0·0059

essential part of transistors, metal rectifiers, thermistors and various other devices. As an example the graph in Fig. 1.5 shows a typical thermistor temperature–resistivity characteristic where the resistivity is plotted to a log scale and indicating a change in resistivity of more than a million to one for a temperature change of from $-100°C$ to $+400°C$. Because of this very large change in resistance with temperature it is simple to use a thermistor as a very sensitive thermometer since a relatively rough measurement of its resistance will be sufficient to determine its temperature to within $\pm0·01°C$ once its calibration is known.
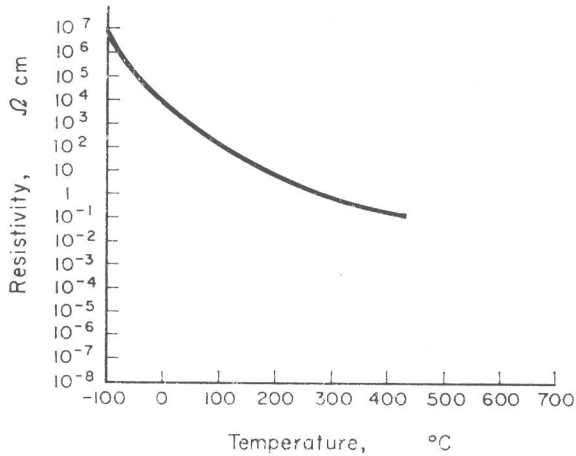
Fig. 1.5

The two most used semiconductor materials are germanium, with a resistivity at room temperature of about 47 ohm-cm, and silicon of resistivity 3×10^5 ohm-cm under the same conditions. Other important materials in use in various devices are gallium arsenide, lead sulphide, indium antimonide and other compounds each of which has some special useful property, but silicon devices have now become the commonest.

To understand the working of a semiconductor device it is first necessary to consider the crystal structure of the basic material. Fig. 1.6 shows in diagram form the arrangement of the nucleus and electrons in a silicon atom, from which it can be seen that the outer shell of electrons has four members known as the valence electrons. The full permitted complement for this shell is eight electrons so that when the atoms are assembled regularly in a crystal of the material it is found, upon examination of the crystal structure, that each of the valence electrons of any one atom is shared with the four neighbouring atoms so that effectively every atom has a *full* shell, as is shown diagrammatically in Fig. 1.7. When of the structure

shown in Fig. 1.7 silicon has a resistivity of 3×10^5 ohm-cm at room temperature, as stated above, when the material is said to be *intrinsic*, the effect of thermal agitation enabling a few electrons to break away from the parent atoms and become current carriers. However, by the addition of an extremely small amount of a pentavalent impurity, which thus has five valence electrons instead of the four for pure silicon, the resistivity can be reduced to only a few ohm-cm, the extra valence electron being very free to move about the lattice. Only one impurity atom for every 10^8 atoms of the pure silicon is necessary to produce this very large change in

• Electrons
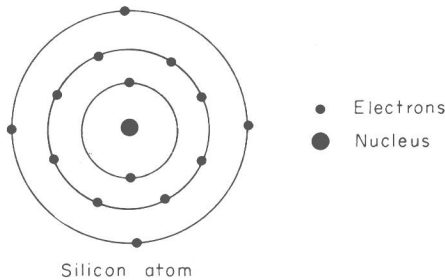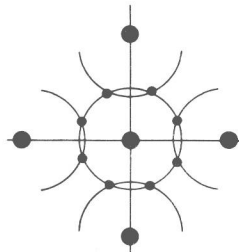● Nucleus

Silicon atom

FIG. 1.6

Electron   sharing
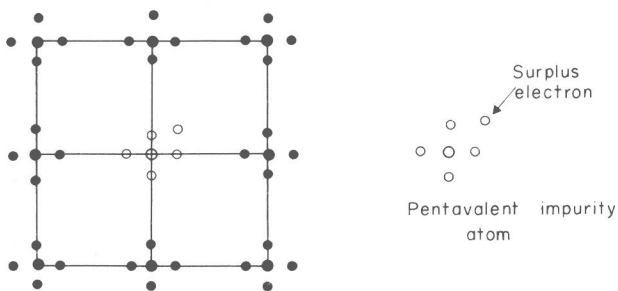in   Germanium
or   Silicon

FIG. 1.7

resistivity and the impurity atom is known as a *donor* atom producing, as it does, an extra electron in the lattice, making the material into n-type. This is shown diagrammatically in Fig. 1.8.

It is also possible to "dope" pure silicon with a few trivalent or *acceptor* atoms having only three electrons in their outer shells. In this case, as shown in Fig. 1.9, there will be a vacant space or "hole" in the lattice electron system and the subsequent material is known as p-type because the lack of an electron is equivalent to a positive charge and the vacant space can be considered to move about because any free electrons in the vicinity which have come from neighbouring atoms can occupy holes and thus effectively transfer these holes to the atoms from which the electrons have come originally. For convenience it is usual to consider n-type material as having a number of free negative charges available to carry current by moving through the lattice and p-type material as having a number of positive charges available to carry current also by moving through the lattice, and this system is shown in Fig. 1.10.
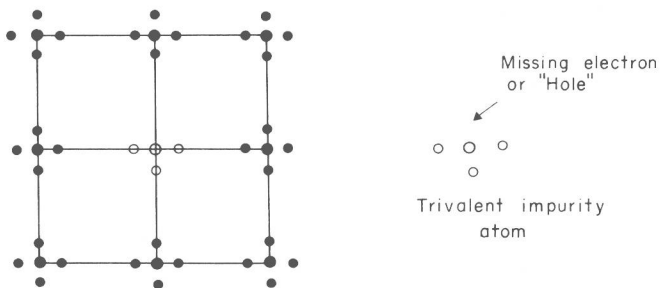
It must be emphasized that the impurity atoms, both of donor and acceptor type, are fixed rigidly in the lattice and will have a charge equal in magnitude to that of one electron, this charge being positive for a donor atom—because it has effectively lost one of its five electrons—and negative for an acceptor atom—because it has accepted an extra electron above its original three. Hence, if by some means the mobile charges can be swept away from some part of the doped material, a space charge will exist in that region due to the charges on the doping atoms.

Under room temperature conditions there will always be a few electrons thermally removed from their parent atoms and thus a few holes simultaneously generated. These charges are known as *minority* carriers to distinguish them from the carriers introduced by doping which are therefore called *majority* carriers as indicated in Fig. 1.10. Thermal generation and recombination can be considered as taking place continuously at all temperatures above absolute zero and this is the main reason for sensitivity to temperature shown by all semiconductor materials.

Surplus
electron

Pentavalent impurity
atom

n – type semiconductor material

FIG. 1.8

Missing electron
or "Hole"

Trivalent impurity
atom

p – type semiconductor material

FIG. 1.9

n – type

p – type

Majority
carriers

Minority
carrier

Minority
carrier

Majority
carriers

● Free electrons

○ Holes

FIG. 1.10