M. Remoissenet

Waves Called Solitons

Concepts and Experiments

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With 127 Figures

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Preface

Nonlinearity is a fascinating element of nature whose importance has been appreciated for many years when considering large-amplitude wave motions observed in various fields ranging from fluids and plasmas to solid-state, chemical, biological, and geological systems. Localized large-amplitude waves called *solitons*, which propagate without spreading and have particle-like properties, represent one of the most striking aspects of nonlinear phenomena. Although a wealth of literature on the subject, including theoretical and numerical studies, is available in good recent books and research journals, very little material has found its way into introductory texbooks and curricula. This is perhaps due to a belief that nonlinear physics is difficult and cannot be taught at an introductory level to undergraduate students and practitioners. Consequently, there is considerable interest in developing practical material suitable for students, at the lowest introductory level.

This book is intended to be an elementary introduction to the physics of solitons, for students, physicists, engineers and practitioners. We present the modeling of nonlinear phenomena where soliton-like waves are involved, together with applications to a wide variety of concrete systems and experiments. This book is designed as a book of physical ideas and basic methods and not as an up-to-the-minute book concerned with the latest research results. The background in physics and the amount of mathematical knowledge assumed of the reader is within that usually accumulated by junior or senior students in physics.

Much of the text of this book is an enlargement of a set of notes and descriptions of laboratory experiments developed over a period of years to supplement lectures on various aspects of wave motion. In spite of the diversity of the material, the book is not a collection of disconnected topics, written for specialists. Instead, I have tried to supply the practical and fundamental background in soliton physics, and to plan the book in order that it should be as much as possible *a self-contained and readable interdisciplinary whole*. Often, the important ideas or results are repeated several times, in different contexts. *Many of my choices of emphasis and examples have been made with experimental aspects in mind. Several experiments described in this book can be performed by the reader.* Although numerical studies play an important role in nonlinear science, I will not consider them in this book because they are described in a considerable body of literature.

In order to facilitate the use of this book, many illustrations have been included in the text and the details of theoretical calculations are relegated to appendices at the end of each chapter. A number of basic references are given as well as references intended to document the historical development of the subject. The referencing is not systematic; the bibliography listed at the end of the book serves only to advise the reader which sources could be used to fill in gaps in his or her basic knowledge and where he or she could turn for further reading.

The text is organized as follows. Our introduction in Chap.1 is devoted to the beautiful historical path of the soliton. The fundamental ideas of wave motion are then set forth in Chap.2 using simple electrical transmission lines and electrical networks as examples. At an elementary level, we review and illustrate the main properties of linear nondispersive and dispersive waves propagating in one spatial dimension. In Chap.3, we consider waves in transmission lines with nonlinearity. These simple physical systems are very useful for a pedagogical introduction to the soliton concept, and they are easy to construct and to model, allowing one to become quickly familiar with the essential aspects of solitary waves and solitons, and their properties. Specifically, we first examine the effect of nonlinearity on the shape of a wave propagating along a nonlinear dispersionless transmission line. Then we consider *the remarkable case where dispersion and nonlinearity can balance to produce a pulse-like wave with a permanent profile.* We describe simple experiments on pulse solitons, which illustrate the important features of such remarkable waves. In Chap.4 we consider the *lattice solitons*, which can propagate on an electrical network; then we examine periodic wavetrains, and modulated waves such as *envelope or hole solitons,* which can travel along electrical transmission lines.

Chapter 5 concentrates on such spectacular waves as the hydrodynamic pulse soliton, which was first observed in the nineteenth century, and the hydrodynamic envelope soliton. Simple water-tank experiments are described.

In Chap. 6, by using a chain of coupled pendulums, that is, a mechanical transmission line, we introduce a new class of large amplitude waves, known as *kink solitons* and *breather solitons* which present remarkable particle-like properties. Simple experiments that allow one to study qualitatively the properties of these solitons are presented.

Chapter 7 deals with a more sophisticated device: the superconductive Josephson junction. Here the physical quantity of interest is a quantum of magnetic flux, or *fluxon,* which behaves like a kink soliton and has properties remarkably similar to the mechanical solitons of Chap.5. In Chap.8 *bright and dark solitons* emerge, which correspond to the optical envelope or optical hole solitons, respectively. They can be observed in optical fibers where exploitation of the typical dispersive and nonlinear effects has stimulated theoretical and experimental studies on nonlinear guided waves.

Whereas the previous chapters are concerned with solitons in the macroworld, Chap.9 deals with nonlinear excitations in the microworld. Specifically, we consider the soliton concept in the study of nonlinear atomic lattices. The nonlinear equations that are encountered in the soliton story and models of several systems described in the text can be solved by using remarkable and powerful mathematical techiques, the main steps of which are given in the last chapter.

If a substantial fraction of users of this book feel that it helped them to approach the fascinating world of nonlinear waves or enlarge their outlook, its purpose will have been fulfilled. I hope the reader will feel encouraged to bring to my notice any remaining errors and other suggestions.

I have greatly benefited from frequent discussions with my colleagues and students. I am particularly grateful to Jean Marie Bilbault, who went over the entire manuscript and gave me invaluable comments.

I would also like to thank Alwyn Scott whose criticism and suggestions helped refine the manuscript.

I also wish to extend my appreciation to Patrick Marquié, Guy Millot, Jean François Paquerot, Michel Peyrard, and Claudine and Gérard Pierre for their comments on various chapters. Special thanks go to Bernard Michaux for his assistance in designing and performing experiments, and improving numerous illustrations throughout the book. Finally, it is a pleasure for me to acknowledge the technical assistance I have received from Dominique Arnoult and Claudine Jonon.

Dijon
October 1993

Michel Remoissenet

Soliton Blues

Music: Michel Remoissenet
Arrangement und transcription:
Michel Thibault

This book is dedicated to all the scientists
who have made the solution concept a reality

Contents

1 Basic Concepts and the Discovery of Solitons

Today, many scientists see nonlinear science as the most deeply important frontier for the fundamental understanding of Nature. The soliton concept was firmly established after a gestation period of about one hundred and fifty years. Since then, different kinds of solitons have been observed experimentally in various real systems, and today, they have captured the imagination of scientists in most physical discipline. They are widely accepted as a structural basis for viewing and understanding the dynamic behavior of complex nonlinear systems. Before introducing the soliton concept via its remarkable and beautiful historical path we compare briefly the linear and nonlinear behavior of a system.

1.1 A look at linear and nonlinear signatures

First, let us consider at time t the response R_1 of a linear system, an amplifier for example, to an input signal $E_1 = A \sin\omega t$ of angular frequency ω, as sketched in Fig. 1.1. In the low amplitude limit the output signal or the response of the system is linear, in other words it is proportional to the excitation

$$R_1 = a_1 E_1. \tag{1.1}$$

Here a_1 is a quantity that we assume to be constant (time independent) to simplify matters. If we double the amplitude of the input signal, the amplitude of the output signal is doubled and so on. The sum of two input signals E_1 and E_2 yields a response which is the superposition of the two output signals,

$$R = a_1(E_1 + E_2) = R_1 + R_2, \tag{1.2}$$

and a similar result holds for the superposition of several signals.

Next, if the amplitude of the input signal gets very large, distortion occurs as a manifestation of overloading. In this case, the response is no longer proportional to the excitation; one has

$$R = a_1 E_1 + a_2 E_1^2 + a_3 E_1^3 + \ldots = a_1 E_1 \left(1 + \frac{a_2}{a_1} E_1 + \frac{a_3}{a_1} E_1^2 + \ldots\right) \tag{1.3}$$

and signals at frequencies 2ω, 3ω, and so on, that is, *harmonics of the input signal are generated.* In some cases a chaotic response can occur: this phenomenon will not be considered in this book. Moreover, the sum of two signals at the input results not only in the sum of responses at the output but also in the product of sums and so on. *The superposition of states is no longer valid.*

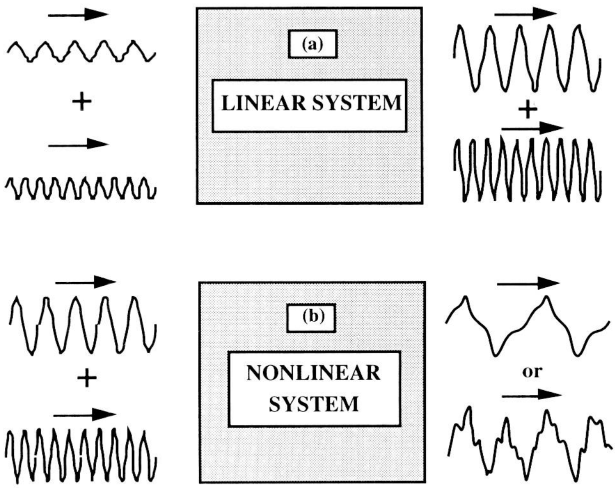

Fig.1.1. Sketch of the linear and nonlinear responses of a system to two input signals.

In equation (1.3) one has $a_1 >> a_2 >> a_3 ...$ and we note that the nonlinear effects increase with the amplitude of the signal and with the coefficients a_2, a_3, ... If the quantities a_2/a_1, a_3/a_1, ... are not very small, say about 0.1, the system or material will be called *strongly nonlinear.*

Now, let us consider waves, that is signals which are not only time dependent but also depend on space, say one space dimension x, only, as will be assumed throughout this book. In this case any arbitrary pulse or disturbance can be regarded as a linear superposition of sinusoidal wave trains with different frequencies. If each of these waves propagates with the same velocity, the system is called *nondispersive* and the pulse travels without deforming its shape, as represented in Fig.1.2. If the velocities of each wave train are different, the pulse spreads out (see Fig.1.3) when propagating and the system is *dispersive.*

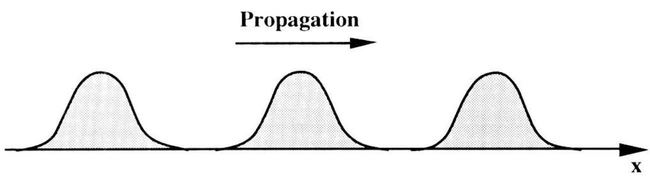

Fig.1.2. Undeformed propagation of a wave pulse in a linear and non dispersive system.

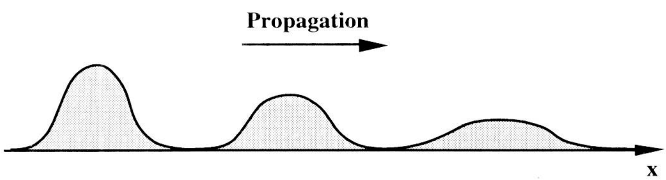

Fig.1.3. Propagation and spreading out of a wave pulse in a linear and dispersive system.

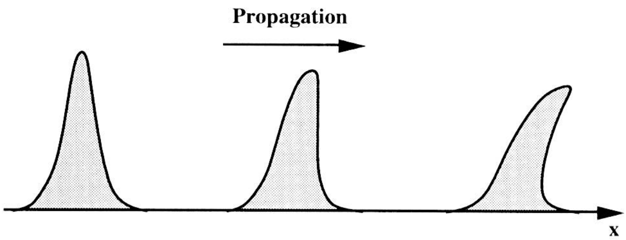

Fig.1.4. Points of large amplitude overtake points of small amplitude for a nonlinear wave pulse.

For waves nonlinearity introduces a new feature which is due to the generation of harmonics: the crest of the wave moves faster than the rest, in other words points of large amplitude overtake points of small amplitude, and the wave shocks and ultimately breaks (see Fig.1.4).

Having outlined typical features of linear and nonlinear systems, or media, let us now look into the beautiful history of nonlinear dispersive waves known as solitary waves and solitons.

1.2 Discovery of the solitary wave

Historically, the first documented observation of a solitary water wave was made by a Scottish engineer, John Scott Russell, in August 1834, when he saw a rounded smooth well-defined heap of water detach itself from the prow of a barge brought to rest and proceed without change of shape or diminution of speed for over two miles along the Union Canal linking Edinburgh with Glasgow. He described his observations in the following delightful terms:

> *I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped-not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation, ...*
>
> John Scott Russell, Report on Waves (1844)

These observations were followed by extensive wave-tank experiments which established the following important properties of *solitary water waves.*
(i) These localized waves are bell-shaped and travel with permanent form and velocity.

(ii) In water of undisturbed depth h a wave of elevation a_m, towards which the crest points , propagates with velocity

$$v = \sqrt{g(h+a_m)}, \tag{1.4}$$

where g is the gravity.
(iii) An initial elevation of water might, depending on the relation between its height and length, evolve into a pure solitary wave, a single solitary wave plus a residual wave train, or two or more solitary waves with or without a residual wave train, as represented in Fig.1.5.
(iv) Solitary waves can cross each other without change of any kind.
(v) Solitary waves of depression are not observed: an initial depression is transformed into an oscillatory wave train of gradually increasing length and decreasing amplitude, as shown in Fig.1.6.

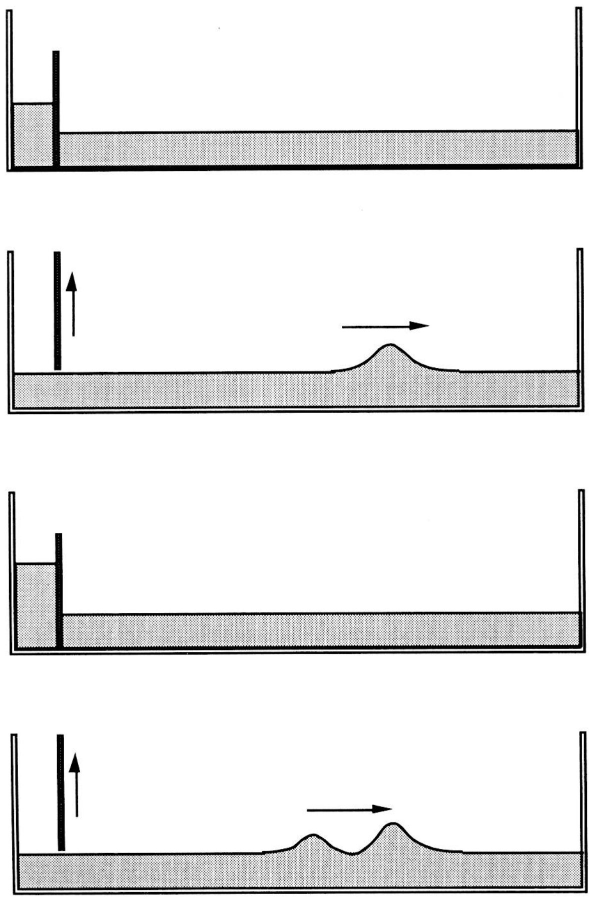

Fig.1.5. (**a**) Evolution of an initial elevation of water into (**b**) a pure solitary wave without a residual wave train (**c**) Evolution of an initial elevation of water into: (**d**) two solitary waves.