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Scientific Fundamentals of Robotics 3

M. Vukobratović · M. Kirćanski

Kinematics and Trajectory Synthesis of Manipulation Robots

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With 66 Figures

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D. Sc., Ph. D. MIOMIR VUKOBRATOVIĆ, corr. member of
Serbian Academy of Sciences and Arts
Institute »Mihailo Pupin«, Belgrade
Volgina 15, POB 15, Yugoslavia

M. Sc., MANJA KIRĆANSKI
Serbian Academy of Sciences and Arts
Institute »Mihailo Pupin«, Belgrade
Volgina 15, POB 15, Yugoslavia

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Preface

A few words about the series "Scientific Fundamentals of Robotics" should be said on the occasion of publication of the present monograph. This six-volume series has been conceived so as to allow the readers to master a contemporary approach to the construction and synthesis of control for manipulation robots. The authors' idea was to show how to use correct mathematical models of the dynamics of active spatial mechanisms for dynamic analysis of robotic systems, optimal design of their mechanical parts based on the accepted criteria and imposed constraints, optimal choice of actuators, synthesis of dynamic control algorithms and their microcomputer implementation. In authors' oppinion this idea has been relatively successfully realized within the six-volume monographic series.

Let us remind the readers of the books of this series. Volumes 1 and 2 are devoted to the dynamics and control algorithms of manipulation robots, respectively. They form the first part of the series which has a certain topic-related autonomy in the domain of the construction and application of the mathematical models of robotic mechanisms' dynamics. Published three years ago, the first part has provided a foundation for both the expansion of the previously achieved results and the generation of new, complementary results which have found their place in the following volumes. This is now the second part of the series, consisting of four volumes, has been created. While Volumes 5 and 6 deal with the problems of nonadaptive and adaptive control and computer-aided design of robots, book 4 presents new results attained in the domain of numeric-symbolic dynamic modelling of robotic systems in real time.

This volume mostly deals with manipulator kinematics. Most efficient methods for evaluating the manipulator kinematic models and the motion generation algorithms for nonredundant and redundant manipulators are considered. The complete material of this monograph is divided into six chapters.

dynamic characteristics of the system are taken into consideration.

In Chapter 5 deals with the manipulator motion synthesis where the system is considered as a dynamic system modelled by the complete, nonlinear dynamic model of the mechanism and the actuators. The complexity of the solution to the general optimal control problem for such systems, is illustrated. Regarding the practical importance of the energy optimal motion synthesis, which simultaneously provides for smooth, jerkless motion and minimal actuators' strains, a particular attention was paid to the energy optimal motion of nonredundant manipulators. An algorithm for determining the energy optimal velocity distribution, given a manipulator end-effector path, was presented too.

In Chapter 6 the problems connected with redundant manipulator motion synthesis are discussed. The various methods, aimed at resolving the ambiguity of the inverse kinematic problem, which have been proposed to date are presented. The motion synthesis in free work space and obstacle-cluttered environment are discussed separately.

Similarly to the previous volumes of the Series, the present book is also intended for students enrolled in postgraduate robotics courses and for engineers engaged in applied robotics, especially in studying kinematics of robotic systems. The authors also think that this, essentially monographic work may, with no great difficulties, be used as the teaching material for subjects from the field of robotics at graduate studies.

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Contents

Chapter 1
Kinematic Equations

1.1. Introduction

In this chapter we will consider some basic kinematic relations describing manipulator motion synthesis. They include the relationship between external (world) coordinates describing manipulator hand motion and joint coordinates (angles or linear displacements). The different approaches to kinematic modelling which have been developed in the last years will be discussed.

The first problem to be considered is that of computing the manipulator tip position with respect to a reference, base coordinate frame, given a vector of joint coordinates. The specification of manipulator hand orientation is of importance in describing manipulation tasks. Therefore, the problem of how to determine hand orientation, given a manipulator configuration in space, will be also described.

The relationship between the linear and angular velocities of the manipulator end-effector and the joint rates is also very important in manipulator control. We will here discuss various types of Jacobian matrices relating between these velocities.

The problem of computing the joint coordinates, given a manipulator position and orientation, will be studied in Chapter 3. The complexity of the inverse kinematic problem, together with various approaches to obtain its solution, will be considered, too.

1.2. Definitions

In this section we will introduce some basic notations and definitions relevant for manipulator kinematics formulation. We will be concerned with the manipulator structure, link, kinematic pair, kinematic chain, the joint coordinate vector and its space, the external coordinate vector and the external coordinate space, direct and inverse kinematic problems and redundancy.

Let us consider the model of a robot mechanism shown in Fig. 1.1. The model consists of n rigid bodies which represent mechanism links. These links are interconnected by revolute or prismatic (sliding) joints, having rotational and translational motion, respectively.

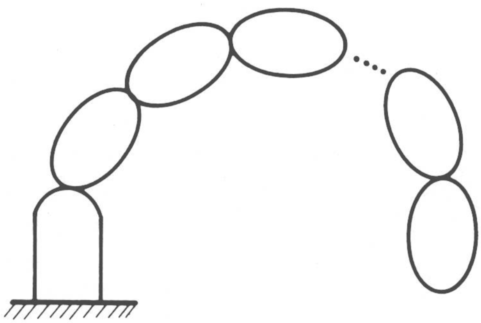

Fig. 1.1. Model of a manipulator with n links and n joints

Manipulator structure

The mechanical structure of the mechanism, represented by an arranged n-tuple $(J_1,\dots,J_n)$, will be termed as manipulator structure, where for each $i \in N = \{1,\dots,n\}$, $J_i \in \{R, T\}$. Here R stands for a revolute joint and T for a prismatic one.

For example, the manipulator structure RTTRRR stands for a mechanism with 6 joints, where the second and third joints are sliding, and the remaining joints are rotational.

Link

A link is defined by an arranged set of parameters $C_i(K_i, \mathcal{D}_i)$, where K_i represents a set of kinematic parameters and $\mathcal{D}_i$ a set of dynamic parameters.

The sets K_i and $\mathcal{D}_i$ may be defined in various ways. Precise definition of kinematic parameters for Rodrigues formula approach and Denavit--Hartenberg notation will be presented in Subsections 1.3.1 and 1.3.2.

The set K_i includes a local coordinate frame attached to link i, a set of distance vectors (parameters) describing the link i and unit vectors of joint axes. On the other hand, the set of dynamic parameters $\mathcal{D}_i$ involves the mass of the link i and its tensor of inertia $\underline{\underline{J}}_i$. If the unit vectors of the local coordinate frame of link i coincide with the main central axes of inertia, this tensor reduces to three moments of inertia $\underline{\underline{J}}_i = (J_{i1}, J_{i2}, J_{i3})$.

Kinematic pair

A kinematic pair P_{ik} represents a set of 2 adjacent links $\{C_i, C_k\}$ interconnected by a joint in point Z_{ik}.

The notion of the class and the subclass of a kinematic pair is introduced depending on the type of joint connection. A *j*th class kinematic pair (j=1,...,5) is defined as a set of 2 adjacent links interconnected by a joint with n = 6-j degrees of freedom. A kinematic pair of *j*th class and ℓth subclass is defined as a pair having r rotational and t prismatic joints in point Z_{ik}, where

$$r = \begin{cases} m-\ell+1\,, & \ell \leqslant m+1 \\ 0\,, & \ell > m+1 \end{cases}$$

$$t = s-r$$

m denotes the maximum possible number of rotational degrees of freedom in the *j*th class. For example, classes 1, 2 and 3 permit 3 rotations (m=3), class 4 - two, and class 5 only one rotation.

Kinematic chain

A kinematic chain Λ_n is a set of n interconnected kinematic pairs, $\Lambda_n = \{P_{ik}\}$, $i \in N$, $k \in N$.

According to the structure of connections, chains are classified into simple, complex, open and closed.

A chain in which no link C_i enters into more than 2 kinematic pairs is said to be a simple kinematic chain. On the other hand, a complex kinematic chain contains at least one link C_i, $i \in N$ which enters into more than 2 kinematic pairs.

An open kinematic chain possesses at least one link which belongs to one kinematic pair only. If each link enters into at least two kinematic pairs, the chain is said to be closed.

In this book we will consider only simple open kinematic chains.

Joint coordinates

Scalar quantities which determine the relative disposition of the links of the kinematic pair $P_{i,k} = \{C_i, C_k\}$ are reffered to as manipulator joint coordinates q^{ℓ}_{ik}. The superscript $\ell \in \{1,\ldots,s\}$, where $s=6-j$ is the number of degrees of freedom, and j is the class of pair P_{ik}.

For the fifth-class pairs, having a single degree of relative motion between links C_{i-1} and C_i, the joint coordinate is q_i. However, a reference disposition of the links which is considered as initial, i.e. where $q_i = 0$, can be chosen in different ways, depending on the manner in which link coordinate frames are attached to the links. We will describe in Subsection 1.3.1 and 1.3.2, how the initial links dispositions, for revolute and sliding joints, are determined, in the two main kinematic modelling techniques.

For an open, simple kinematic chain with n degrees of freedom, the joint coordinates form an n-dimensional vector q

$$q = [q_1 \; q_2 \; \cdots \; q_n]^T \in R^n.$$

Joint coordinate space

Joint coordinate space is the n-dimensional space $Q \subset R^n$, $Q = \{q: q_{imin} \leq q_i \leq q_{imax}\}$, where q is the joint coordinate vector, q_{imin} and q_{imax}, $i=1,\ldots,n$ are boundary values of joint coordinate q_i, defined by physical constraints of the manipulator mechanical structure, and n is the number of degrees of freedom. The manipulator location in the work space is uniquely defined, given a vector $q \in Q$ (often referred to as manipulator configuration). Joint coordinate space is also termed as configuration space.

External coordinates

External (or world) coordinates x_{ei}, i=1,...,m describe position and orientation (completely or partially) of the manipulator hand with respect to some reference coordinate system. The reference system is chosen to suit a particular application. Most frequently, a fixed coordinate frame attached to manipulator base is considered as the reference system. More detailed discussion about the reference system with respect to which the manipulation task is described, will be presented in Chapter 4.

External coordinates x_{ei} form an n-dimensional vector $x_e = [x_{e1} \cdots x_{em}]^T \in R^m$.

The choice of the external coordinate vector as well as its dimension m, are highly dependant on the given manipulation task and the manipulator itself. For practical, industrial manipulators, the case m=6 is the most general case, since it makes specification of any payload position and orientation possible. Therefore, we will consider 6-dimensional external coordinate vector (less number of external coordinates is obtained simply by rejecting some of them). A common partition is that the first three elements of the external coordinate vector x_e define the position of the end-effector, while the rest of them define orientation with respect to a reference coordinate system. Thus we will distribute vector into two parts

$$x_e = [x_{eI}^T \; x_{eII}^T]^T \tag{1.2.1}$$

where $x_{eI} \in R^3$ specifies hand position and $x_{eII} \in R^{m-3}$ - hand orientation. Usually, position of the manipulator hand is specified by

$$x_{eI} = [x \; y \; z]^T \tag{1.2.2}$$

$$x_{eI} = [r \; \beta \; \alpha]^T \tag{1.2.3}$$

$$x_{eI} = [r \; \alpha \; z]^T \tag{1.2.4}$$

where x, y and z are Cartesian coordinates (Fig. 1.2), r, β and α are spherical coordinates (Fig. 1.3) and, finally, r, α and z are cylindrical coordinates (Fig. 1.4). In these figures $\vec{q}_{n1}$, $\vec{q}_{n2}$ and $\vec{q}_{n3}$ denote the unit vectors of the coordinate frame attached to the link n (last link in the chain).

Cartesian coordinates are of primary importance in industrial practice, while spherical and cylindrical coordinates are convenient for specific tasks. Spherical coordinates correspond to a translation r along the z axis followed by a rotation β about y axis, and then a rotation α about z axis. Specifying the position of the manipulator tip in cylindrical coordinates, corresponds to a translation r along the x axis, followed by a rotation α about the z axis, and finally a translation z along z axis.

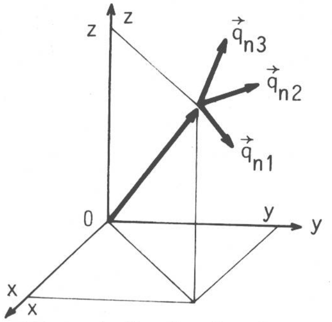

Fig. 1.2. Cartesian coordinates

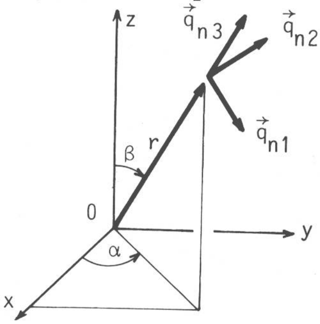

Fig. 1.3. Spherical coordinates

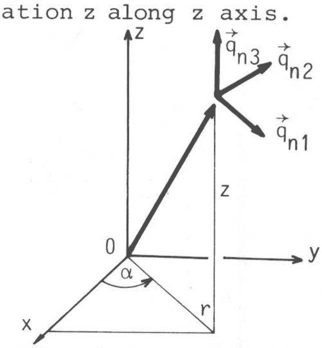

Fig. 1.4. Cylindrical coordinates

The orientation of the end-effector has been specified in several ways in the references on the subject. Orientation vector x_{eII} may have the form

$$x_{eII} = [\psi \ \theta \ \varphi]^T \tag{1.2.5}$$

where ψ, θ and φ are Euler angles. Several types of Euler angles have been adopted [1, 3], depending upon the sequence of rotations about the x, y and z axes. Here, we shall consider only yaw, pitch and roll angles (Fig. 1.5), since they seem to be the most appropriate for specifying manipulator hand orientation. Yaw angle corresponds to a rotation ψ about the z axis, pitch corresponds to a rotation about the new y axis, and roll corresponds to a rotation φ about the x axis.

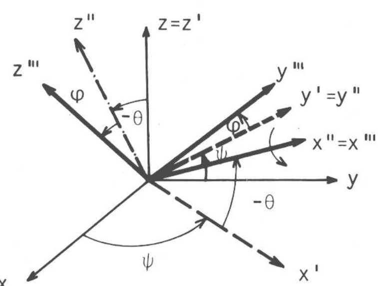

Fig. 1.5. Yaw, pitch and roll angles

Another way to specify hand orientation by means of three magnitudes is the use of Euler parameters [4]

$$x_{eII} = [\lambda_1 \ \lambda_2 \ \lambda_3]^T \tag{1.2.6}$$

A more detailed discussion on manipulator hand orientation will be given in Section 1.4.

External coordinate space

The external coordinates space is defined as m-dimensional space $X_e \subset R^m$, $x_e = \{x_e: x_e = f(q),\ q \in Q\}$, where f is a nonlinear, continuous and differentiable vector function, which maps joint coordinates vector q = $= [q_1 \cdots q_n]^T \in Q$ into the external coordinates vector $x_e = [x_{e1} \cdots x_{em}]^T$ in a unique way. The space X_e is at the same time a generalized region of reachable manipulator work space.

Direct kinematic problem

Manipulator position and orientation in space, i.e. external coordinates vector is uniquely defined, given a joint coordinates vector $q \in Q$. Solving the equation

$$x_e = f(q) \tag{1.2.7}$$

is known as the direct kinematic problem. This solution differs depending on the type of external coordinates. This will be discussed in Sections 1.3 and 1.4.

Inverse kinematic problem

Determining joint coordinates, given a vector of external coordinates, i.e. solving the equation

$$q = f^{-1}(x_e) \tag{1.2.8}$$

is known as the inverse kinematic problem. This problem is far more complex than the direct kinematic problem, since it is equivalent to obtaining solutions to a set of nonlinear trigonometric equations. This problem will be considered in Chapter 3 in detail.

Redundancy

Depending on the number of degrees of freedom and a given manipulation