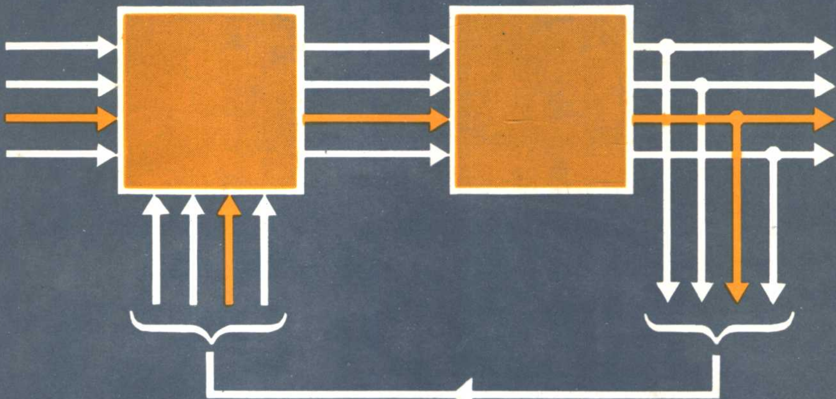
MODERN CONTROL SYSTEM THEORY

SECOND EDITION



M. GOPAL

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PREFACE TO SECOND EDITION

Since the printing of the first edition, I have received input from several teachers and students giving me suggestions to strengthen various sections and include answers to problems. Most of these suggestions have been incorporated in the second edition.

In the last few years, personal computers have become so powerful that it is now possible to use them to solve complex control system problems easily and efficiently. This development should make the study of control systems easier, as the student can go through large numbers of analysis and design runs in order to learn its theory. The book offers computer-aided learning environment with any commercially available CAD (Computer-Aided-Design) software.

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PREFACE TO FIRST EDITION

Over the past two decades, Modern Control System Theory has been gaining great importance, for being potentially applicable to an increasing number of widely different disciplines of human activity. Basically, the Modern Control System Theory has involved the study of analysis and control of any dynamical system—whether engineering, economic, managerial, medical, social or even political. This theory first gained considerable maturity in the discipline of engineering and has been successfully applied in a variety of branches of engineering, particularly receiving great impetus from aerospace engineering. Recently, it has been applied in economics and other disciplines as well, and has proved very promising. A fascinating fact is that all these widely different disciplines of application depend upon a common core of mathematical techniques of the Modern Control System Theory. It is these techniques that I have exposed in this book, emphasizing their application in the engineering discipline.

Modern control theory has no doubt been presented in varying depths by many prominent authors. Among many excellent presentations, a few stand out in my mind because of the repeated use I have made of them over the years: *Linear System Theory*—C. T. Chen; *Linear Optimal Control Systems* —H. Kwakernaak and R. Sivan; *An Introduction to Linear Control Systems* —T. E. Fortmann and K. L. Hitz; *Optimal Control Theory: An Introduction*— D. E. Kirk; *Optimum Systems Control*—A. P. Sage and C. C. White, III However, despite having these and many other good books on modern control theory, while teaching undergraduate and postgraduate students of various engineering branches and while guiding doctoral students, I have experienced a strong need for a book that meets the following requirements:

1. A thorough exposure of modern control theory through the application of its potential concepts consistently to a variety of practical system examples drawn from various engineering disciplines.
2. A significant provision of the necessary topics that enables a research student to comprehend various technical papers in which modern control theory techniques are employed to solve many systems and control problems.
3. A flexibility which enables a teacher to add recent potential topics of Linear Multivariable System Theory.

I have therefore made an attempt to meet these requirements which are eagerly sought after, but have not been met in any of the existing works. The present book is the result of such an attempt.

This book presents both continuous-time and discrete-time systems, and brings out, in particular, the similarities which reinforce many ideas. Also, an attempt has been made to point out many exceptions that occur which warrant a careful study in each case. Special attention has been given to important control problems such as deadbeat control, non-zero set points, and external disturbances and sensitivity problems in optimal linear regulators. Further, at appropriate places, the recent proposals for system design have been emphasized, such as modal control, state observers and estimators, suboptimal control, etc.

Specifically, the book provides lucid illustrations of modern control system theory concepts by applying them repeatedly to five practical control problems from various engineering disciplines. It was not possible to avoid simplification of the real-life problems for the sake of tractability; yet, particular care has been taken to retain the essence of the real-life problems in their simplified versions. In fact, the same chosen five real-life problems have been thoroughly developed over several chapters to reinforce the concepts.

The book is organised as follows:
Chapter 2 presents the basic core background, namely: linear spaces and linear operators. Chapters 3 and 4 deal with the issues of modelling of systems. Chapters 5 through 8 primarily present methods of analysis; also, they project some of the potential design techniques that evolve from analysis. Finally, Chapters 9 through 12 completely address themselves to the design of controllers for several classes of plants.

Most of the theoretical results have been presented in a manner suitable for digital computer programming, along with the necessary algorithms for numerical computations. However, detailed discussion of these algorithms has been deliberately avoided; instead, suitable references for further study have been suggested. Exercise problems, tailored particularly to help the reader understand and apply the results presented in the text, have been given at the end of each chapter. In fact, some of these problems also serve the purpose of extending the subject matter of the text.

A basic working understanding in the following areas has been assumed on the part of the reader: calculus, linear differential and difference equations, transform theory, matrix theory and probability theory. It would be additionally facilitating, though not necessary, for the reader to have taken a course on classical control theory.

For teaching the material covered in this book, I suggest—based on the successful class-testing of the material in the courses EE 658 and EE 660 at the Indian Institute of Technology, Bombay—that two courses may be offered:

1. A one-semester course at senior undergraduate or postgraduate level covering Chapters 1 through 8.

2. A one-semester course at postgraduate level covering Chapters 9 through 12.

I have great pleasure in expressing the acknowledgements which I owe to many persons in writing this book. I warmly recognize the continuing debt to my 'mentor' in Control System Theory Prof. I. J. Nagrath of the Birla Institute of Technology and Science, Pilani. It is in him that I have found my teacher, friend, and source of inspiration. At the Indian Institute of Technology, Bombay, I have been influenced and assisted by a great many people while preparing this book. I acknowledge pleasant association with Dr. M. C. Srisailam, Dr. S. D. Agashe, Dr. H. Narayanan, Prof. V. V. Athani, Dr. (Mrs.) Y. S. Apte and Dr. M. P. R. Vittal Rao. I must record, separately, my appreciation of the help given by my doctoral students, Mr. J. G. Ghodekar, Mr. P. Pratapchandran Nair and Dr. S. I. Mehta during the crucial period of the growth of this book. Finally, I wish to express my gratitude to my wife Lakshmi, my son Ashwani and daughter Anshu for their interest, encouragement, and understanding and for bearing with me through the project.

I am grateful to the authorities of the Indian Institute of Technology, Bombay for aiding this book writing project through Curriculum Development Cell.

I warmly welcome suggestions and criticism from the users of this book. I shall consider it a pleasure to respond to specific questions concerning the use of the material in this book.

M. Gopal

CONTENTS

1. INTRODUCTION

1.1 SYSTEMS: MODELLING, ANALYSIS AND CONTROL

The word 'system' implies essentially two concepts: (1) interaction within a set of given or chosen entities, and (2) a boundary (real or imaginary), separating the entities inside the system from its outside entities. The interaction is among entities inside the system that influence or get influenced by those outside the system. The boundary is. however, completely flexible. For instance, one may choose to confine oneself to only a constituent part of an original system as a system itself; or, on the contrary, one may choose to strech the boundary of the original system to include new entities as well.

In this book, we confine ourselves to only *physical systems*. In the study of physical systems, the entities of interest are certain physical quantities; these quantities being interrelated in accordance with some principles based on fundamental physical laws. Under the influence of external *inputs* (entities outside the boundary), the interactions arise in the system in a manner entirely attributable to the character of the inputs and the bonds of interaction. The inputs, while affecting the system behaviour, are usually not reciprocally affected and therefore are arbitrary in their time-behaviour.

In dealing with control systems, we will invariably be concerned with the dynamic characteristics of the system. Our main interest will be focussed upon only some of the entities of *dynamic* systems, namely those whose behaviour we wish to control. These entities—the *outputs* of the system, are normally accessible for purposes of measurement.

The study of physical systems often broadly consists of the following stages: (1) Modelling (2) Analysis (3) Design and synthesis.

Zadeh and Desoer (1963) have defined a 'system' as a collection of all of its *input-output pairs*. For a quantitative analysis of a system, we determine mathematical relations which can be used to generate all input-output pairs belonging to the system. In our terminology, the mathematical relations used for generating all possible input-output pairs of a system will be referred to as the *mathematical model* of the system. The mathematical model that we develop for a system should account for the fact that to each input to a dynamical system, there are, in general, a number of possible outputs. As we shall see in later chapters, the nonuniqueness of response to a given input reflects the dependence of output not merely on the input but also on the *initial status* (*initial conditions*) of the dynamical system. A set of differential/difference equations is a well-known form of mathematical description of a dynamical system which accounts for initial conditions of the system and gives rise to set of all possible input-output pairs.

Another useful form of mathematical description of a dynamical system is *state variable formulation*. As we shall see in later chapters of this book, the *state* of a system is a mathematical entity that mediates between the inputs and outputs. For given inputs and initial status of the dynamic system, the change in state variables with time is first determined and therefrom the values of outputs are obtained. The reader will observe the fact that the set of state variables, in general, is not a quantity that is directly measurable; it is introduced merely as a mathematical convenience. The only variables that have physical meaning are those that we can generate or observe, namely the inputs and outputs.

It is important to note that for the purpose of mathematical modelling, certain idealizing assumptions are always made, since a system, generally, cannot be represented in its full intricacies. An idealized physical system is often called *physical model* which is a conceptual physical system resembling the actual system in certain salient features but which is simpler and therefore more amenable to analytical studies. In many cases, the idealizing assumptions involve neglecting effects which are clearly negligible. Many of these effects are in fact neglected as a matter of course without a clear statement of implied assumptions. For example, the effect of mechanical vibrations on the performance of an electronic circuit is ordinarily not considered. Similarly, in electric power equipment the induced voltage due to surrounding time-varying electromagnetic fields (radio waves etc.) is usually neglected.

The above mentioned examples concern factors which are completely negligible in situations of interest. However, in many other situations there are more important effects which will still be neglected frequently to define a problem so that it can be handled mathematically without much complexity. For example, one early approximation which is usually made is to consider a system with *distributed parameters* as an equivalent system with *lumped parameters*. (If it is required to control the temperature of a room, it will first be assumed that the room is isothermal; to do otherwise would lead to very complicated partial differential equations representing the heat flow, both conductive and convective, between any two points in the room). Similarly, in physical systems we are uncertain to varying degrees about the values of parameters, measurements, expected inputs and disturbances (*stochastic systems*). In many practical applications, the uncertainties can be neglected and we proceed as though all quantities have definite values that are known precisely. This assumption gives us a *deterministic model* of the system.

Generally, crude approximations are made in a first attack on the problem so as to get a quick feel of the predominant effects. These assumptions are then gradually given up to obtain a more accurate physical model. A point of diminishing return is reached when the gain in accuracy of representation