

Edited by J.W. Bull



# Numerical Analysis and Modelling of Composite Materials

Edited by

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#### **Preface**

If it were possible to use a single non-composite material for a specific function, the material would have basic properties directly applicable to the design calculations. The material would also have many other properties that would rank it against alternate materials. For example, Young's modulus is a basic material property used in design, while properties such as fire resistance, impact resistance and jointability would be ranking properties. Once the material has been manufactured into a component, other ranking properties must be considered, such as the method of material failure: ductile collapse, brittle failure or fatigue? These questions are asked of individual materials.

If a suitable material does not exist for a function, then two or more materials are combined to form a composite. Steel bars, which are strong in tension, are added to the weak tensile section of a concrete beam to increase the beam's tensile resistance. Consequently, composite materials are used where there are specified needs, for example in the building industry and in aerospace engineering.

Each new use of a composite will have different material requirements. The composite will be required to resist such events as impact loading, vibrational loading, delamination, cracking and fatigue, to name but a few. Each composite material property must be assessed and quantified, ideally in real time and in the real world environment, although this is not always possible.

Research workers, engineers and designers need to use the latest and finest available techniques to assess the properties of composite materials. This means, in the vast majority of cases, carrying out numerical analysis and modelling of the composites, subjected to a variety of loading. However, many of these analyses are carried out in isolation of others working in a similar area of research. Many of these areas have their own journals and cross-fertilisation of ideas and results do not always take place.

This book brings together a wide range of composite material disciplines. The book gives the reader ideas and references to the best practice, and to areas of common problems. The book shows where common solutions of

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numerical analysis and modelling have been used successfully. The thirteen chapters in the book are divided into eight topic areas: aerospace industry; cylindrical shells and panels; practical construction problems; damage tolerance; interface regions; temperature effects; fibre debonding and woven fabrics. Each topic area can be briefly described as follows:

#### The aerospace industry

Chapter 1 provides an insight into the behaviour of composite rotor-blades, their numerical modelling and the associated analysis methodologies. Chapter 2 describes where advanced composite materials are used in airframes, the types of defects experienced in composite materials, generalised defect type and defect repair modelling.

#### Cylindrical shells and panels

Chapter 3 develops a numerical procedure for the free vibrational analysis of fibre reinforced laminated cylindrical shells. Numerical results are developed. Chapter 4 analyses and models cylindrical pressure vessels consisting of advanced fibre/resin composites and equipped with metallic liners. The optimal design of multi-layer composite pressure vessels is also discussed. Chapter 5 considers the analysis of the stress distribution around openings in finite-width composite panels. Design aspects such as strength prediction, shape optimisation and delamination are also considered.

#### Practical construction problems

Chapter 6 models the repair of a crater produced by a chemical explosion in the support material under a runway. The model is used to assess and design a runway crater repair. Chapter 7 shows that it is feasible to derive the macroscopic nonlinear behaviour of masonry from the knowledge of the constituent materials, through the homogenisation theory for periodic media. The softening character of the constitutive damage and plasticity laws is also modelled.

#### Damage tolerance

Chapter 8 describes an approach to identify and calibrate damage and fracture mechanics models for composites. Several constitutive laws suitable to characterise composite linear and nonlinear behaviour are outlined. The modelling of composite, monolayered, multilayered and sandwich shells and solids are included. Chapter 9 discusses approaches to damaged composites to estimate stresses, stiffness reductions and cumulative non-elastic strains. Cumulative damage models are then presented, followed by a discussion of a fatigue life prediction model.

#### Interface regions

Chapter 10 employs micromechanics to characterise the stress states and underlying mechanics of the interface regions of composites. The role of a

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ductile interlayer in promoting fracture resistance of brittle polymer matrix composites is also investigated.

#### Temperature effects

Chapter 11 reviews the literature in the area of metal matrix composites subjected to creep due to moderate temperature cycles. The chapter also discusses the relevant data required to make accurate calculations of composite thermal-cycling creep rates. Three approaches to the modelling are discussed and applied to the design of high temperature composites.

#### Fibre debonding

Chapter 12 reviews experimental techniques of fibre debonding and sliding, used in the determination of the interfacial properties of ceramic matrix composites. Detailed descriptions of tests are presented, including discussion of the effects of fibre surface topography and fibre surface treatments on the frictional behaviour of fibres in ceramic composites.

#### Woven fabrics

Chapter 13 presents a composite stiffness model for the prediction of thermoelastic properties of orthogonal plain weave fabric laminates.

My thanks go to my family for their support, to the chapter authors for their contributions and to Blackie A&P for their help, guidance, understanding and unfailing ability to remember deadlines!

J.W.B

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# 1 Analysis of composite rotor blades OMRI RAND

#### 1.1 INTRODUCTION

The purpose of this chapter is to provide an insight into the behavior of composite rotor blades, their modeling and the associated analysis methodologies. From a structural point of view, blades are usually treated as slender structures. This category includes all structures where one of their dimensions is much larger than the others. The discussion in this chapter will be focused on the analysis of helicopter blades, however, the structural modeling is also applicable to general machinery blades or any other similar slender structure. Due to their slenderness, the notion 'beam' is frequently used to describe the blade structure. The assumption of a slender structure influences the deformation presumptions, the boundary conditions and the treatment of the distributed and the tip loads.

The initial motivation to design helicopter blades using composites emerged from their preferable fatigue characteristics and the simplicity they offer in the design and manufacturing of various aerodynamic surface geometries. However, the possibility to improve the blade structural dynamics response, by applying appropriate fiber orientations that will induce advantageous structural couplings, seems to be more attractive. Currently, such 'aeroelastic tailoring' is one of the main research goals and has already proved to be feasible.

Generally, the structural analysis of composite beams poses many modeling and computational challenges. Analysis tools for isotropic beams are inappropriate in this case. This is because the well-known Bernoulli-Euler assumptions are not valid for composite beams, since composite materials, and in particular orthotropic materials, couple axial stress with shear strain and couple shear stress with axial strain. Thus, a detailed description of the warping (known also as 'shear deformation'), and predominantly its out-of-plane component, is inevitable. This requirement for a detailed warping description is added to other modeling issues that characterize the structural dynamics of rotating blades. Among these, the most important are the geometrical nonlinearities and the significant loads—deformation dependency. Both of the above issues force a nonlinear (and sometimes iterative) solution.

In reality, rotating blades undergo small strains, although moderate rotations and large displacements may take place. Thus, since the focus of this chapter is in the composite related structural effects, the involved geometrical nonlinearities will not be discussed here. However, in order to