GASDYNAMICS SERIES

Volume 2

SIMILARITY LAWS AND MODELING

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SIMILARITY LAWS AND MODELING

GASDYNAMICS

A Series of Monographs

Edited by PETER P. WEGENER

Yale University

VOLUME 1: Nonequilibrium Flows, Parts I and II, *edited by Peter P. Wegener*

VOLUME [redacted]

FOREWORD

This is the second volume of a series of monographs in the field of gasdynamics. The first volume appeared in two parts in 1969 and 1970, and it was devoted to the subject of nonequilibrium flows. It may be helpful to recall some of the thoughts that were set out in the Preface for the Series. There the intention was expressed to interpret the subject of gasdynamics broadly, to encompass all aspects of the dynamics of compressible media rather than concentration on high-speed flow alone. It was further announced that, in addition to theory, fundamentals of experimention will be of interest. In sum, it was hoped that the potential audience would include graduate students. Therefore, in addition to recent advances, the plan remains to include the underlying fundamentals of a given field as well. However, the emphasis will remain on the fluid mechanics of problems rather than on a discussion of the details of the physics and chemistry of certain flows. Finally, it was planned to group articles in a given volume, or a part of one volume, about a single major subject.

Turning to the volume at hand, we are now preparing the ground for discussions of experimental methods as planned. Volume 2 will first be devoted to the question of similitude. We were most fortunate to enlist Professor J. Zierep of Karlsruhe, Germany, in the venture, and it became a natural part of the manuscript that fields other than gasdynamics were covered. Similarity here extends to the problem of dimensional analysis, dimensional groups, and similarity laws in the entire field of fluid mechanics. For this reason it was specifically decided to publish the work as a separate volume in order to be able to address the book to a wider audience, including those who may not be specifically interested in compressible fluid mechanics. Coverage includes the subjects of heat transfer, chemical reactions, and relaxation times, and therefore the content has an additional relation to the previously published nonequilibrium discussions.

Volume 3, just as Volume 1, will be published in two parts, discussing experimental methods as well as the wide range of facilities available or planned for the study of high-speed flows. The editor will gladly receive criticisms, suggestions, recommendations of hitherto neglected topics.

New Haven, Connecticut

Peter P. Wegener

PREFACE

Similarity laws have become an essential and integral part of fluid mechanics. Dimensionless parameters have achieved great importance, indeed in some problems they have had an effect akin to black magic! One does not cease to marvel at the simplicity of thought and calculation by which interesting results in physics and technology may be found from dimensional analysis. However, it appears there is a lack of comprehensive and simple discussions of the entire field including instructive examples. Roughly speaking two classes of papers are found in the literature. For one the classical mechanical and thermodynamic similarity laws with their dimensionless groups are derived. On the other hand we find a discussion of the similarity laws of gasdynamics. A systematic description of both groups of practically important problems and their mutual relationships does not seem to exist. It will be attempted in the unifying work at hand to show the natural connection between both fields. Among other devices to achieve this goal, results of dimensional analysis and gasdynamic laws, respectively, will be confronted with each other. Furthermore, a survey of the different methods for finding such relations will be provided.

Concerning the structure of the work, Chapter 1 contains some historical remarks and answers the question of the nature of similarity laws. The latter answer is not straightforward since many different relationships are hidden under the general heading of similitude. Next some selected examples follow to stress the utility of the reasoning given. Chapter 2 contains a survey of the four most important methods of derivation of such laws. The schemes provided differ in the amount of information contained in them, and they will be discussed critically in each instance. The gasdynamic similarity laws for bodies whose geometry can be related to each other by affine transformation are discussed in Chapter 3. The material given includes linearized subsonic flow and supersonic flow, as well as transonic and hypersonic flows, respectively. Numerous applications to given problems are included. Finally, Chapter 4 provides a brief overview for corresponding flows with relaxation. Unfortunately, little is known on similarity of nonequilibrium flows, and a challenging field for further research is open. The given law of similarity for affine bodies perturbed about the equilibrium state appears to be new.

The level of the discussion is such that students with a general background in fluid mechanics ought to be able to follow the text. It is further hoped that the material is useful for the practicing engineer. Finally, the author has the hope that the specialist may occasionally encounter some new results.

I wish to express my sincere appreciation to the editor of this series, Professor Peter P. Wegener, for critically reading the whole manuscript and for

proposing many improvements. I am furthermore grateful to Dr. G. Ernst for translating the work into English.

Karlsruhe, Germany

J. Zierep

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Chapter 1
THE PROBLEM AND EXAMPLES

1.1 HISTORICAL REMARKS

The beginnings of considerations of similarity date far back. In a book by Sommerfeld (1957), the first investigations of this kind are ascribed to Galilei. Going to the roots, one recognizes that studies of this subject are as old as the equations that describe physical phenomena. The laws of fluid dynamics, the subject of this treatise, however, have been worked out more recently. The first systematic investigation of the theoretical basis of this subject was done by Helmholtz (1873). It is remarkable that in this work all similarity parameters that result from the continuity and momentum equations, respectively, are more or less explicitly given. Of course, in the case of gases, only isothermal processes are considered and so the applicability of the considerations is limited. In this early work, but in an unfamiliar form, the similarity parameters which were later designated to honor the names of Reynolds, Froude, and Euler appear. Even the statement of similarity resulting from a constant Mach number is contained in principle. It is stated that similarity requires that the sound velocities should change in the same ratio as the flow velocities. The work that contains theoretical as well as experimental investigations and that became of fundamental importance was done by Reynolds (1883). The discussion of the problem of stability in the change from laminar to turbulent flow was originated by Reynolds. He clearly recognized that the basic hydrodynamic properties depend only on the dimensionless quantity, later to bear his name,[1]

$$\mathrm{Re} = \frac{w\ell}{\nu}, \tag{1.1}$$

as suggested by Sommerfeld (1908). Another important quantity was introduced by Moritz Weber (1919) as the Froude number[2]:

$$\mathrm{Fr} = \frac{w^2}{g\ell}. \tag{1.2}$$

Froude in 1869 had been the first to determine the drag of ships by means of similarity laws. His investigations became of basic importance for shipbuilding. Furthermore, Eq. (1.2) is important for all flows influenced by gravity, and we shall return to this subject later.

From Eqs. (1.1) and (1.2) we can recognize that difficulties appear if two similarity laws must be satisfied simultaneously. If two fields are considered in which the respective Reynolds and Froude numbers have the same values, there follows

$$\mathrm{Re}\ \mathrm{Fr} = \frac{w^3}{g\nu} = \text{const.}$$

If the kinematic viscosities ν are also equal in both fields with g=const there results w=const as a condition of similarity. Such similarity implies identity since ℓ=const and consequently ordinarily Reynolds and Froude number cannot be satisfied simultaneously in the same model experiment. This fact can be avoided by taking a different ν for each flow. However, this causes difficulties in practice, as can easily be seen.

The ratio of the velocity w to the sound velocity a is a fundamental parameter for compressible flow. To honor Ernst Mach this ratio was called Mach number[3] by Ackeret in his habilitation thesis (1928) by

$$M = \frac{w}{a}. \tag{1.3}$$

Another dimensionless form is closely connected with this quantity and is often called the Euler number[4]

$$Eu = \frac{p}{\rho w^2}. \tag{1.4}$$

For a thermally perfect gas we have

$$a^2 = \gamma \frac{p}{\rho}, \tag{1.5}$$

and therefore the relation

$$Eu = \frac{1}{\gamma M^2}. \tag{1.6}$$

In gasdynamics Eq. (1.4) usually occurs in a slightly modified form. In the numerator an appropriate pressure difference appears, for instance, the pressure difference with respect to the free stream pressure, and in the denominator an additional factor 1/2 occurs. The result is the pressure coefficient

$$C_p = \frac{\Delta p}{\frac{1}{2}\rho w^2}. \tag{1.7}$$

von Kármán (1923) was the first to point out that there is an interesting relation between the Mach number and the Reynolds number. From the elementary kinetic theory of gases we have

$$\nu \sim a\Lambda, \tag{1.8}$$

where Λ is the mean free path of the molecules. There follows, accordingly, for the ratio of the Mach number to the Reynolds number

$$\frac{M}{Re} \sim \frac{\Lambda}{\ell} = Kn. \tag{1.9}$$

This ratio of the mean free path to a typical reference length ℓ of the flow field is called the Knudsen number.[5] This parameter plays an important part in modern fluid dynamics. The Mach number can therefore be interpreted physically as the product of the Reynolds number and the Knudsen number. We can see immediately that the concept of the continuum in the flow field is valid only if

$$M \ll Re \quad \text{or} \quad Kn \ll 1.$$

Aside from the dimensionless quantities considered above, there often occurs a similarity number that contains the surface tension σ. Moritz Weber (1919), in a fundamental study, was the first to deal with this parameter. He suggested an expression "Kapillar-Kenngrösse." Today it is generally called the Weber number[6]

$$We = \frac{\rho w^2 \ell}{\sigma}. \tag{1.10}$$

Weber has worked out many contributions to the theory of similarity. A comprehensive review of the whole subject by Weber (1930) should be specially mentioned.

Periodic flows are characterized by a regular vibration period, or a frequency. If these quantities are made dimensionless, there arises another number of similarity. In this case the nomenclature in the literature is again not uniform. Some authors speak of a reduced frequency while most other authors use the name Strouhal number,[7] because Strouhal (1878) has shown the significance of that number. We agree with the latter group and set, with t being the vibration period

$$Str = \frac{\ell}{wt}. \tag{1.11}$$

The above discussion is purposely limited to entirely mechanical quantities. Later, in the systematic treatment, we shall encounter a large number of additional similarity numbers where this restriction will be dropped. The most important historical dates will be reported at that time as far as possible.

At this point, a difficulty with the notation of the similarity numbers should be mentioned. Thus far we have listed Re, Fr, M, Eu, We, and Str. It would be reasonable to use as few letters as possible to describe each parameter unambiguously. However, this principle has not been followed in the literature and considerable confusion in notation is found. The author could not restrict himself to the use of two letters only, such as Re, Fr, and Ma. Confusion arises for "Ma" because "a" stands for the sound velocity in the English literature and, therefore, this letter should not be used in this connection. The use of three letters for the Strouhal number should prevent confusion with the Stokes number, Sto, and the Stanton number, Sta, which will be used later.

As stated, the similarity numbers enumerated and the resulting laws of mechanical similarity have been known for a long time. This is different in the important field of gasdynamics whose similarity laws become of basic importance in aeronautics. This subject is an essential extension of older considerations given by Prandtl (1922) and Glauert (1928) for linear subsonic and supersonic flows. These laws permit the calculation of the influence of compres-