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The Measurement of Turbulent Fluctuations

An Introduction to Hot-Wire Anemometry and Related Transducers

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Translated by S. Chomet, King's College London
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With 95 Figures

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Foreword

Smol'yakov and Tkachenko's book is a very thorough and detailed survey of the response of hot wires and related transducers to a fluctuating flow field. Now that the electronic equipment needed for hot-wire anemometry is so easy to make or cheap to buy, transducer response is the most critical part of the subject - except for the fragility of the sensing element , for which textbooks are no remedy! We hope that this book will be useful to all students and research workers concerned with the theory or practice of these devices or the interpretation of results.

Peter Bradshaw

Imperial College London

Preface

"The importance of experimental data and of experimentally established general properties is often underestimated in the study of turbulence.....The most direct path is to use experimentally established properties as the foundation upon which models explaining these properties can be constructed."

M. D. Millionshchikov

Turbulence belongs to a class of physical phenomena that are very frequently encountered in both nature and technology. It is the most common and also the most complicated form of motion of real liquids and gases. It is observed in the oceans, in the atmosphere, and in a very wide range of systems in engineering. The rational design of airplanes, rockets, ships, dams, hydroelectric plant, canals, turbines, ventilators, and many other technological systems must involve the consideration of turbulence. The study of turbulence is vital in relation to marine flows, weather and climate forcasting, wind distributions, and ecological problems associated with environmental pollution. The theory of stellar evolution and general cosmological theories must also take account of the properties of turbulent motion. It is thus clear that turbulence is indeed involved in a very wide range of phenomena, and this is essentially the reason why turbulence has been attracting the increasing attention of major teams of scientists and engineers.

At the same time, turbulence is an exceedingly complex phenomenon to investigate. As in the kinetic theory of gases, one cannot describe the motion of the individual elements of a medium that participate in its turbulent motion. Fortunately, this degree of detail is in fact unnecessary and turbulent motion is described as a statistical ensemble of random fields. However, the resulting equations of motion do not form a closed system, and cannot therefore be solved without the use of additional assumptions or of experimental data. This means that one has to use experiment as a direct source of information on turbulence or as a means of empirical accumulation of information necessary for the closure of the original equations.

The measurement of the statistical parameters of turbulence is not at all a simple matter, especially for novice experimenters. The difficulties flow largely from the following three circumstances Firstly, the experimenter must be at least reasonably familiar with the theory of random processes and fields so that he can appreciate the relative usefulness of the different statistical parameters and also the functional relationships between them. This is required if the parameters to be measured are to be chosen judiciously.

Secondly, turbulence is a system with an exceedingly large number of degrees of freedom. It usually involves a wide-band set of different components of motion and internal forces, including essential contributions due to small-scale and high-frequency components. This imposes stringent conditions on the measuring equipment in relation to its resolution in both space and time. The latter requirement is readily satisfied by modern electronics (if used correctly) whose response can be made very rapid indeed, but this point must always be borne in mind because older equipment may have inferior time resolution.

The situation is much less satisfactory in relation to spatial resolution. The necessary miniaturization of primary transducers (input devices) is often a difficult technological problem that is far from a satisfactory solution. Efficient spatial filtration, for which transducer miniaturization is a necessary but not a sufficient condition, is also a problem that remains unsolved.

The necessity for efficient spatial filtration of wavenumber components of the random turbulent fields is the origin of the third difficulty in the experimental investigation of small-scale turbulence. It can be formulated as a problem in the interpretation of the experimental data which are always distorted by the averaging effect of the transducers. This means that the form of the signal generated by a transducer is different from that of the turbulent fluctuations acting upon it. The problem of reconstruction of the true fluctuations from the recorded transducer signal belongs to the class of the so-called ill-posed problems, and can only be solved approximately. In fact, the **a priori** knowledge about the turbulence field that is necessary for the solution of the problem must be greater than the amount of information obtained as a result of our measurements. Unfortunately, the experimenter cannot readily break out of this vicious circle.

The aim of the present book is to present a systematic account of those aspects of the measurement of turbulent pulsations that relate to the above three difficulties. The book can therefore be divided into three parts that deal, in turn, with the following three questions: what to measure, how to measure, and how to interpret the results of measurement ? We shall concentrate on the last question which has previously attracted least attention but is fundamental to the measurement of turbulent fluctuations.

The first part of our book (Chapter 1) lays down the fundamental ideas of the statistical approach to the description of turbulence. It introduces concepts that are important to the experimenter, namely, those of correlation and spectral analysis, it explains the wavenumber-frequency representation of the random turbulent field, it describes the classification of the different measurable statistical parameters of turbulence, and it points out the relationships between these parameters and their relative information content. The presentation of this largely classical material is different from that which a competent reader will find in the excellent monographs by Batchelor [4], Hinze [47], and Monin and Yaglom [19]. We have tried to achieve maximum simplicity and clarity in the presentaion of the basic ideas of the theory of random fields without introducing, whenever possible, excessive mathematical complication. We considered that the experimenter facing the difficult task of measurement requires a clear appreciation of the essence of the relevant phenomena more than the full rigor of the analytical proof. In line with the modern practice, we have stressed the importance of multidimensional wavenumber-frequency spectra rather than the separate wavenumber and frequency spectra. Cross-spectra are therefore extensively used throughout the book, in contrast to previous monographs on turbulence.

The second part of the book (Chapter 2) presents the basic ideas underlying experimental methods of determining the different statistical parameters of turbulent fields of velocity, pressure, shear stress, impurity concentration, and certain other physical variables. Topics covered include the elements of the theory of dimensions and similarity, the principles of operation of specialized equipment, and the specific experimental uncertainties that characterize the measurement of turbulent fluctuations. Since the range of topics covered in this part of the book is very wide, the account is necessarily restricted to fundamentals. More detailed accounts will be found in the very extensive specialist literature that is now available and is cited throughout the text. Unfortunately, this literature takes the form of individual papers scattered over a large number of periodicals. There are only two monographs in which the technique of measurement of turbulent fluctuations is examined in detail. These are the books of Hinze [4] and Bradshaw [3], which are mainly concerned with hot-wire anemometry. We consider that a brief but thorough-going review of modern experimental methods and techniques, and of the associated experimental uncertainties, should fill a gap in the current literature on turbulence.

The third part of our book (Chapters 3, 4, and 5) is devoted to the very difficult and important problem of interpretation of experimental results that are distorted by the averaging effect of transducers of finite size. Here we encounter a specific experimental uncertainty that is very important and very difficult to avoid or take into account. It was first considered in relation to turbulence by Uberoi and Kovasznay [87] and was further examined by Corcos [64 - 66] and by Petrovskii [26]. However, many new results have been obtained (some by the present authors) since the publication of these papers. The principal method which is at present uniquely suited to the problem of extracting the true picture from experimental data (and thus escaping from the vicious circle mentioned above) is that based on the preliminary introduction of more or less likely models of the turbulent field, which are then used to find the required correction functions. These functions do not necessarily lead to complete success, but the extent to which they are inadequate can be exploited as the starting point for the subsequent improvement of the original models. This process is continuing, so that it is not surprising that we have not provided in this book any hard and fast recipes suitable for all cases. This approach follows the lines suggested by the above quotation from the paper by Academician D.M. Millionshchikov at the 1972 Moscow Symposium on Turbulence.

Statistical models of turbulent fields had to be included in this book. Models of turbulent pressure fluctuations, the measurement of which is subject to greater uncertainties than that of velocity fluctuations, are examined in some detail. In the case of the latter fluctuations, we have confined our attention to correction functions for isotropic turbulence. Experimental data (our own and those due to others) are cited only to the extent to which they help in explaining the principles involved in measurement.

It is, of course, impossible to review all aspects of turbulence measurement between the hard covers of a single book. For example, we have not been able to cover the measurements of magnetohydrodynamic turbulence, turbulence with clearly defined stratification of physical properties, and turbulence accompanied by chemical reactions. We have concentrated our attention on measurements of the most commonly employed, or the most highly distorted, parameters of incompressible liquids. In many cases, the ideas and methods discussed are of a more general significance and can be extended to other flows and parameters.

The book is designed for a wide range of specialists involved in the experimental investigation of turbulence with the view to finding new relationships or solving technological problems. It may also be found useful by researchers with theoretical interests, who need to develop models of turbulent fields on the basis of experimental data. It is probable that the book will be found most useful to engineers, researchers, and more advanced students facing for the first time the problem of measuring the statistical parameters of turbulence or of developing the apparatus and instrumentaion for such measurements.

Chapters 1 and 2, and also Sections 4.2 and 4.4, were written by A.V Smol'yakov, Chapters 3 and 4 (apart from the Sections just mentioned) and Sections 5.1 and 5.2 were written by V.M. Tkachenko, and Section 5.3 was written jointly.

The authors are greatly indebted to V.S. Petrovskii for his stimulating advice and for his unfailing interest throughout the writing of this book.

Contents

Chapter 1

Statistical Description of Turbulence

1.1 Turbulence as a Random Process

The most immediate impression of turbulent flow is, probably, that it is an exceedingly complicated, tortuous and chaotic phenomenon. The experimenter equipped with instruments capable of recording the parameters of a turbulent flow will soon conclude that all these parameters fluctuate in an irregular fashion.

Turbulent flows are very different from the smooth, laminar flow demonstrated by the classical experiments of Osborne Reynolds [80] in which a small filament of dye introduced into a laminar flow showed practically no lateral spreading. However, laminar flow is very unusual both in nature and in technology since it occurs only at very low velocities and on a very small spatial scale. Sutton [37] has remarked that "We might proceed by attempting to define laminar motion as the exceptional case and then say that other motions of liquids are turbulent." In this book, we shall confine our attention to turbulent flows.

The following property of turbulent flows will have to be borne in mind in the ensuing account: when an experiment is repeated, one cannot reproduce exactly all the details of the turbulent velocity field or other fluctuating variables, such as pressure, temperature, density, impurity concentration, and so on. For example, if we produce a turbulent flow in a tube connecting two reservoirs, we find that the instantaneous flow velocity at a given point in the interior of the tube at a given time (measured, for example, from the beginning of the experiment) is appreciably different when the experiment is repeated under seemingly identical conditions. We note particularly that the instantaneous values of turbulent fluctuations cannot be predicted for any point within the flow or for any instant of time.

The obvious chaotic character of turbulent motion and the unpredictability of its details do not, however, mean that turbulent fluctuations do not follow certain cause-and-effect regularities. On the contrary, the modern theory of turbulence takes as its starting point the assumption that all the details of the flow are completely determined by the equations of motion of a viscous fluid (the Navier-Stokes equations), i.e., it is assumed that, if one could find the exact solutions of the Navier-Stokes equations for each particular situation, and if all the data on the initial and boundary conditions were available for the flow, one could predict all the fluctuations in a turbulent flow at each point at any time.

However, the general solutions of the Navier-Stokes equations have not as yet been found. The boundary conditions are also partly unknown because they involve such uncontrollable parameters as weak, random variations in the position of the walls bounding the flow and small perturbations in density, pressure, and temperature of the medium, which are due to a variety of subsidiary and practically unavoidable factors. In other words, variations in the fluctuating parameters of turbulence obtained as a result of repeated and seemingly identical experiments are, in reality, due to uncontrollable variations in the boundary and initial conditions governing the flow. The unpredictability of turbulent fluctuations is simply the result of our ignorance of these conditions. Moreover, even if the boundary conditions were to be known, exact predictions would still be impossible because we are still unable to solve the complicated, nonlinear Navier-Stokes equations.

The actual situation is often even worse. Many types of turbulent flow are complicated by various additional factors that are not taken into account in the Navier-Stokes equations and, generally speaking, we do not have at our disposal a rigorous mathematical formulation of the problem. These factors include the effect of humidity, solar radiation, and other thermodynamic effects in problems involving atmospheric turbulence, departures from the continuity of the medium in technological processes due to the presence of solid, gel-like and, generally, foreign impurities, non-standard rheological behavior of liquids based on new materials, and so on. The Navier-Stokes equations must then be modified by introducing additional hypotheses, the validity of which may be in some doubt and their direct experimental verification not always possible.

Greater familiarity with the problem of turbulence shows, however, that it is not as hopeless as might appear at first sight although the associated theoretical and experimental difficulties are undoubtedly very considerable. Turbulence is commonly studied by statistical, probabilistic methods because of the random nature of turbulent fluctuations. When this approach is adopted, and it is the only possible approach at the present time, neither the theoretician nor the experimenter is interested in the instantaneous values of the fluctuating variables, which are not needed in practise. The achievable aim (which is also of practical interest) is to determine only the statistical parameters of turbulence.

Measurements will yield the average statistical parameters if we deliberately arrange for the measuring devices and transducers to react in a particular way to the action of the turbulent fluctuations. It is important to ensure that the measuring system and the underlying methods do, in fact, perform the desired function, i.e., that they are capable of yielding data on the average statistical variables of interest.

In practise, one often cannot avoid instrumental uncertainties, above all, those associated with measuring transducers, and uncertainties associated with the specific experimental conditions. Because of this, rational processing of experimental data for these random variables, and the interpretation of these data, are at least as important for the problem of turbulence as the measurements themselves. This is greatly assisted by performing measurements in parallel with an analytic approach to the problem, based on physical models on the one hand and the mathematical formalism of probability theory on the other.

We shall therefore begin our account with certain simple ideas and concepts in probability theory, which are used both in measurements on random turbulent fluctuations and in the processing and analysis of the results obtained from these measurements.

Probability distributions. The experimenter frequently has to deal with different turbulence parameters which, at a given instant of time and a fixed point in the flow, form a set of unpredictable and, in that sense, random quantities. These may include both scalar (temperature, pressure, and density) and vector (velocity and vorticity) variables.

In the statistical approach, the random variables are characterized by probability distributions which provide a quantitative measure of the probability of appearance of a random turbulent fluctuation in a particular range of values of a given variable.

Suppose, for example, that $p(\underline{x}, t)$ is the pressure at time t at the point $\underline{x}(x_1, x_2, x_3)$ in a liquid or gas undergoing turbulent motion (x_1, x_2, x_3 are the components of the vector $\underline{x}$ defining the position of the point of observation). Repeated measurements of this random variable in a large number of outwardly identical experiments will show that some values of $p(\underline{x}, t)$ are encountered more frequently than others. As soon as a sufficiently large number of such observations becomes available, we can construct the probability distribution $w(p)$ for the random variable p. An example of this distribution is shown in Fig. 1.1.

The dimensionality of the function w(p) is always the reciprocal of the dimensionality of the random variable itself, and the dimensionless number.

$$S(p_0,\ \Delta p) = \int_{p_0}^{p_0+\Delta p} w(p)\,dp \qquad (1.1)$$

is a measure of the fraction of all experiments in which the random variable p is found to lie between p_0 and $p_0 + \Delta p$.

If the interval p is small enough, the density w(p) may be assumed to be constant within this interval, and we have the approximate result

$$S(p_0,\ \Delta p) \simeq w(p_0)\Delta p$$

The "measure of the fraction" for all experiments that have been performed (whatever their outcome) is assumed to be equal to unity:

$$\int_{-\infty}^{\infty} w(p)\,dp = 1 \qquad (1.2)$$

which can be looked upon as the normalization condition for the function w(p). The condition given by (1.2) can also be looked upon as the trivial expression of the fact that each experiment must record *some* value of the random variable p (whatever it is). The turbulent pressure $p(\underline{x},\ t)$ and its distribution w(p) were, of course, chosen only as a simple example (we shall occasionally use it again in the ensuing account). All that we have said is equally valid for the distributions of other random turbulent fluctuations.

If, by virtue of some physical restrictions, the random variable can assume values only within a certain limited interval, the limits of the interval will also be the limits of integration in (1.2).

The infinite limits in (1.2) must not be interpreted as meaning that turbulent fluctuations can have infinite (positive or negative) values: such events have, of course, zero probability. Nevertheless, if the actual maximum possible values of the turbulent fluctuations are unknown, the infinite limits can be retained in (1.2). This does not lead to an error because regions with the zero probability do not contribute to the integral.

It was assumed in the above discussion that the random variable could assume any value within a finite or infinite interval, and that these values filled the interval in a continuous fashion. Such random variables are referred to as *continuous*, and turbulent fluctuations belong to this particular category. We shall not, therefore, consider the so-called discrete random variables which are encountered in other applications of probability theory and can assume only certain definite values within a given interval, i.e., do not fill it continuously.

The function $w(p)$ shown in Fig. 1.1 is often referred to as the probability density or the differential distribution. The latter designation is intended to emphasize the difference between this function and the integral distribution given by

$$F(p_0) = \int_{-\infty}^{p_0} w(p)\,dp \tag{1.3}$$

which gives the probability that the random variable does not exceed some given arbitrary level p .

It follows from (1.3) that $F(-\infty) = 0$ and $F(\infty) = 1$, where the latter relation is equivalent to the normalization condition (1.2) for the density $w(p)$.

The function $F(p_0)$ is called the integral distribution because of the form of (1.3), whereas the differential distribution is defined by the inverse of (1.3):

$$w(p) = \lim_{\Delta p \to 0} \frac{F(p + \Delta p) - F(p)}{\Delta p} = \frac{dF(p)}{dp}$$

A random variable is considered fully determined in the statistical sense if its differential or integral distribution is known.

It is quite obvious that the correct determination of the distribution of a random variable can only be based on observations resulting from a sufficiently large number of experiments (realizations). In fact, the theory demands an infinitely large ensemble of realizations. On the other hand, the experimenter has no option but to accept a finite set of experiments. However, the number of experiments must not be less than a certain admissible limit if one is to achieve the required precision in the determination of the distribution law and to preserve the information content of the data. We shall return to this point in Section 2.7 in connection with measurements of the statistical parameters of turbulence.

1.2 Statistical Averages of Random Variables

Distribution moments. The most common object of experimental and theoretical study of turbulence is not the probability distribution itself but its numerical characteristics, known as the *distribution moments*. These are defined by

$$m_k(p) = \overline{p^k(\underline{x},\ t)} = \int_{-\infty}^{\infty} p^k w(p)\,dp \qquad k = 1,\ 2,\ \ldots,\ \infty \tag{1.4}$$

and are determinate, nonrandom numbers.

Determination of the complete set ($k = 1, 2, \ldots, \infty$) of the distribution moments is equivalent, from the point of view of information content, to the determination of the differential or integral probability distributions.

The integration of a certain power k of the random variable $p(\underline{x}, t)$ weighted by the probability density $w(p)$ in (1.4) may be looked upon as a probabilistic or statistical operation yielding the average of the random variable $p^k(\underline{x}, t)$. This is indicated by the second designation of the distribution moment in (1.4), in which the bar above the variable is a more compact representation of statistical averaging than the integral sign. The notation $\langle p^k(\underline{x}, t)\rangle$ is often used instead of $\overline{p^k(\underline{x}, t)}$.

It is readily seen that the first-order moment ($k = 1$) or, more briefly, the first moment, is given by

$$m_1(p) = \langle p(\underline{x},\ t)\rangle = \int_{-\infty}^{\infty} pw(p)\,dp \tag{1.5}$$

and is simply the average hydrodynamic pressure at a given space-time point $(\underline{x}, t)$ in the turbulent flow. The first moment of a random variable is often referred to as its mathematical expectation. The mathematical expectation can be represented geometrically by the abscissa of the center of gravity of the area under the curve representing the probability density (Fig. 1.1). This can readily be verified by inspection of (1.5).

If the random variable is taken to be the fluctuating velocity of a turbulent flow, the first moment is equal to the average velocity.

Early work on turbulent flows was concerned with the first moments or the mathematical expectations of velocity, pressure, frictional stress and certain other random parameters of turbulence. The second, third,

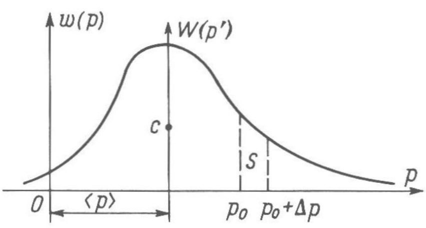

Figure 1.1
Example of the probability density of a random turbulent fluctuation: c is the "center of gravity" of the area under the curve

and higher-order moments subsequently became accessible to measurement and theoretical analysis. The current tradition is that only such work is part of statistical research into turbulence. This, however, is not entirely correct because studies of the mathematical expectations of turbulent fluctuations are also statistical studies of the simplest characteristics. On the other hand, we have already noted that the statistical approach is the only possible one in the study of any parameters of random turbulent fields.

When turbulence and many other random processes are investigated, it is common practise to resolve the corresponding random variable into two components, namely, the nonrandom, determined mathematical expectation and the random fluctuation about this mathematical expectation:

$$p = m_1(p) + p' \quad \text{or} \quad p = \langle p \rangle + p'$$

i.e., the random variable is now taken to be not the total value of p but only the fluctuation $p' = p - \langle p \rangle$ about the average value $\langle p \rangle = m_1(p)$.

It is clear that the first moment of p' will be zero, by definition, and the center of gravity of the area under the probability density curve will lie on the ordinate axis (Fig. 1.1). In this sense, the random variable p' may be referred to as centered, and its distribution function $W(p')$ and its moment $M_k(p')$ as the central distribution and moments, respectively. We shall largely be concerned with centered variables and will omit the prime for the sake of simplicity.

The second central moment, or the *variance*, of turbulent pressure fluctuations, i.e.,

$$M_2 = \langle p^2 \rangle = \int_{-\infty}^{\infty} p^2 W(p)\,dp$$

is a measure of their intensity. The variance of turbulent velocity fluctuations has the dimensions of the square of velocity, and is proportional to the kinetic energy of the fluctuating part of the turbulent motion.

The square root of the variance, or the *standard deviation* of the random variable from the average,

$$\sigma = [M_2(p)]^{1/2}$$

can be represented geometrically by a certain effective width of the region under the probability density curve (Fig. 1.1). The greater the standard deviation, the more uncertain is the random value of the given fluctuation, and vice versa. As $\sigma \to 0$, the random fluctuations vanish