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10th International IPCO Conference
New York, NY, USA, June 2004
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Preface

This volume contains the papers accepted for publication at IPCO X, the Tenth International Conference on Integer Programming and Combinatorial Optimization, held in New York City, New York, USA, June 7–11, 2004. The IPCO series of conferences presents recent results in theory, computation and applications of integer programming and combinatorial optimization.

These conferences are sponsored by the Mathematical Programming Society, and are held in those years in which no International Symposium on Mathematical Programming takes place. IPCO VIII was held in Utrecht (The Netherlands) and IPCO IX was held in Cambridge (USA).

A total of 109 abstracts, mostly of very high quality, were submitted. The Program Committee accepted 32, in order to meet the goal of having three days of talks with no parallel sessions. Thus, many excellent abstracts could not be accepted.

The papers in this volume have not been refereed. It is expected that revised versions of the accepted papers will be submitted to standard scientific journals for publication.

The Program Committee thanks all authors of submitted manuscripts for their support of IPCO.

March 2004

George Nemhauser
Daniel Bienstock

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Robust Branch-and-Cut-and-Price for the Capacitated Vehicle Routing Problem

Ricardo Fukasawa[1], Jens Lysgaard[2], Marcus Poggi de Aragão[3], Marcelo Reis[3], Eduardo Uchoa[4]*, and Renato F. Werneck[5]

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Abstract. The best exact algorithms for the Capacitated Vehicle Routing Problem (CVRP) have been based on either branch-and-cut or Lagrangean relaxation/column generation. This paper presents an algorithm that combines both approaches: it works over the intersection of two polytopes, one associated with a traditional Lagrangean relaxation over q-routes, the other defined by bound, degree and capacity constraints. This is equivalent to a linear program with exponentially many variables and constraints that can lead to lower bounds that are superior to those given by previous methods. The resulting branch-and-cut-and-price algorithm can solve to optimality all instances from the literature with up to 135 vertices. This doubles the size of the instances that can be consistently solved.

1 Introduction

Let $G = (V, E)$ be an undirected graph with vertices $V = \{0, 1, \ldots, n\}$. Vertex 0 represents the *depot*, whereas all others represent *clients*, each with an associated demand $d(\cdot)$. Each edge $e \in E$ has a nonnegative length $\ell(e)$. Given G and two positive integers (K and C), the *Capacitated Vehicle Routing Problem* (CVRP) consists of finding routes for K vehicles satisfying the following constraints: (i) each route starts and ends at the depot, (ii) each client is visited by a single vehicle, and (iii) the total demand of all clients in any route is at most C. The goal is to minimize the sum of the lengths of all routes. This classical NP-hard problem is a natural generalization of the Travelling Salesman Problem (TSP),

* Corresponding author.

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and has widespread application itself. The CVRP was first proposed in 1959 by Dantzig and Ramser [13] and has received close attention from the optimization community since then.

A landmark exact algorithm for the CVRP, presented in 1981 by Christofides, Mingozzi and Toth [11], uses a Lagrangean bound from minimum q-route subproblems. A *q-route* is a walk that starts at the depot, traverses a sequence of clients with total demand at most C, and returns to the depot. Some clients may be visited more than once, so the set of valid CVRP routes is strictly contained in the set of q-routes. The resulting branch-and-bound algorithm could solve instances with up to 25 vertices, a respectful size at the time.

Several other algorithms using Lagrangean relaxation appear in the literature. Christofides et al. [11] also describe a lower bound based on k-degree center trees, which are minimum spanning trees having degree $K \leq k \leq 2K$ on the depot, plus $2K - k$ least-cost edges. Lagrangean bounds based on K-trees (sets of $n + K - 1$ edges spanning V) having degree $2K$ in the depot were used by Fisher [14] and by Martinhon, Lucena, and Maculan [24], among others. Miller [25] presented an algorithm based on minimum b-matchings having degree $2K$ at the depot and 2 on the remaining vertices. Lagrangean bounds can be improved by dualizing capacity inequalities [14,25] and also comb and multistar inequalities [24].

Another family of exact algorithms stems from the formulation of the CVRP as a set partitioning problem by Balinski and Quandt [8]. A column covers a set of vertices S with total demand not exceeding C and has the cost of a minimum route over $\{0\} \cup S$. Unfortunately, the formulation is not practical because pricing over the exponential number of columns requires the solution of capacitated prize-collecting TSPs, a problem almost as difficult as the CVRP itself. Agarwal, Marthur and Salkin [7] proposed a column generation algorithm on a modified set partitioning problem where column costs are given by a linear function over the vertices yielding a lower bound on the actual route cost. Columns with the modified cost can be priced by solving easy knapsack problems. Hadjiconstantinou et al. [17] derive lower bounds from heuristic solutions to the dual of the set partitioning formulation. The dual solutions are obtained by the so-called additive approach, combining the q-route and k-shortest path relaxations.

For further information and comparative results on the algorithms mentioned above, we refer the reader to the surveys by Toth and Vigo [31,32].

Recent research on the CVRP has been concentrated on the polyhedral description of the convex hull of the edge incidence vectors that correspond to K feasible routes and on the development of effective separation procedures [1,3,5,6,12,20,26]. In particular, Araque et al. [4], Augerat et al. [6], Blasum and Hochstättler [9], Ralphs et al. [30], Achuthan, Caccetta, and Hill [2] and Lysgaard, Letchford, and Eglese [23] describe complete branch-and-cut (BC) algorithms. These are the best exact methods currently available for the CVRP. However, the addition of several elaborate classes of cuts does not guarantee tight lower bounds, especially for large values of K ($K \geq 7$, say). Closing the resulting duality gap usually requires exploring several nodes in the branch-and-cut tree.

Even resorting to massive computational power (up to 80 processors running in parallel in a recent work by Ralphs [29,30]) several instances with fewer than 80 vertices, including some proposed more than 30 years ago by Christofides and Eilon [10], can not be solved at all. In fact, branch-and-cut algorithms for the CVRP seem to be experiencing a "diminishing returns" stage, where substantial theoretical and implementation efforts achieve practical results that are only marginally better than those of previous works.

We present a new exact algorithm for the CVRP that seems to break through this situation. The main idea is to combine the branch-and-cut approach with the q-routes approach (which we interpret as column generation instead of the original Lagrangean relaxation) to derive superior lower bounds. Since the resulting formulation has an exponential number of both columns and rows, this leads to a branch-and-cut-and-price (BCP) algorithm. Computational experiments over the main instances from the literature show that this algorithm is very consistent on solving instances with up to 100 vertices. Eighteen open instances were solved for the first time.

The idea of combining column and cut generation to improve lower bounds has rarely been used, since new dual variables corresponding to separated cuts may have the undesirable effect of changing the structure of the pricing subproblem. However, if cuts are expressed in terms of variables from a suitable original formulation, they can be incorporated into the column generation process without disturbing the pricing. We refer to branch-and-bound procedures based on such formulations as *robust branch-and-cut-and-price* algorithms. Poggi de Aragão and Uchoa [28] present a detailed discussion on this subject, including new reformulation techniques that extend the applicability of robust branch-and-cut-and-price algorithms to virtually any combinatorial optimization problem. This article on the CVRP is part of a larger effort to demonstrate that these methods lead to significant improvements on a wide variety of problems. Major advances have already been reported on two other problems: capacitated minimum spanning tree [15] and generalized assignment [27].

This article is organized as follows. Section 2 describes the integer programming formulation we will deal with. Section 3 gives a general description of our algorithm, including its two main components: column and cut generation. Following the work of Irnich and Villeneuve [18] on the CVRP with time windows, our column generation procedure eliminates q-routes with small cycles. The separation routines are based on the families of inequalities recently discussed by Letchford, Eglese, and Lysgaard [20,23]. Section 4 presents an empirical analysis of our method. Final remarks are made in Sect. 5.

2 The New Formulation

A classical formulation for the CVRP [19] represents by x_{ij} the number of times a vehicle traverses the edge $(i, j) \in E$. The set of client vertices is denoted by $V_+ = \{1, \ldots, n\}$. Given a set $S \subseteq V_+$, let $d(S)$ be the sum of the demands of all vertices in S, and let $\delta(S)$ be the cut-set defined by S. Also, let $k(S) = \lceil d(S)/C \rceil$. Define the following polytope in $\mathbb{R}^{|E|}$:

$$P_1 = \begin{cases} \sum_{e \in \delta(\{i\})} x_e = 2 & \forall i \in V_+ & (1) \\ \sum_{e \in \delta(\{0\})} x_e = 2 \cdot K & & (2) \\ \sum_{e \in \delta(S)} x_e \geq 2 \cdot k(S) & \forall S \subseteq V_+ & (3) \\ x_e \leq 1 & \forall e \in E \setminus \delta(\{0\}) & (4) \\ x_e \geq 0 & \forall e \in E \ . \end{cases}$$

Constraints (1) state that each client is visited once by some vehicle, whereas (2) states that K vehicles must leave and enter the depot. Constraints (3) are rounded capacity inequalities, which require all subsets to be served by enough vehicles. Constraints (4) enforce that each edge not adjacent to the depot is traversed at most once (edges adjacent to the depot can be used twice when a route serves only one client). The integer vectors x in P_1 define all feasible solutions for the CVRP. There are exponentially many inequalities of type (3), so the lower bound given by

$$L_1 = \min_{x \in P_1} \sum_{e \in E} \ell_e x_e$$

must be computed by a cutting plane algorithm.

Alternatively, a formulation with an exponential number of columns can be obtained by defining variables (columns) that correspond to q-routes without 2-cycles (subpaths $i \to j \to i$, $i \neq 0$). Restricting the q-routes to those without such cycles improves the formulation and does not change the complexity of the pricing [11]. Let Q be an $m \times p$ matrix where the columns are the edge incidence vectors of all p such q-routes. Let q_j^e be the coefficient associated with edge e in the j-th column of Q. Consider the following polytope in $\mathbb{R}^{|E|}$, defined as the projection of a polytope in $\mathbb{R}^{p+|E|}$:

$$P_2 = \text{proj}_x \begin{cases} \sum_{j=1}^{p} q_j^e \cdot \lambda_j - x_e = 0 \quad \forall e \in E & (5) \\ \sum_{j=1}^{p} \lambda_j = K & (6) \\ \sum_{e \in \delta(\{i\})} x_e = 2 \quad \forall i \in V_+ & (1) \\ x_e \geq 0 \quad \forall e \in E & \\ \lambda_j \geq 0 \quad \forall j \in \{1, \ldots, p\} \ . & \end{cases}$$

Constraints (5) define the coupling between variables x and λ. Constraint (6) defines the number of vehicles to use. It can be shown that the set of integer vectors in P_2 also defines all feasible solutions for the CVRP. Due to the exponential number of variables λ, the lower bound given by

$$L_2 = \min_{x \in P_2} \sum_{e \in E} \ell_e x_e$$

must be computed using column generation or Lagrangean relaxation.

The description of polyhedra associated with column generation or Lagrangean relaxation in terms of two sets of variables, λ and x, used in the definition of P_2, is called *Explicit Master* in [28]. The main contribution of this article is a formulation that amounts to optimizing over the intersection of polytopes P_1 and P_2. The Explicit Master format describes such intersection as follows:

$$P_3 = P_1 \cap P_2 = \text{proj}_x \begin{cases} \sum_{e\in\delta(\{i\})} x_e = 2 & \forall i \in V_+ & (1) \\ \sum_{e\in\delta(\{0\})} x_e = 2 \cdot K & & (2) \\ \sum_{e\in\delta(S)} x_e \geq 2 \cdot k(S) & \forall S \subseteq V_+ & (3) \\ x_e \leq 1 & \forall e \in E \setminus \delta(\{0\}) & (4) \\ \sum_{j=1}^{p} q_j^e \cdot \lambda_j - x_e = 0 & \forall e \in E & (5) \\ \sum_{j=1}^{p} \lambda_j = K & & (6) \\ x_e \geq 0 & \forall e \in E & \\ \lambda_j \geq 0 & \forall j \in \{1, \ldots, p\} \ . & \end{cases}$$

Constraint (6) can be discarded, since it is implied by (2) and (5). Computing the improved lower bound

$$L_3 = \min_{x \in P_3} \sum_{e\in E} \ell_e x_e$$

requires solving a linear program with an exponential number of both variables and constraints. A more compact LP is obtained if every occurrence x_e in (1)–(4) is replaced by its equivalent given by (5). The resulting LP will be referred to as the *Dantzig-Wolfe Master problem* (DWM):

$$\text{DWM} = \begin{cases} L_3 = \min \sum_{j=1}^{p} \sum_{e\in E} \ell_e \cdot q_j^e \cdot \lambda_j & & (7) \\ \text{s.t.} \quad \sum_{j=1}^{p} \sum_{e\in\delta(\{i\})} q_j^e \cdot \lambda_j = 2 & \forall i \in V_+ & (8) \\ \sum_{j=1}^{p} \sum_{e\in\delta(\{0\})} q_j^e \cdot \lambda_j = 2 \cdot K & & (9) \\ \sum_{j=1}^{p} \sum_{e\in\delta(S)} q_j^e \cdot \lambda_j \geq 2 \cdot k(S) & \forall S \subseteq V_+ & (10) \\ \sum_{j=1}^{p} q_j^e \cdot \lambda_j \leq 1 & \forall e \in E \setminus \delta(\{0\}) & (11) \\ \lambda_j \geq 0 & \forall j \in \{1, \ldots, p\} \ . & \end{cases}$$

Capacity inequalities are not the only ones that can appear in the DWM. A generic cut $\sum_{e\in E} a_e x_e \geq b$ can be included as $\sum_{j=1}^{p}(\sum_{e\in E} a_e q_j^e) \cdot \lambda_j \geq b$. In fact, we added all classes of cuts described in [23].