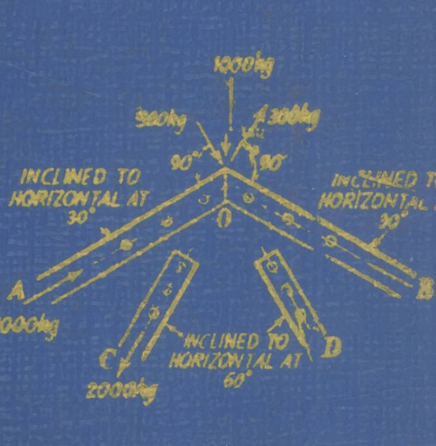
A TEXT BOOK OF APPLIED MECHANICS

I. B. PRASAD



KHANNA PUBLISHERS, DELHI

A TEXTBOOK OF

APPLIED MECHANICS

For Engineering Students of Degree and Diploma Classes

By

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"*A Text-Book of Strength of Materials*"

"*Applied Mech. and S.O.M.*"

1980

KHANNA PUBLISHERS

2-B, NATH MARKET, NAI SARAK

DELHI-110006.

Phone : **26 23 80**

Published by :
Romesh Chander Khanna
KHANNA PUBLISHERS,
DELHI-110006 (India).

First Edition 1965
Second Edition 1967
Third Edition 1969
Fourth Edition 1972
Fifth Edition 1974
Sixth Edition 1976
Seventh Edition 1978
Eighth Edition 1980

Price : **Rs. 28·00**

Printed at :
Sharma Printers,
Paharganj, New Delhi.

PREFACE TO THE EIGHTH EDITION

The book has been brought upto-date by including the latest questions set at the various engineering examinations. Many new problems on simple harmonic motion and other topics have also been added.

15th February, 1980 *I.B. PRASAD*

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Statics of a Particle

1·1. Force and Its Characteristics

Force is that which produces or tends to produce change in the state of rest or of uniform motion in a straight line, of a body.

Let a horizontal force P be applied to a body placed on a rough horizontal plane. When P is small, the body does not move. When P is increased, the body will start moving in a straight line if the line of action of P passes through the centre of gravity (c.g.) of the body ; there will be motion of translation as well as of rotation if the line of action of P does not pass through the c.g. of the body.

Thus we see that the effect of a force depends on three characteristics—(1) magnitude, (2) direction, (3) position or line of action. The complete effect of a force can be found only if we know all these three characteristics.

If we draw a straight line parallel to the line of action of the force, whose length is proportional to the magnitude of the force, the line is said to represent the force in *magnitude and direction*. Thus let the force P be 15 kilograms acting in the north-east direction. Let 1 cm length represent 5 kilograms. Then a straight line AB of length 3 cm drawn in the north-east direction will represent the force P in direction and magnitude. An arrow is placed on the line with the arrow-head pointing north-east to show the sense of the force, *i.e.* the force is acting from A towards B. The force represented by the line AB is written as $\overrightarrow{AB}$.

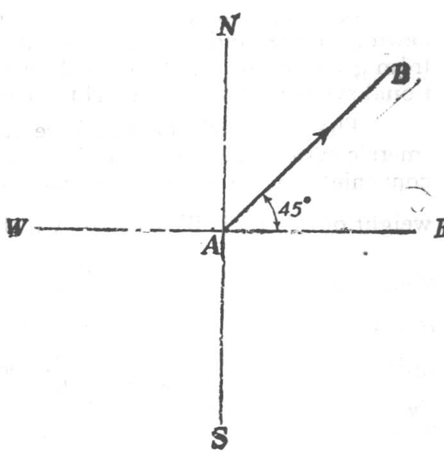

Fig. 1·1

If the line AB is drawn through the point at which P acts, then AB is said to represent P in ***direction***, ***magnitude*** and ***position***, or, in short, AB represents P completely.

Any quantity which possesses magnitude as well as direction is called a *vector quantity*. Some examples of vector quantities are force, velocity and acceleration. Any vector quantity can be represented by means of a straight line which is called a *vector*. Thus the force P is a vector quantity which is represented by the vector $\overrightarrow{AB}$.

1·2. Units of Mass and Force

In the *mathematical* or *academic system*, the unit of mass is the kilogram (metre kilogram-second unit), or the gram (centimetre-gramme-second unit) or the pound (foot-pound-second unit). The weight of a body is the earth's attraction on it and is equal to the product of its mass and acceleration due to gravity (g). If m be the mass of a body, its weight is mg. If m is measured in kilograms (kg) and g in metres per second per second, then the weight mg is in **newtons** ; if m is measured in grammes (gm) and g in centimetres per second per second, then the weight mg is in **dynes** ; if m is measured in pounds (lb) and g in feet per second per second, then the weight mg is in **poundals**. Thus the unit of weight or force is newton (M.K.S. system) or dyne (C.G.S. system) or poundal (F.P.S. system).

Now a force of mg newtons is equal to the weight of a body whose mass is m kg ; hence a force of mg newtons is also written as a force of m kg-weight. Similarly a force of mg dynes is equal to m gm-weight and a force of mg poundals is equal to m lb weight. A force of m kg-weight is also written as m kg_f.

The value of g is approximately 981 cm/sec² or 9·81 metres/sec² or 32·2 ft/sec².

kg wt., gm wt., lb wt. are called gravitational units, while newton, dyne and poundal are called absolute units. To convert from gravitational unit to absolute unit, multiply by g. In gravitational system, mass and weight are equal numerically.

The *Engineer's units* of force or weight are the *kilogram weight* (metric system) and the *pound weight* (British system), which are for convenience written as *kilogram* and *pound* respectively. If the weight of a body is W, its mass will be $\frac{W}{g}$. If W is measured in kilograms, the unit of mass, $\frac{W}{g}$, is $\frac{\text{kg}}{\text{metres/sec}^2}$ or $\frac{\text{kg}}{\text{metres}}\,\text{sec}^2$ which is also known as a *slug* (kg) unit. If W is measured in pounds the unit of mass, $\frac{W}{g}$, is $\frac{\text{lb}}{\text{ft/sec}^2}$ or $\frac{\text{lb}}{\text{ft}}\,\text{sec}^2$ which is also known as a *slug* (lb) unit. The Engineer's units have not found favour with Engineers and Scientists and hence all problems is this book have been solved in mathematical units.

1·3. Resultant and Components

If the combined effect of several forces $P_1, P_2, P_3 \ldots$ acting on a body is the same as that of a single force R, then R is called the *resultant* of P_1, P_2, P_3 ..., and the forces P_1, P_2, P_3...... are called the *components* of R.

1·4. Law of Parallelogram of Forces

The resultant of two forces acting at a point can be found by the application of the law of parallelogram of forces, which is stated below.

If two forces, acting at a point O, be represented in direction and magnitude by straight lines OA and OB, and the parallelogram OACB be completed, then their resultant acts through O and is represented in magnitude and direction by the diagonal OC of the parallelogram which passes through O.

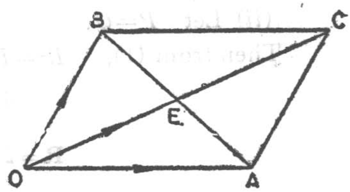

Fig. 1·2

Cor. Let the diagonals of the parallelogram intersect at E. Then E is the middle point of each AB and OC.

The resultant of forces $\vec{OA}$ and $\vec{OB} = \vec{OC} = 2\vec{OE}$.

Hence the resultant of forces represented in direction and magnitude by $\vec{OA}$ *and* $\vec{OB}$ *is represented by* $2\vec{OE}$, *where E is the middle point of AB.*

1·5. Resultant of Two Forces Acting at a Point

Let two forces P and Q, acting at O, be represented in direction and magnitude by the sides OA and OB respectively of the parallelogram $OACB$. Then OC represents their resultant R.

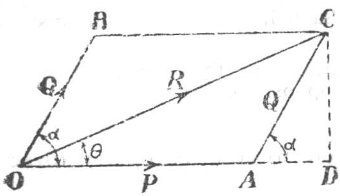

Fig. 1·3

Let $\angle AOB = \alpha$, $\angle AOC = \theta$.

Draw CD perpendicular to OA.

Since AC is equal and parallel to OB, we get $AC = Q$.

Also $\angle CAD = \angle AOB = \alpha$.

$$\therefore \quad AD = Q \cos\alpha, \qquad DC = Q \sin\alpha$$

$$OC^2 = OD^2 + DC^2 = (OA + AD)^2 + DC^2$$

$$R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2 + 2PQ\cos\alpha. \qquad ...(1)$$

Equation (1) gives the magnitude of R.

$$\tan\theta = \frac{DC}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha} \qquad ...(2)$$

Equation (2) gives the direction of R.

Particular Cases

(*i*) Let $\alpha=90°$, *i.e.* let the forces act at right angles. Then parallelogram $OACB$ becomes a rectangle.

From (1) and (2), or directly,

$$R^2=P^2+Q^2,$$

$$\tan\theta=\frac{Q}{P}.$$

(*ii*) Let $P=Q$.

Then from (1), $R^2=P^2+P^2+2P^2\cos\alpha=2P^2(1+\cos\alpha)$

$$=4P^2\cos^2\frac{\alpha}{2}$$

$$\mathbf{R=2P\cos\frac{\alpha}{2}}$$

From (2), $\tan\theta=\dfrac{P\sin\alpha}{P+P\cos\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}$

$$=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}=\tan\frac{\alpha}{2}$$

$$\therefore \qquad \theta=\frac{\alpha}{2}.$$

i.e. the resultant bisects the angle between the forces, a result which is quite obvious also from first principles.

Note. It is easy to see that the greatest resultant of two forces P and Q is $P+Q$ when the two forces act in the same line and same sense and that the least resultant is $P-Q$ when they have the same line of action and opposite senses.

If two forces acting at a point are in equilibrium, they must be equal in magnitude, have the same line of action and opposite senses.

1·6. Resolution of a Force

Finding the components of a given force in two given directions is called *resolution*.

Let the given force be R, and let it be required to find its components in directions making angles α and β with its line of action.

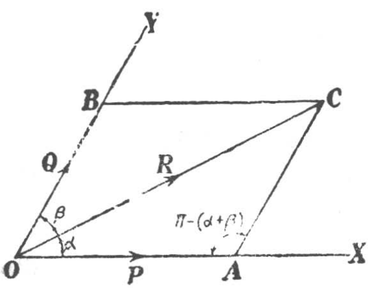

Fig. 1·4

Let OC represent R in magnitude and direction and let the lines OX and OY make angles α and β respectively with OC. Through C, draw CA parallel to OY, meeting OX at A, and CB parallel to OX, meeting OY at B. Then OA and OB represent the components of R along OX and OY respectively.

Let $\overrightarrow{OA}=P$, $\overrightarrow{OB}=Q$.

$\angle OCA = \angle BOC$ (alternate angles)

$= \beta$

$\therefore \quad \angle OAC = \pi - (\alpha + \beta)$

In $\triangle OAC$, by trigonometry,

$$\frac{OA}{\sin \beta} = \frac{AC}{\sin \alpha} = \frac{OC}{\sin [\pi - (\alpha+\beta)]}$$

$$\therefore \quad \frac{P}{\sin \beta} = \frac{Q}{\sin \alpha} = \frac{R}{\sin (\alpha+\beta)}$$

for AC is equal to OB which is proportional to Q.

Hence $$\mathbf{P} = \frac{\mathbf{R} \sin \beta}{\sin (\alpha+\beta)}$$

$$\mathbf{Q} = \frac{\mathbf{R} \sin \alpha}{\sin (\alpha+\beta)}.$$

Particular case. Let OX and OY be at right angles. Then $OACB$ becomes a rectangle and

$\alpha + \beta = 90°$

or $\beta = 90° - \alpha$

$\therefore \quad \frac{P}{R} = \cos \alpha$

or $\mathbf{P} = \mathbf{R} \cos \alpha$

$\frac{Q}{R} = \cos (90° - \alpha)$

$= \sin \alpha$

or $\mathbf{Q} = \mathbf{R} \sin \alpha$.

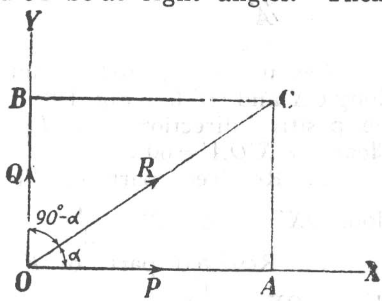

Fig. 1·5

When the components P and Q are at right angles, they are called the **resolved parts** of R.

We see that the resolved part of R in a direction inclined at angle α to R (*i.e.* along OX)

$$= P = R \cos \alpha.$$

This result is important. *To find the resolved part of a force in a given direction, multiply the force with the cosine of the angle between the line of action of the force and the given direction.*

In the application of this rule, care must be exercised in the measurement of the angle between the line of action of the force and the given direction. Let R be the force and $X'X$ the given direction. Let O be their point of intersection. *The positive direction of R is that in which the arrow-head points away from O.* If it is required to find the resolved part of R along OX, then multiply R with cos XOA, where OA is the positive direction of R. Let $\angle XOA = \alpha$. Then resolved part of R along $OX = R \cos \alpha$.

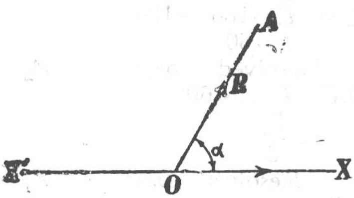

Fig. 1·6

Clearly, angle $X'OA=180°-\alpha$. Hence the resolved part of R along $OX'=R\cos(180°-\alpha)=-R\cos\alpha$. We can also find the resolved part of R along OX' by first finding the resolved part of R along OX and then reversing its sign. Thus the resolved part of R along OX is $R\cos\alpha$; reversing the sign, we see that the resolved part of R along OX' is $-R\cos\alpha$. This method is often convenient, since the use of obtuse angles is avoided.

Consider the case shown in Fig 1·7.

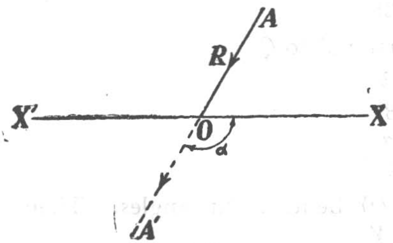

Fig. 1·7

The angle between R and OX is **not** XOA. Produce AO to A'; *then OA' is the positive direction of R* and the angle between R and OX is XOA'. Hence the resolved part of R along OX is $R\cos XOA'=R\cos\alpha$, and the resolved part of R along OX' is $R\cos(180°-\alpha)=-R\cos\alpha$.

Let it be required to find the resolved parts of a force F_1 along OX and OY (*see* Fig. 1·8). Produce AO to A'. Then OA' is the positive direction of F_1.

Clearly $\angle X'OA'=60°$.

∴ Resolved part of F_1 along $OX'=F_1\cos 60=\frac{F_1}{2}$.

∴ Resolved part of F_1 along $OX=-\frac{F_1}{2}$.

$\angle A'OY'=30°$.

∴ Resolved part of F_1 along $OY'=F_1\cos 30$

$=\frac{F_1\sqrt{3}}{2}$.

∴ Resolved part of F_1 along $OY=-\frac{F_1\sqrt{3}}{2}$.

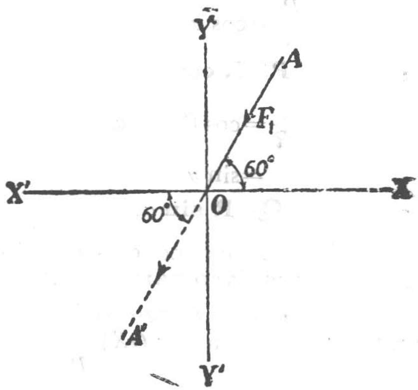

Fig. 1·8

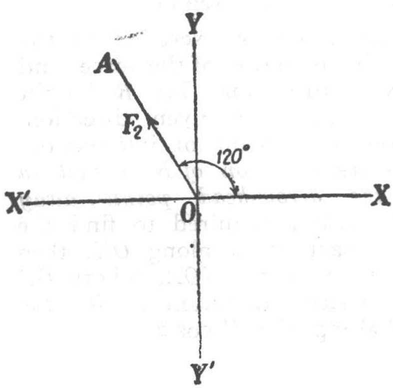

Fig. 1·9

Next, let us find the resolved parts of F_2 along OX and OY (*see* Fig. 1·9).

$\angle AOX'=180°-120°$.
$=60°$

∴ Resolved part of F_2 along $OX'=F_2\cos 60°$

$=\frac{F_2}{2}$

∴ Resolved part of F_2 along OX

$=-\frac{F_2}{2}$

(which is also equal to $F_2\cos 120°$)

$\angle AOY=120°-90°=30°$

∴ Resolved part of F_2 along OY

$$=F_2 \cos 30°$$

$$=\frac{F_2\sqrt{3}}{2}$$

(which is also equal to $F_2 \sin 120°$).

Similarly, resolved part of F_3 along OX (*see* Fig 1·10)

$$=\frac{F_3}{2}$$

and revolved part of F_3 along OY

$$=-\frac{F_3\sqrt{3}}{2}.$$

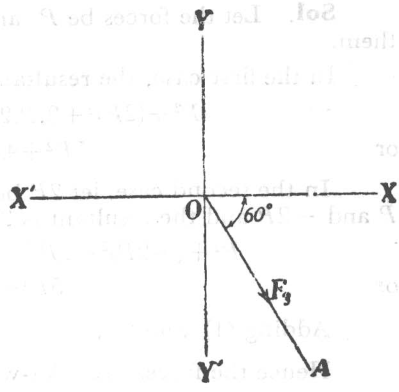

Fig. 1·10

From the above discussion, we derive the following rule which may be found helpful.

If the positive direction of a force F makes with a line OX an angle θ, and OY is perpendicular to OX, the angles being measured in the same sense, then the resolved parts of F along OX and OY are respectively F cos θ and F sin θ.

Cor. The resolved part of a force F in its own direction is F, and the resolved part in a perpendicular direction is zero.

Ex. 1. *Find the magnitude of two forces such that, if they act at right angles, their resultant is $\sqrt{10}$ kgf, whilst when they act at an angle of 60°, their resultant is $\sqrt{13}$ kgf.*

Sol. Let the forces be P and Q.

When the forces are at right angles, their resultant is $\sqrt{10}$ kgf.

$$\therefore \quad P^2+Q^2=10 \quad \ldots(1)$$

When the forces act at 60°, their resultant is $\sqrt{13}$ kgf.

$$P^2+Q^2+2PQ\cos 60°=13$$

i.e. $$P^2+Q^2+PQ=13 \quad \ldots(2)$$

From (1) and (2), by subtraction,

$$\therefore \quad PQ=3 \quad \ldots(3)$$

Multiply (3) by 2 and add to (1), we get

$$P^2+Q^2+2PQ=16$$

$$P+Q=4 \quad \ldots(4)$$

Solving (3) and (4), $P=$**3 kgf**, $Q=$**1 kgf.**

Ex. 2. *The resultant of two forces, one of which is double the other, is 13 kg-wt. If one of them is reversed, the other remaining unaltered, resultant becomes 9 kg-wt. Find the magnitudes of the forces and the cosine of the angle at which the forces are inclined in first case.*

Sol. Let the forces be P and $2P$ and α the angle between them.

In the first case, the resultant is 13 kg-wt.

$$\because \quad P^2+(2P)^2+2.P.2P \cos \alpha=13^2=169$$

or
$$5P^2+4P^2 \cos \alpha=169 \quad ...(1)$$

In the second case, let $2P$ be reversed, so that the forces are P and $-2P$ and the resultant is 9 kg-wt.

$$P^2+(-2P)^2+2P.(-2P). \cos \alpha=9^2=81$$

or
$$5P^2-4P^2 \cos \alpha=81 \quad ...(2)$$

Adding (1) and (2), $\quad 10P^2=250$ whence $P=5$.

Hence the forces are 5 kg-wt and 10 kg-wt.

Substituting in (1),

$$5\times25+4\times25\times \cos \alpha=169$$

$$\cos \alpha=0·44.$$

Ex. 3. *The resultant of two forces is 8 kg-wt and its direction is inclined at 60° to one of the forces whose magnitude is 4 kg wt. Find the magnitude and direction of the other force.*

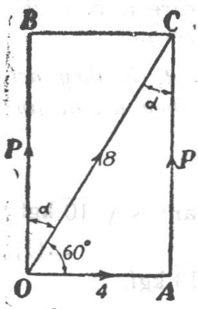

Fig. 1·11

Sol. Let $\vec{OA}=4$ kg wt, $\vec{OC}=8$ kg wt.

Let $\vec{OB}=P$ be the other force inclined at $\angle\alpha$ to OC.

Clearly $\angle OCA=\alpha$,

$$\vec{AC}=P.$$

Also $\quad \angle OAC=180°-(60°+\alpha)$

From $\triangle OCA$,

$$\frac{4}{\sin \alpha}=\frac{P}{\sin 60°}=\frac{8}{\sin [180°-(60°+\alpha)]}.$$

Taking the first and third terms,

$$\frac{4}{\sin \alpha}=\frac{8}{\sin (60°+\alpha)}$$

$$2 \sin \alpha=\sin (60°+\alpha)$$

$$=\sin 60° \cos \alpha+\cos 60° \sin \alpha$$

$$=\frac{\sqrt{3}}{2} \cos \alpha+\frac{1}{2} \sin$$

$$3 \sin \alpha=\sqrt{3} \cos \alpha, \tan \alpha=\frac{1}{\sqrt{3}}$$

$$\therefore \quad \alpha=30°.$$

Taking the first and second terms and substituting the value of α,

$$\frac{4}{\sin 30^\circ}=\frac{P}{\sin 60^\circ}$$

$$P=\frac{4 \sin 60^\circ}{\sin 30}=4\sqrt{3}\ \textbf{kg wt.}$$

Hence the other force is $4\sqrt{3}$ kg wt at right angles to the force of 4 kg wt.

Ex. 4. *The resultant of two forces P, Q acting at an angle* θ, *is equal to* $(2m+1)\sqrt{P^2+Q^2}$; *when they act at an angle* $\frac{\pi}{2}-\theta$, *it is equal to* $(2m-1)\sqrt{P^2+Q^2}$. *Show that* $\tan\theta=\frac{m-1}{m+1}$.

Sol. When the angle between the forces is θ, the sesultant is $(2m+1)\sqrt{P^2+Q^2}$.

$$\therefore\quad (2m+1)^2(P^2+Q^2)=P^2+Q^2+2PQ\cos\theta \qquad \ldots(1)$$

When the angle between forces is $\frac{\pi}{2}-\theta$, the resultant is $(2m-1)\sqrt{P^2+Q^2}$.

$$\therefore\quad (2m-1)^2(P^2+Q^2)=P^2+Q^2+2PQ\cos\left(\frac{\pi}{2}-\theta\right)$$

$$=P^2+Q^2+2PQ\sin\theta \qquad \ldots(2)$$

Equation (1) can be written as

$$(P^2+Q^2)[(2m+1)^2-1]=2PQ\cos\theta \qquad \ldots(3)$$

Equation (2) can be written as

$$(P^2+Q^2)[(2m-1)^2-1]=2PQ\sin\theta \qquad \ldots(4)$$

Dividing (4) by (3)

$$\tan\theta=\frac{(2m-1)^2-1}{(2m+1)^2-1}$$

$$=\frac{4m^2-4m}{4m^2+4m}=\frac{m-1}{m+1}.$$

Ex. 5. *The resultant of two forces P and Q is R ; if Q be doubled, R is doubled, whilst, if Q be reversed, R is again doubled ; show that*

$$P:Q:R::\sqrt{2}:\sqrt{3}:\sqrt{2}.$$

Sol. Let the angle between the forces be α in each case. The resultant of P and Q is R.

$$\therefore\quad P^2+Q^2+2PQ\cos\alpha=R^2 \qquad \ldots(1)$$

The resultant of P and $2Q$ is $2R$.

$$\therefore\quad P^2+4Q^2+4PQ\cos\alpha=4R^2 \qquad \ldots(2)$$

The resultant of P and $-Q$ is $2R$.

$$\therefore\quad P^2+Q^2-2PQ\cos\alpha=4R^2 \qquad \ldots(3)$$