

9462457

Turbulence in Fluids

Stochastic and Numerical Modelling

L629
E.2

9462457

Turbulence in Fluids

Stochastic and Numerical Modelling

by

MARCEL LESIEUR

*National Polytechnic Institute,
School of Hydraulics and Mechanics,
Grenoble, France*

Second revised edition

ACADEMIC PUBLISHERS

BOSTON / LONDON

Library of Congress Cataloging in Publication Data

Lesteur, Marcel.

Turbulence in fluids : Stochastic and numerical modelling / Marcel
Lesteur. -- 2nd rev. ed.

p. cm. -- (Fluid mechanics and its applications ; v. 1)

Includes bibliographical references.

Includes index.

ISBN 0-7923-0645-7 (alk. paper)

1. Fluid mechanics. 2. Turbulence. 3. Transport theory.

I. Title. II. Series.

QC145.2.L47 1991

532'.0527--dc20

90-46046

ISBN 0-7923-0645-7

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates
the publishing programmes of
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

All Rights Reserved

© 1990 by Kluwer Academic Publishers

No part of the material protected by this copyright notice may be reproduced or
utilized in any form or by any means, electronic or mechanical,
including photocopying, recording or by any information storage and
retrieval system, without written permission from the copyright owner.

Printed in the Netherlands

FLUID MECHANICS AND ITS APPLICATIONS

Volume 1

Series Editor: R. MOREAU

MADYLAM

Ecole Nationale Supérieure d'Hydraulique de Grenoble

Boîte Postale 95

38402 Saint Martin d'Hères Cedex, France

Aims and Scope of the Series

The purpose of this series is to focus on subjects in which fluid mechanics plays a fundamental role.

As well as the more traditional applications of aeronautics, hydraulics, heat and mass transfer etc., books will be published dealing with topics which are currently in a state of rapid development, such as turbulence, suspensions and multiphase fluids, super and hypersonic flows and numerical modelling techniques.

It is a widely held view that it is the interdisciplinary subjects that will receive intense scientific attention, bringing them to the forefront of technological advancement. Fluids have the ability to transport matter and its properties as well as transmit force, therefore fluid mechanics is a subject that is particularly open to cross fertilisation with other sciences and disciplines of engineering. The subject of fluid mechanics will be highly relevant in domains such as chemical, metallurgical, biological and ecological engineering. This series is particularly open to such new multidisciplinary domains.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of a field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

Preface to the first edition

Turbulence is a dangerous topic which is often at the origin of serious fights in the scientific meetings devoted to it since it represents extremely different points of view, all of which have in common their complexity, as well as an inability to solve the problem. It is even difficult to agree on what exactly is the problem to be solved.

Extremely schematically, two opposing points of view have been advocated during these last ten years: the first one is "statistical", and tries to model the evolution of averaged quantities of the flow. This community, which has followed the glorious trail of Taylor and Kolmogorov, believes in the phenomenology of cascades, and strongly disputes the possibility of any coherence or order associated to turbulence.

On the other bank of the river stands the "coherence among chaos" community, which considers turbulence from a purely deterministic point of view, by studying either the behaviour of dynamical systems, or the stability of flows in various situations. To this community are also associated the experimentalists who seek to identify coherent structures in shear flows.

My personal experience in turbulence was acquired in the first group since I spent several years studying the stochastic models of turbulence, applied to various situations such as helical or two-dimensional turbulence and turbulent diffusion. These techniques were certainly not the ultimate solution to the problem, but they allowed me to get acquainted with various disciplines such as astrophysics, meteorology, oceanography and aeronautics, which were all, for different reasons, interested in turbulence. It is certainly true that I discovered the fascination of Fluid Dynamics through the somewhat abstract studies of turbulence.

This monograph is then an attempt to reconcile the statistical point of view and the basic concepts of fluid mechanics which determine the evolution of flows arising in the various fields envisaged above. It is true that these basic principles, accompanied by the predictions of the instability theory, give valuable information on the behaviour of turbulence

and of the structures which compose it. But a statistical analysis of these structures can, at the same time, supply information about strong non-linear energy transfers within the flow.

I have tried to present here a synthesis between two graduate courses given in Grenoble during these last few years, namely a "Turbulence" course and a "Geophysical Fluid Dynamics" course. I would like to thank my colleagues of the Ecole Nationale d'Hydraulique et Mécanique and Université Scientifique et Médicale de Grenoble, who offered me the opportunity of giving these two courses. The students who attended these classes were, through their questions and remarks, of great help. I took advantage of a sabbatical year spent at the Department of Aerospace Engineering of the University of Southern California to write the first draft of this monograph: this was rendered possible by the generous hospitality of John Laufer and his collaborators. Finally, I am grateful to numerous friends around the world who encouraged me to undertake this work.

I am greatly indebted to Frances Métais who corrected the English style of the manuscript. I am uniquely responsible for the remaining mistakes, due to last minute modifications. I ask for the indulgence of the English speaking reader, thinking that he might not have been delighted by a text written in perfect French. I hope also that this monograph will help the diffusion of some French contributions to turbulence research.

Ms Van Thai was of great help for the drawings. I am also extremely grateful to Jean-Pierre Chollet, Yves Gagne and Olivier Métais for their contribution to the contents of the book and their help during its achievement, and to Sherwin Maslowe who edited several Chapters.

This book was written using the TEX system. This would not have been possible without the constant help of Evelyne Tournier, of Grenoble Applied Mathematics Institute, and of Claude Goutorbe, of the University computing center.

Finally I thank Martinus Nijhoff Publishers for offering me the possibility of presenting these ideas.

Grenoble, October 1986

Marcel Lesieur

Foreword to the second edition

Four years seems to be a good period of time to assess one's old points of view in such rapidly evolving field as Turbulence and Fluid Mechanics. The new possibilities offered by direct-numerical simulations have provided a lot of information on vortex dynamics, coherent structures and transition, compressible or rotating flows. The third chapter now gives a basic presentation of the linear-instability theory applied to shear or thermally unstable flows. A substantial part of the phenomenology in Chapter VI is devoted to mixing-length theory applied to turbulent shear flows. Concerning the stochastic models, it seemed necessary to include more information on the D.I.A. and R.N.G. theories. New calculations and experiments on stratified or shear flows have been incorporated, with emphasis put on the three-dimensional structures topology. Recent results on the intermittency of isotropic turbulence, and on passive scalar diffusion are also included. Finally, I rewrote Chapter XII on large-eddy simulation in order to make it more general and accessible to graduate students. This is the general point of view which has been my guideline during the write up for this second edition.

All this makes for a much more substantial book. I hope the original spirit of the first edition has not been lost, but I think it has resisted well to my attacks. Particular thanks are extended to Pierre Comte and the graduate students of our group for their important visual contribution which illustrates so well coherent structures and transition. Many thanks also to Olivier Métais who contributed greatly to certain of the new numerical results shown here, and for his permanent and total support and interest. Jim Riley was unlucky enough to spend his sabbatical with us in Grenoble during these last few months, and influenced many of my conclusions, while pretending to correct the language of a few chapters. Finally, I am indebted to all the sponsoring agencies and companies who showed a continuous interest during all these years in the development of fundamental and numerical research on Turbulence in Grenoble.

Grenoble, June 1990

Marcel Lesieur

Contents

I Introduction to turbulence in fluid mechanics	1
1 Is it possible to define turbulence?	1
2 Examples of turbulent flows	6
3 Fully developed turbulence	9
4 Fluid turbulence and "chaos"	11
5 "Deterministic" and statistical approaches	13
6 Why study isotropic turbulence?	15
7 One-point closure modelling	16
8 Outline of the following chapters	17
II Basic fluid dynamics	19
1 Eulerian notation and Lagrangian derivatives	19
2 The continuity equation	20
3 The conservation of momentum	21
4 The thermodynamic equation	24
5 The incompressibility assumption	28
6 The dynamics of vorticity	30
7 The generalized Kelvin theorem	32
8 The Boussinesq approximation	36
9 Internal inertial-gravity waves	39
10 Barré de Saint-Venant equations	43
2.10.1 Derivation of the equations	43
2.10.2 The potential vorticity	45
2.10.3 Inertial-gravity waves	46
2.10.3.1 Analogy with two-dimensional compressible gas	46
11 Gravity waves in a fluid of arbitrary depth	47
III Transition to turbulence	49
1 The Reynolds number	50
2 Linear-instability theory	52
3.2.1 The Orr-Sommerfeld equation	53
3.2.2 The Rayleigh equation	54
3.2.2.1 Kuo equation	56
3 Transition in shear flows	57
3.3.1 Free-shear flows	57
3.3.1.1 Mixing layers	57
3.3.1.2 Mixing layer with differential rotation	63

3.3.1.3 Plane jets and wakes	63
3.3.2 Wall flows	67
3.3.2.1 The boundary layer	67
3.3.2.2 Poiseuille flow	69
3.3.3 Transition, coherent structures and Kolmogorov spectra	69
3.3.3.1 Linear stability of a vortex filament within a shear	71
3.3.4 Compressible turbulence	72
3.3.4.1 Compressible mixing layer	73
3.3.4.2 Compressible wake	75
3.3.4.3 Compressible boundary layer	75
4 The Rayleigh number	76
5 The Rossby number	80
3.5.1 Quasi-two-dimensional flow submitted to rotation	81
3.5.1.1 Linear analysis	81
3.5.1.2 The straining of absolute vorticity	82
6 The Froude Number	84
7 Turbulence, order and chaos	86
IV The Fourier space	89
1 Fourier representation of a flow	89
4.1.1 Flow "within a box":	89
4.1.2 Integral Fourier representation	90
2 Navier-Stokes equations in Fourier space	92
3 Boussinesq approximation in the Fourier space	94
4 Craya decomposition	95
5 Complex helical waves decomposition	97
V Kinematics of homogeneous turbulence	101
1 Utilization of random functions	101
2 Moments of the velocity field, homogeneity and stationarity	102
3 Isotropic	104
4 The spectral tensor of an isotropic turbulence	109
5 Energy, helicity, enstrophy and scalar spectra	110
6 Alternative expressions of the spectral tensor	113
7 Axisymmetric turbulence	116
VI Phenomenological theories	119
1 Inhomogeneous turbulence	119
6.1.1 The mixing-length theory	120
6.1.2 Application of mixing-length to turbulent-shear flows	120
6.1.2.1 The plane jet	120
6.1.2.2 The round jet	123
6.1.2.3 The plane wake	124
6.1.2.4 The round wake	126
6.1.2.5 The plane mixing layer	126
6.1.2.6 The boundary layer	128

2	Triad interactions and detailed conservation	128
6.2.1	Quadratic invariants in physical space	131
6.2.1.1	Kinetic energy	131
6.2.1.2	Helicity	132
6.2.1.3	Passive scalar	133
3	Transfer and Flux	133
4	The Kolmogorov theory	137
6.4.1	Oboukhov's theory	139
5	The Richardson law	141
6	Characteristic scales of turbulence	141
6.6.1	The degrees of freedom of turbulence	142
6.6.1.1	The dimension of the attractor	143
6.6.2	The Taylor microscale	143
6.6.3	Self-similar decay	144
7	Skewness factor and enstrophy divergence	145
6.7.1	The skewness factor	145
6.7.2	Does enstrophy blow up at a finite time?	146
6.7.2.1	The constant skewness model	147
6.7.2.2	Positiveness of the skewness	147
6.7.2.3	Enstrophy blow up theorem	148
6.7.2.4	A self-similar model	149
6.7.2.5	Oboukhov's enstrophy blow up model	151
6.7.2.6	Discussion	151
6.7.3	The viscous case	152
8	The internal intermittency	154
6.8.1	The Kolmogorov-Oboukhov-Yaglom theory	155
6.8.2	The Novikov-Stewart (1964) model	156
6.8.3	Experimental and numerical results	157
6.8.4	Temperature and velocity intermittency	158
VII	Analytical theories and stochastic models	161
1	Introduction	161
2	The Quasi-Normal approximation	164
3	The Eddy-Damped Quasi-Normal type theories	167
4	The stochastic models	169
5	Phenomenology of the closures	175
6	Numerical resolution of the closure equations	178
7	The enstrophy divergence and energy catastrophe	183
8	The Burgers- <i>M.R.C.M.</i> model	186
9	Isotropic helical turbulence	188
10	The decay of kinetic energy	192
11	The Renormalization-Group techniques	197
7.11.1	The <i>R.N.G.</i> algebra	198
7.11.2	Two-point closure and <i>R.N.G.</i> techniques	202

7.11.2.1 The $k^{-5/3}$ range	202
7.11.2.2 The infrared spectrum	203
VIII Diffusion of passive scalars	205
1 Introduction	205
2 Phenomenology of the homogeneous passive scalar diffusion	206
8.2.1 The inertial-convective range	207
8.2.2 The inertial-conductive range	208
8.2.3 The viscous-convective range	210
3 The <i>E.D.Q.N.M.</i> isotropic passive scalar	212
8.3.1 A simplified <i>E.D.Q.N.M.</i> model	216
8.3.2 <i>E.D.Q.N.M.</i> scalar-entropy blow up	217
4 The decay of temperature fluctuations	222
8.4.1 Phenomenology	222
8.4.1.1 Non-local interactions theory	223
8.4.1.2 Self-similar decay	225
8.4.1.3 Anomalous temperature decay	228
8.4.2 Experimental temperature decay data	230
8.4.3 Discussion of the <i>L.E.S.</i> results	231
8.4.4 Diffusion in stationary turbulence	232
5 Lagrangian particle pair dispersion	233
6 Single-particle diffusion	235
8.6.1 Taylor's diffusion law	235
8.6.2 <i>E.D.Q.N.M.</i> approach to single-particle diffusion	237
IX Two-dimensional and quasi-geostrophic turbulence	243
1 Introduction	243
2 The quasi-geostrophic theory	246
9.2.1 The geostrophic approximation	248
9.2.2 The quasi-geostrophic potential vorticity equation	250
9.2.3 The n -layer quasi-geostrophic model	255
9.2.4 Interaction with an Ekman layer	259
9.2.4.1 Geostrophic flow above an Ekman layer	260
9.2.4.2 The upper Ekman layer	262
9.2.5 Barotropic and baroclinic waves	264
3 Two-dimensional isotropic turbulence	266
9.3.1 Fjortoft's theorem	267
9.3.2 The enstrophy cascade	268
9.3.3 The inverse energy cascade	272
9.3.4 The two-dimensional <i>E.D.Q.N.M.</i> model	276
9.3.5 Freely-decaying turbulence	279
4 Diffusion of a passive scalar	283
5 Geostrophic turbulence	287
9.5.1 Rapidly-rotating stratified fluid of arbitrary depth	292

X Absolute equilibrium ensembles	295
1 Truncated Euler Equations	295
2 Liouville's theorem in the phase space	296
3 The application to two-dimensional turbulence	300
4 Two-dimensional turbulence over topography	302
XI The statistical predictability theory	305
1 Introduction	305
2 The <i>E.D.Q.N.M.</i> predictability equations	309
3 Predictability of three-dimensional turbulence	311
4 Predictability of two-dimensional turbulence	313
XII Large-eddy simulations	317
1 The direct-numerical simulation of turbulence	317
2 The Large Eddy Simulations	319
12.2.1 Large and subgrid scales	319
12.2.2 L.E.S. and the predictability problem	320
3 The Smagorinsky model	321
4 L.E.S. of 3-D isotropic turbulence	322
12.4.1 Spectral eddy-viscosity and diffusivity	323
12.4.2 Spectral large-eddy simulations	325
12.4.3 The anomalous spectral eddy-diffusivity	329
12.4.4 Alternative approaches	331
12.4.5 A local formulation of the spectral eddy-viscosity	332
5 L.E.S. of two-dimensional turbulence	334
XIII Towards real-world turbulence	337
1 Introduction	337
2 Stably-stratified turbulence	338
13.2.1 The so-called "collapse" problem	338
13.2.2 A numerical approach to the collapse	340
3 The two-dimensional mixing layer	349
13.3.1 Generalities	349
13.3.2 Two-dimensional turbulence in the mixing layer	351
13.3.3 Two-dimensional unpredictability	354
13.3.4 Two-dimensional unpredictability and 3D growth	355
4 3D numerical simulations of the mixing layer	359
13.4.1 Direct-numerical simulations	359
13.4.2 Large-eddy simulations of mixing layers	360
13.4.3 Recreation of the coherent structures	361
13.4.4 Rotating mixing layers	362
5 Conclusion	363
References	367
Index	395

Chapter I

INTRODUCTION TO TURBULENCE IN FLUID MECHANICS

1 - Is it possible to define turbulence?

Everyday life gives us an intuitive knowledge of turbulence in fluids: the smoke of a cigarette or over a fire exhibits a disordered behaviour characteristic of the motion of the air which transports it. The wind is subject to abrupt changes in direction and velocity, which may have dramatic consequences for the seafarer or the hang-glider. During air travel, one often hears the word turbulence generally associated with the fastening of seat-belts. Turbulence is also mentioned to describe the flow of a stream, and in a river it has important consequences concerning the sediment transport and the motion of the bed. The rapid flow of any fluid passing an obstacle or an airfoil creates turbulence in the boundary layers and develops a turbulent wake which will generally increase the drag exerted by the flow on the obstacle (and measured by the famous C_x coefficient): so turbulence has to be avoided in order to obtain better aerodynamic performance for cars or planes. The majority of atmospheric or oceanic currents cannot be predicted accurately and fall into the category of turbulent flows, even in the large planetary scales. Small-scale turbulence in the atmosphere can be an obstacle towards the accuracy of astronomic observations, and observatory locations have to be chosen in consequence. The atmospheres of planets such as Jupiter and Saturn, the solar atmosphere or the Earth's outer core are turbulent. Galaxies look strikingly like the eddies which are observed in turbulent flows such as the mixing layer between two flows of different velocity, and are, in a manner of speaking, the eddies of a turbulent universe. Turbulence is also produced in the Earth's outer magnetosphere, due to the development of instabilities caused by the interaction of the solar

wind with the magnetosphere. Numerous other examples of turbulent flows arise in aeronautics, hydraulics, nuclear and chemical engineering, oceanography, meteorology, astrophysics and internal geophysics.

It can be said that a turbulent flow is a flow which is disordered in time and space. But this, of course, is not a precise mathematical definition. The flows one calls "turbulent" may possess fairly different dynamics, may be three-dimensional or sometimes quasi-two-dimensional, may exhibit well organized structures or otherwise. A common property which is required of them is that they should be able to mix transported quantities much more rapidly than if only molecular diffusion processes were involved. It is this latter property which is certainly the more important for people interested in turbulence because of its practical applications: the engineer, for instance, is mainly concerned with the knowledge of turbulent heat diffusion coefficients, or the turbulent drag (depending on turbulent momentum diffusion in the flow). The following definition of turbulence can thus be tentatively proposed and may contribute to avoiding the somewhat semantic discussions on this matter:

- a) Firstly, a turbulent flow must be unpredictable, in the sense that a small uncertainty as to its knowledge at a given initial time will amplify so as to render impossible a precise deterministic prediction of its evolution.

- b) Secondly, it has to satisfy the increased mixing property defined above.

- c) Thirdly, it must involve a wide range of spatial wave lengths. Such a definition allows in particular an application of the term "turbulent" to some two-dimensional flows. It also implies that certain non dimensional parameters characteristic of the flow should be much greater than one: indeed, let l be a characteristic length associated to the large energetic eddies of turbulence, and v a characteristic fluctuating velocity; a very rough analogy between the mixing processes due to turbulence and the incoherent random walk allows one to define a turbulent diffusion coefficient proportional to lv . As will be seen later on, l is also called the integral scale. Thus, if ν and κ are respectively the molecular diffusion coefficients¹ of momentum (called below the kinematic molecular viscosity) and heat (the molecular conductivity), the increased mixing property for these two transported quantities implies that the two dimensionless parameters $R_l = lv/\nu$ and lv/κ should be much greater than one. The first of these parameters is called the Reynolds number, and the second one the Peclet number. Notice finally that the existence of a large Reynolds number implies, from the phenomenology developed in Chapter VI, that the ratio of the largest to the smallest scale may be

¹ These coefficients will be accurately defined in Chapter II.

of the order of $R_i^{3/4}$. In this respect, the property b) stressed above implies c).

A turbulent flow is by nature unstable: a small perturbation will generally, due to the nonlinearities of the equations of motion, amplify. The contrary occurs in a "laminar" flow, as can be seen on Figure I-1, where the streamlines, perturbed by the small obstacle, reform downstream. The Reynolds number of this flow, defined as

$$Re = [\text{fluid velocity}] \times [\text{size of the obstacle}] / \nu$$

is in this experiment equal to $2.26 \cdot 10^{-2}$. This Reynolds number is different from the turbulent Reynolds number introduced above, but it will be shown in chapter III that they both characterize the relative importance of inertial forces over viscous forces in the flow. Here the viscous forces are preponderant and will damp any perturbation, preventing a turbulent wake from developing.



Figure I-1: Stokes flow of glycerin past a triangular obstacle (picture by S. Taneda, Kyushu University; from Lesieur (1982), courtesy S. Taneda and "La Recherche")

There is a lot of experimental or numerical evidence showing that turbulent flows are rotational, that is, their vorticity $\vec{\omega} = \vec{\nabla} \times \vec{u}$ is non zero, at least in certain regions of space. Therefore, it is interesting to ask oneself how turbulence does in fact arise in a flow which is irrotational upstream². It is obviously due to the viscosity, since an immediate consequence of Kelvin's theorem, demonstrated in Chapter II, is that zero-vorticity

² for instance, a uniform flow

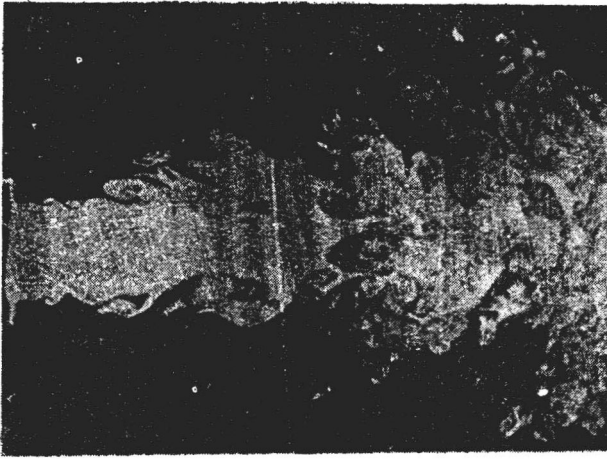


Figure I-2: turbulent jet (picture by J.L. Balint, M. Ayrault and J.P. Schon, Ecole Centrale de Lyon; from Lesieur (1982), courtesy J.P. Schon and "La Recherche")

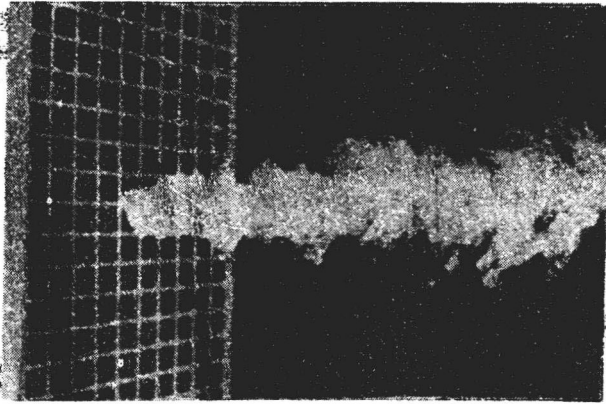


Figure I-3: turbulence created in a wind tunnel behind a grid. Here turbulence fills the whole apparatus, and a localized source of smoke has been placed on the grid to visualize the development of turbulence (picture by J.L. Balint, M. Ayrault and J.P. Schon, Ecole Centrale de Lyon; from Lesieur (1982), courtesy "La Recherche")

is conserved following the motion in a perfect fluid³: the presence of

³ The perfect fluid is an approximation of the flow where molecular



Figure I-4: turbulence in a mixing layer (Brown and Roshko, 1974). In I-4a, the Reynolds number (based on the velocity difference and the width of the layer at a given downstream position) is twice Figure I-4b's (courtesy A.Roshko and J. Fluid Mech.)

boundaries or obstacles imposes a zero-velocity condition which produces vorticity. Production of vorticity will then be increased, due to the vortex filaments stretching mechanism to be described later, to such a point that the flow will generally become turbulent in the rotational regions. In what is called grid turbulence for instance, which is produced in the laboratory by letting a flow go through a fixed grid, the rotational "vortex streets" behind the grid rods interact together and degenerate into turbulence. Notice that the same effect would be obtained by pulling a grid through a fluid initially at rest. In some situations, the vorticity is created in the interior of the flow itself through some external forcing or rotational-initial conditions (as in the example of the temporal mixing layer presented later on).

viscous effects are ignored.