

**INTRODUCTION
TO
EXPERIMENTAL
NUCLEAR PHYSICS**

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TO
EXPERIMENTAL NUCLEAR PHYSICS

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PREFACE

This book is designed primarily as a companion volume for the several textbooks available on nuclear physics. It is aimed at the level of M.Sc. in Indian universities but it can be useful for research students using various experimental methods from nuclear physics in their studies. To get the best advantage from this book, a familiarity with basic nuclear physics is desirable.

The idea of writing such a book came to me when I was supervising an experimental nuclear physics project for the National Science Talent Summer School participants. Along with these participants I discovered that the basic experimental methods employed in nuclear physics are not described in a single way in an *elementary* way. Most of these methods are introduced in the familiar textbooks on nuclear physics but in these books the treatment is usually limited in depth and is often disjointed in sequence. There are, of course, advanced treatises where the subject is developed very elaborately upto research level. Outstanding among such advanced books are "Nuclear Radiation Detection" by W.J. Price and, of course, the much-respected "Alpha-, Beta-, and Gamma-Ray Spectroscopy" edited by K. Siegbahn. However, the former book involves more of nuclear physics while the latter book, in two volumes, is rather formidable for a beginner although encyclopedic in nature. I therefore thought that a book written at a more elementary level could assist students in understanding how nuclear physics is experimentally studied.

Today, nuclear methods are being used in many different areas. In India itself many universities are in a position to develop teaching laboratories in nuclear physics, thanks to the nuclear detection systems developed by the Bhabha Atomic Research Centre and the Electronics Corporation of India. With the development of various centres of nuclear research, atomic reactors, the forthcoming variable energy cyclotron at Calcutta and so on, training in experimental nuclear physics is going to be more important.

Again, new research frontiers like ESCA, ion-channeling in crystals, Mossbauer spectroscopy, perturbed angular correction, positron annihilation studies etc. are attracting research workers from other areas to employ these nuclear techniques for their measurements. The field of nuclear medicine is growing rapidly in the Western countries and it will not be long before these developments are followed in India. It therefore appears that familiarity with experimental methods of nuclear physics should form an essential part of the M.Sc. syllabus in India.

Chapter 1 introduces basic properties of nuclei, nuclear radiation and nuclear decay while Chapter 2 briefly summarises the main features of nuclear

reactions. The interaction of nuclear radiation with matter is described in Chapter 3. An introduction to various nuclear radiation detectors is provided in the next chapter. The newly developed semiconductor radiation detectors are described with proper emphasis. Chapters 5, 6 and 7 describe the various techniques used to measure alpha rays, beta rays and gamma rays. Methods for detection of other heavy charged particles (e.g. protons etc.) are not discussed separately. Chapter 8 summarises the special techniques that are used to render the nuclear techniques more powerful. Chapter 9 provides an interface between the experimental nuclear techniques and nuclear physics studied by these techniques. Chapter 10 introduces areas like positron annihilation studies, Mossbauer spectroscopy and perturbed angular correlation where nuclear techniques form the methodology.

While illustrating various nuclear methods in this book there has been a greater emphasis on radioactive nuclear decay processes rather than on nuclear reactions. This emphasis has been purposeful with a hope that in a University teaching laboratory it is much easier to study radioactive nuclear decay.

It is hoped that this book will serve the purpose of preparing a student upto an intermediate level from where he can appreciate advanced books on this subject with a better preparation.

I would like to take this opportunity to express my appreciation to my teachers Professor R.M. Steffen and Professor S. Jha and my colleague Dr. H.G. Devare who have taught me this subject. Of course, any deficiency in my education, as revealed in the following pages, is entirely my responsibility.

The first draft of this book was written during my visit to the Tata Institute of Fundamental Research, Bombay in 1970. I am grateful to Professor B.V. Thosar and Dr. S.K. Mitra for their kind hospitality during my stay there. I wish to thank the Indian Institute of Technology, Kanpur for providing various facilities for completing this book. Valuable suggestions made by Dr. M. M. Chaudhri about the cover design are thankfully appreciated. I wish to express my thanks to Mr. R.K. Bajpai for preparing the drawings and Mr. R.N. Srivastava for the excellent job of typing the manuscript.

The co-operation and guidance provided by the publishers has been a valuable and pleasant experience and I wish to record my gratitude to them. Finally, I wish to express my appreciation to my wife Sushama for her encouragement during this project.

Kanpur, India
March 23, 1972

R. M. SINGRU

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Chapter 1

REVIEW OF NUCLEAR PROPERTIES AND NUCLEAR DECAY

1.1 INTRODUCTION

Nuclear radiations originate in atomic nucleus as a result of radioactive decay of the nucleus. Before we understand the radioactive decay, let us briefly review the structure of atoms and nuclei.

An atom consists of a positively charged nucleus with negatively charged electrons orbiting around it. Almost the whole mass of the atom is contributed by its nucleus.

The nucleus itself is composed of protons and neutrons held together by specifically nuclear forces. Protons have a mass equal to $m_p = 1835.9 m_0$ where m_0 is the rest mass of the electron. It has unit positive charge and spin $1/2$. Neutrons have a mass equal to $m_n = 1838.6 m_0$ and spin $1/2$ but they are electrically neutral. An atomic nucleus is usually denoted by two numbers, A and Z . The first number A indicates the total number of nucleons (protons and neutrons) inside the nucleus and this number is called the mass number. The number Z , called the atomic number, indicates the number of protons in the nucleus. In this model, an atomic nucleus (A, Z) is composed of Z protons and $A-Z$ neutrons. This makes the total positive charge of the nucleus as $+Ze$ which is balanced by the total negative charge $-Ze$ contributed by the Z atomic electrons outside the nucleus to make the atom as a whole electrically neutral. According to the new convention the mass number, A , is shown as a left superscript on the chemical symbol of the atom while the atomic number, though included in the chemical symbol, is sometimes shown as a left subscript. Thus $^{137}_{55}\text{Cs}$ stands for the isotope of Caesium ($Z = 55$) having $A = 137$. For almost all elements there are more than one isotope differing in their mass number.

The electrons in the atom orbit around the nucleus and depending on the radii of these orbits they are said to belong to the K, L, M, N ... shells. The K-shell orbits are closest to the nucleus, the L-shell being the next outer orbit and so on. The chemical properties of the atom are determined by the number and configuration of the atomic electrons.

For stable (non-radioactive) nuclei the plot of neutron number $N (= A - Z)$ versus the proton number Z shows a behaviour as shown in Fig. 1.1.

For stable nuclei, the neutron and proton numbers are favourable for stability. For unstable nuclei the neutron and proton numbers are not favourable for stability and this

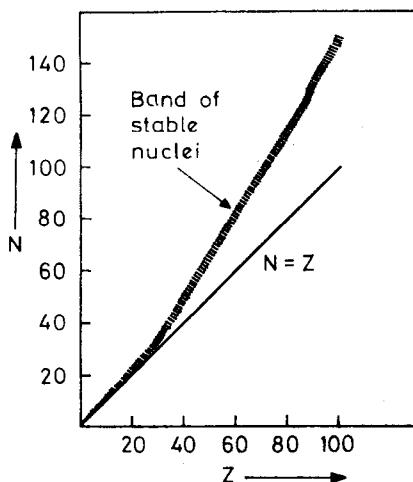


Fig. 1.1 A plot of N versus Z for stable nuclei.

relative number tries to reach a correct stability ratio by undergoing radioactive decay of the nucleus. Three kinds of radiations can accompany radioactive decay of the nucleus. These are alpha rays, beta rays and gamma rays. Let us understand the basic properties of nuclear energy levels before we study the properties of each type of nuclear radiation.

1.2 PROPERTIES OF NUCLEAR STATES

In the subsequent chapters we shall see how the study of nuclear radiations help us to know the properties of the nuclei involved. Such a study of nuclear energy levels is called nuclear spectroscopy. Before we study the various techniques of nuclear spectroscopy, let us briefly review those properties of nuclei which are important for understanding nuclear structure.

a) energy: A bound system such as a nucleus always has discrete energy states. The state having minimum energy amongst these is called the ground state and the other higher states (usually unstable against radioactive decay, particularly

gamma decay) are called the excited states. These states are indicated in the so-called energy level diagram (see Fig. 1.2) where the ground state is arbitrarily given energy value as zero and all other excited states are shown as higher levels, their energy being measured from the ground state.

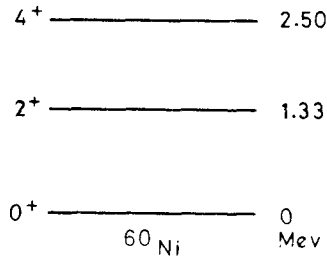


Fig. 1.2 Energy level diagram of ^{60}Ni .

b) angular momentum: In each energy state the nuclei possess angular momentum. Each nucleon (proton or neutron) has the intrinsic spin ($s = 1/2$) as well as an orbital angular momentum l . Thus the nucleus, which is a system of Z protons and $(A-Z)$ neutrons will have a total angular momentum resulting from the proper vector addition of intrinsic spins s_i and orbital angular momenta l_i . The total angular momentum of the nucleus is usually denoted by J and loosely called as nuclear 'spin'. Like all angular momenta, J is a vector. Its absolute value will be $\hbar[J(J+1)]^{1/2}$ and its projection on z -axis will have the values $m_J = J, J-1, \dots, -(J-1), -J$. Thus the nuclear 'spin' is the largest value of $m_J (= J)$ in units of \hbar . For nuclei with even mass number the nuclear spin is always a multiple of \hbar and for nuclei with odd mass number the nuclear spin is always a multiple of $\hbar/2$. Thus for odd A , $J = 1/2, 3/2, 5/2, \dots$, and for even A , $J = 0, 1, 2, 3, \dots$.

c) parity: Nuclei exhibit another definite property in each energy state. This is a property of the nuclear wave function (Ψ) under space inversion and is called 'parity' of the nucleus in an energy state. It can be shown that under space inversion, i.e. under the inversion of coordinate system so that $x \rightarrow -x$; $y \rightarrow -y$ and $z \rightarrow -z$, the wave function $\Psi(x, y, z)$ would change into $\Psi(-x, -y, -z)$ and

$$\Psi(-x, -y, -z) = \pm \Psi(x, y, z) \quad \dots (1.1)$$

If the wave function transforms with a positive sign, i.e. $\Psi(-x, -y, -z) = + \Psi(x, y, z)$ the wave function is said to have even parity. On the other hand, if Ψ transforms as

$\Psi(-x, -y, -z) = -\Psi(x, y, z)$, the wave function is said to have odd parity. For a nucleus in any state, its 'parity' is usually denoted as a superscript, either + (even) or - (odd), on the value of nuclear spin. Thus a designation 2^+ indicates that the particular nuclear state has $J = 2$ with even parity. Parity is also conserved during nuclear decay and nuclear reactions.

d) nuclear dipole magnetic moment: Associated with its spin a nucleus may possess a magnetic dipole moment in an energy state. The magnetic (dipole) moment of the nucleus is usually expressed in the units of nuclear magnetons $\mu_N (\mu_N = eh/2mp = 5.05 \times 10^{-27} \text{ J} \cdot \text{m}^{-2} \cdot \text{Wb}^{-1})$. Also, by convention, the quoted magnetic moment value is the maximum value of the magnetic moment in the direction of an external magnetic field. This value will, therefore, correspond to the maximum value of the projection quantum number ($m_J = J$) along the quantization direction. Thus the magnetic moment $\bar{\mu}_J$ of nucleus corresponds to value $\bar{\mu}_J = g_J J \mu_N$ where g_J is the nuclear g-factor.

e) nuclear electric quadrupole moment: A nucleus may possess electric quadrupole (or higher) moment if it has a non-spherical charge distribution. The nuclear quadrupole moment is defined as

$$Q = \frac{1}{e} \int \rho(r) (3z^2 - r^2) dv \quad \dots (1.2)$$

where $\rho(r)$ is the charge density inside the nucleus. Quadrupole moments are therefore expressed in units of barns (10^{-28} m^2). For nuclei with spherically symmetric charge distribution $Q = 0$ and this is the reason why all even-even nuclei ($J = 0$ in the ground state) have $Q = 0$ in their ground states. Positive quadrupole moments indicate an oblate spheroidal shape for nuclear charge distribution. Similarly negative values of Q indicate a prolate shape for the nuclear charge distribution.

f) lifetime: A given nuclear state can be stable or unstable. In the latter case it will decay by α , β or γ ray emission and this decay will be characterised by a decay constant λ or a meanlife $\tau (\tau = 1/\lambda)$. For a general case when the nuclear state can decay in more than one fashion (say by β decay as well as γ decay) we denote the total decay constant λ as

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \dots \quad \dots (1.3)$$

where $\lambda_1, \lambda_2, \lambda_3$ etc. are partial decay constants of each specific mode. We can also write

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \dots \quad \dots (1.4)$$

and call τ as the total mean lifetime and $\tau_1, \tau_2 \dots$ etc. as

partial mean lifetimes. According to Heisenberg's uncertainty principle we can associate with a nuclear state an energy indeterminacy ΔE corresponding to a mean lifetime τ such that $\Delta E = \Gamma = \hbar/\tau$. This indeterminacy ΔE in the energy of the state is called the level width Γ . Thus every nuclear level will have a finite level width Γ unless it has an infinite lifetime.

g) isospin: One can describe every nuclear state with an isotopic spin or isospin. However, we shall not concern ourselves with this description in this book.

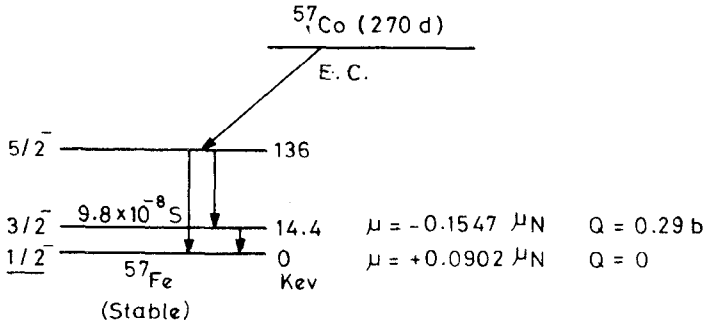


Fig. 1.3 Energy level diagram of ^{57}Fe with all assignments of spin, parity etc.

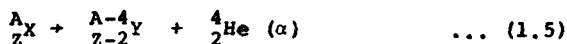
h) summary: We thus see that each nuclear state is characterised by certain values of energy, spin, parity, magnetic dipole moment, electric quadrupole moment and lifetime. This is illustrated with the help of Fig. 1.3 which shows the energy level diagram of ^{57}Fe . One of the main aims of experimental nuclear physics is to experimentally determine these numbers for energy levels of various nuclei. This experimentally measured data can then be compared with the predictions of nuclear structure theory so that our understanding of nuclear structure is more complete. In the following chapters we shall discuss various experimental methods of determining these properties of nuclei.

1.3 ALPHA RAYS

Alpha rays are positively charged particles and are

identical with doubly ionised helium atoms (He^{++}). Thus they have a mass of 4 amu and a charge of $+2e$. Their charge to mass ratio is $e/m = 4823 \text{ emu/gm}$. Alpha rays strongly ionize the medium through which they travel and their rate of energy loss being rapid, they come to rest in short distances.

Alpha rays are emitted by nuclei as a result of alpha decay. Mostly heavy nuclei ($Z > 82$) undergo natural alpha decay. Usually alpha decay is accompanied by beta decay and/or gamma decay. In alpha decay, parent nucleus transforms into a daughter nucleus and an alpha particle; thus the mass number of the parent nucleus decreases by 4 units while the atomic number decreases by 2 units. Such a nuclear transformation can be described by an equation



where X stands for parent atom and Y stands for a daughter atom (which will be chemically different from X). Some examples of alpha decay are



A plot of $N(\alpha)$ the number of alpha rays emitted per unit time against E_α the energy of alpha rays is called alpha ray spectra and it usually shows a plot similar to Fig. 1.4.

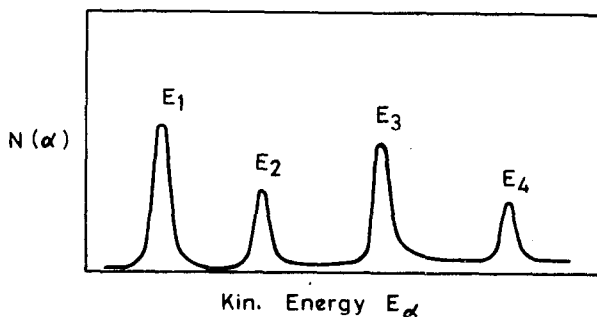


Fig. 1.4 A typical alpha ray spectra.

The observed line spectrum of alpha rays shows that alpha rays come out of the nucleus with single energies. These energy values are characteristic of a particular decay involved. The various lines are attributed to the alpha decay leading to various excited states of the daughter nucleus as shown in Fig. 1.5.

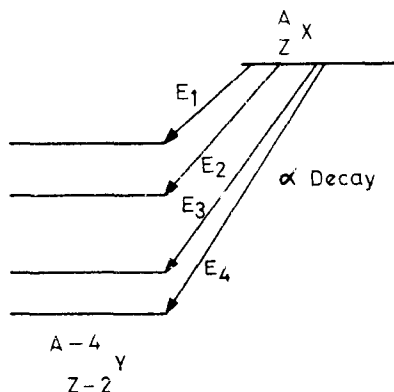


Fig. 1.5 Alpha decay leading to various excited states of the daughter nucleus.

1.4 BETA RAYS

Beta rays are also corpuscular in nature and their name collectively stands for positrons or electrons. Thus beta rays have electrical charge either positive or negative and a mass which is m_0 (the rest mass of electron). The ionising nature of beta rays is discussed in Chapter 3.

The energy spectrum of beta rays is continuous in nature (Fig. 1.6) in contrast to the line spectrum of alpha rays. The beta ray spectrum rises with energy, reaches a smooth maximum and then comes down to meet the energy axis at a point E_0 , the maximum energy. The maximum energy, E_0 , also called the end-point energy, is characteristic of a particular beta transition.

The continuous, bell-shaped nature of beta ray spectra is explained by assuming that during beta decay a neutron (proton) transforms into a proton (neutron) with the emission of electron (positron) and an antineutrino (neutrino). Symbolically beta decay is described by

$$n \rightarrow p + e^- + \bar{\nu} \quad \dots (1.7a)$$

$$p \rightarrow n + e^+ + \nu \quad \dots (1.7b)$$

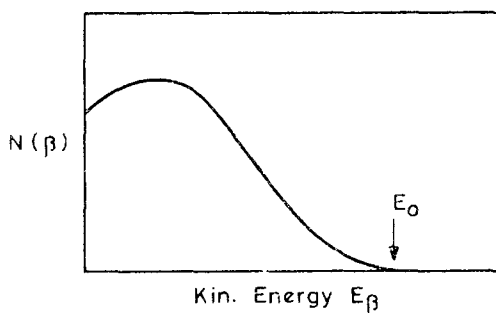
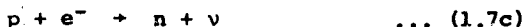


Fig. 1.6 A typical beta ray spectrum

Here ν is a neutral particle called neutrino which has almost zero mass, no charge and spin $1/2$. The antiparticle of neutrino, called antineutrino is denoted by $\bar{\nu}$ and is identical with the neutrino except for the sign of its magnetic moment and helicity. The neutrino interacts with matter in a very weak way.

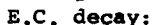
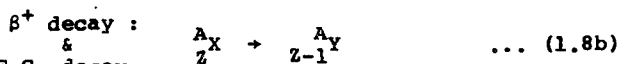
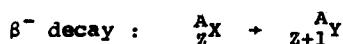
Thus the available disintegration energy of the beta decay is shared between the recoiling nucleus, beta ray and neutrino and according to laws of conservation of energy and linear momentum, this gives rise to a continuous nature of the beta ray spectrum. The energy E_0 corresponds to the case where almost all the disintegration energy is carried away by the beta particle.

There is yet another mode of beta decay called electron capture. In this process the nucleus captures one of the orbital atomic electron (usually the K-shell electron) through the following transformation:



Thus in electron capture (denoted by E.C.) the only particle that is emitted is the monoenergetic neutrino.

A parent atom A_ZX will be transformed during beta decay according to the following equations:



We thus see that for all beta decays the mass numbers of the parent and daughter nuclei are same. For β^- decay the atomic number increases by one unit while for β^+ and E.C. decay the atomic number decreases by one unit. On a typical N versus Z chart these decays will, therefore, be indicated by transformations as shown in Fig. 1.7.

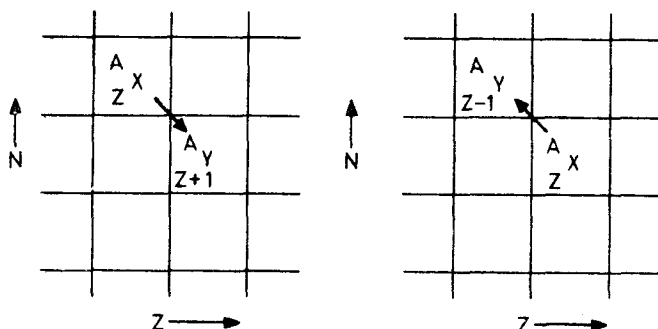
(a) β^- decay.(b) β^+ and E.C. decay.

Fig. 1.7

For each of the above decay the disintegration energy or Q-value will be given by the following formulae:

$$\beta^- \text{ decay} \quad Q = [M(A, Z) - M(A, Z + 1)]c^2 \quad \dots (1.9a)$$

$$\beta^+ \text{ decay} \quad Q = [M(A, Z) - M(A, Z - 1) - 2m_0]c^2 \quad \dots (1.9b)$$

$$\text{E.C. decay} \quad Q = [M(A, Z) - M(A, Z - 1)]c^2 \quad \dots (1.9c)$$

where the M's stand for respective atomic masses.

For a beta decay to occur Q must be positive. Thus Eqs. (1.9) give the condition on the atomic masses of the parent and daughter atom for a particular beta decay to take place.

1.5 GAMMA RAYS

Gamma rays are nothing but electromagnetic radiations of very small (10^{-10} to 10^{-12} m) wavelength. Thus we can call gamma rays as hard X-rays. However, the origin of gamma rays is different. They are emitted by the nucleus when it makes a transition from a higher excited state to a lower excited state. In radioactive decay, daughter nucleus is usually left in an excited state as a result of the alpha or beta decay of the parent nucleus. Subsequently the daughter nucleus de-excites from these higher levels by emitting gamma rays. Thus gamma rays usually follow the alpha or beta decay.

Gamma rays being electromagnetic radiation, travel with the velocity of light $c = 2.997925 \times 10^8 \text{ m s}^{-1}$. As we know, gamma rays can be considered as photons having a corpuscular nature with their energy being quantised. Thus a gamma ray having a wave frequency ν will have a quantum of energy $h\nu$ (where h = Planck's constant $= 6.626 \times 10^{-34} \text{ J s} = 4.134 \times 10^{-15} \text{ eV s}$).

If a gamma ray comes out as a result of transition from an initial nuclear state of energy E_i to a final nuclear state of energy E_f , then the energy of the gamma ray is given by

$$h\nu = E_i - E_f \quad \dots (1.10)$$

The intensity of gamma rays is given by the number of photons (quanta) passing through unit area per unit time.

Since gamma rays result from the transitions between excited nuclear states, they are monoenergetic as shown by Eq. 1.10. A general case of gamma decay following a beta decay is shown in Fig. 1.8.

In the case shown a parent nucleus A_ZX undergoes beta decay to the daughter nucleus ${}^{Z+1}_Y$. The beta transitions land in two groups (β_1 and β_2) on the excited states E_2 , and E_1 respectively of the daughter nucleus. These excited states will de-excite by emission of gamma rays. A direct transition from the second excited state to the ground state will give rise to a gamma ray γ_1 having an energy $E(\gamma_1) = E_2 - E_0$. Such a direct transition, bypassing the intermediate excited state is called a cross-over gamma transition. The other way in which the second excited level can decay is via the intermediate (first) excited state by the emission of two successive gamma rays γ_2 and γ_3 . The emission of γ_2 leaves the nucleus in its first excited state which de-excites to the ground state by emission of γ_3 . The energies of γ_2 and γ_3 are given by

$$E(\gamma_2) = E_2 - E_1 \quad \dots (1.11a)$$