Wing-Kuen Ling

NONLINEAR DIGITAL FILTERS

Analysis and Applications

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Nonlinear Digital Filters

Preface

Digital filters may be one of the most important building blocks in electrical and electronic engineering. They are widely employed in signal processing, communications, control, circuits design, electrical engineering and biomedical engineering communities. However, as digital filters are linear time invariant systems, any nonlinear behaviors occur in digital filters should be avoided. The first observed nonlinear phenomenon was the limit cycle behavior discovered in 1965 by implementing a digital filter via a finite state machine. Since then, engineers have tried to avoid the occurrence of the limit cycle behavior. In 1988, L. O. Chua and T. Lin (Chaos in Digital Filters, *IEEE Transactions on Circuits and Systems*, Vol. 35, no. 6, pp. 648–658) observed that besides the occurrence of the limit cycle behavior, digital filter may exhibit fractal behaviors if implemented via the two's complement arithmetic. This observation implies that digital filters associated with nonlinearities may exhibit chaotic behaviors and this property may be utilized in some applications. While avoiding the occurrence of nonlinear behaviors, engineers began investigating the applications of digital filters with nonlinearities and found that many applications – such as computer cryptography – secure communications, etc. Hence, the subject of nonlinear digital filters plays an increasingly important role in electrical and electronic engineering.

However, most nonlinearities associated with digital filters are discontinuous. For examples, quantization, saturation and two's complement arithmetic, all involve discontinuous nonlinear function. The analysis of systems with discontinuous nonlinearities is difficult and not many existing techniques can be applied for the analysis of these systems. Hence, the objective of this book is to introduce techniques for the analysis of digital filters with various nonlinearities as well as to explore applications using digital filters associated with nonlinearities. From Chapter 3 through to Chapter 9, techniques for the analysis of digital filters associated with various nonlinearities will be introduced. In Chapter 10, application of digital filters associated with nonlinearities is explored.

I believe that this book would be very useful for those engineers who deal with nonlinearities of digital filters. Also, it starts from very simple and fundamental techniques (initially introduced in first or second year courses) and is suitable for both the third year undergraduate and postgraduate student. In order to improve the readability of this book, examples are presented in each section or subsection and these examples directly illustrate the main concepts found in that section or subsection.

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In addition to the material covered in the text, Matlab source codes for the figures can be found on the textbook website: http://books.elsevier.com/companions/798123725363.

Contents

1

INTRODUCTION

WHY ARE DIGITAL FILTERS ASSOCIATED WITH NONLINEARITIES?

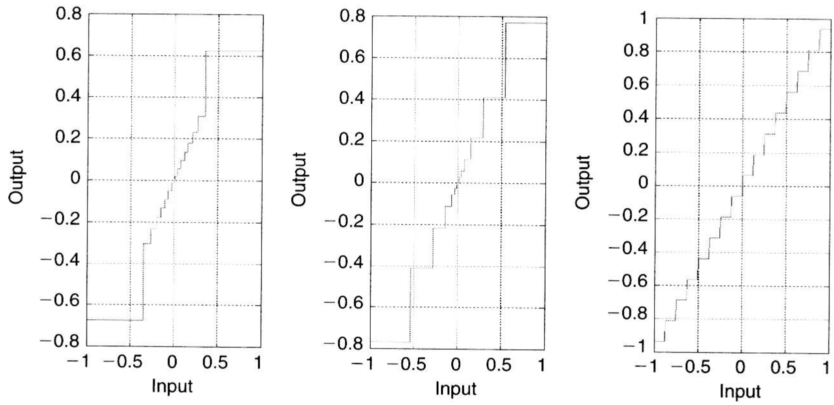
Nonlinearities are associated with digital filters mainly for implementation reasons and are tailor-made for many applications.

Nonlinearities due to implementation reasons

The most common nonlinearities associated with digital filters due to implementation reasons are quantization, saturation and two's complement. Quantization occurs because of the finite word length effects. Saturation occurs because of a constraint being imposed on the maximum bound of signals. Two's complement operation occurs because of the overflow of signals to their sign bits. Although most computers these days are more than 64 bits—and floating point arithmetic is employed for the implementation—the cost of using computers to implement a simple digital filter is very high. Thus, many simple digital filters are still implemented using very simple circuits or microcontrollers because of their low cost. In these situations, only 8, or even lower, bits fixed point arithmetic are employed for the implementation. As a result, effects due to the quantization, saturation and two's complement would be significant. Hence, the analysis and design of digital filters under these nonlinearities are important.

Quantization

Quantization is a nonlinear map that partitions the whole space and represents all of the values in each subspace by a single value. For example, for real input signals, if the input to the quantizer is nonnegative, then the output of the quantizer is represented by the value '1', and '−1' for other values. In this example, the set of real numbers is partitioned into two subsets, nonnegative and negative. '1' and '−1' are used for the representation of all values in these two subsets. It is worth noting that quantization is a noninvertible map. Hence, once



(a) (b) (c)

Figure 1.1 Input output relationships of 4 bit (a) Lloyd Max quantizer with Gaussian input statistics, (b) μ law quantizer with $\mu = 100$ and (c) uniform quantizer.

a quantization is applied, information is lost and error would be introduced. As a result, one of the most important issues in quantization is to minimize the quantization error.

Quantization can be classified as uniform quantization and nonuniform quantization. Uniform quantization partitions the whole space in a uniform manner, and vice versa for the nonuniform quantization. The most common nonuniform quantizers are the Lloyd Max quantizer and the μ law quantizer, as shown in Figure 1.1a and 1.1b, respectively. It can be seen from this figure that the quantization step sizes are unevenly distributed, while that of the uniform quantizer shown in Figure 1.1c is evenly distributed. Another type of classification of quantization is based on the number of subspaces that are partitioned. For an N bit quantization, the whole space can be partitioned into 2^N subspaces. In Figure 1.2, three 4-bit quantizers are shown, so there are exactly 16 quantization levels in each of the quantizers. In general, more bits of the quantizers would give less quantization error. However, the implementation complexity would be increased. Quantizers can also be classified as midrise or midthread quantizer. A midrise quantizer is the one that has a transition at the origin, and vice versa for the midthread quantizer. Figure 1.2a and 1.2b show the midrise and midthread quantizers, respectively.

Saturation

Saturation maps the whole space within a bounded subspace. The boundary of the bounded subspace is characterized by the saturation level. For example, an

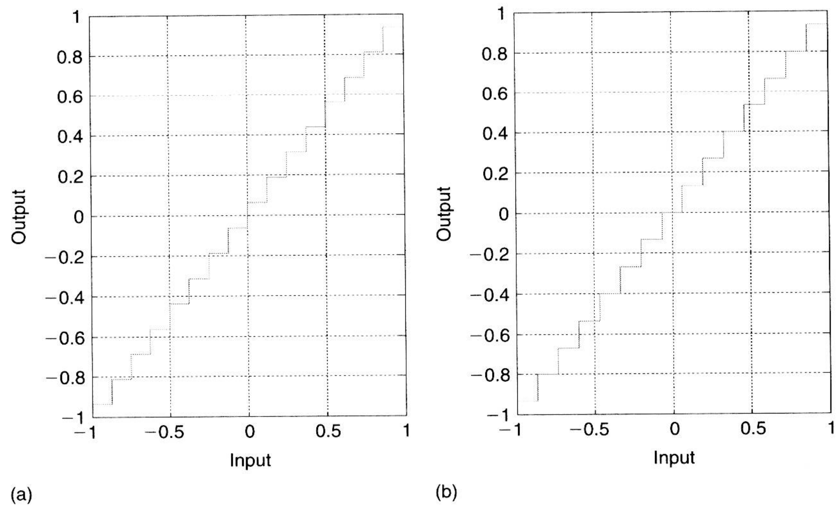


Figure 1.2 Input output relationships of 4 bit (a) midrise quantizer and (b) midthread quantizer.

output of a saturator is 1 and −1 if the input is greater than 1 and smaller than −1, respectively, and the saturation level of this saturator is 1. Figure 1.3 shows the input output relationship for this saturator.

Two's complement

Two's complement partitions the whole space into periodic subspaces and maps all subspaces into a single subspace. For example, the set of real numbers is divided into subsets with periodic 2 and all real values are mapped to values between −1 and 1 as shown in Figure 1.4.

Nonlinearities due to tailor-made applications

Digital filters are widely used in many applications in signal processing, communications, control, electrical and biomedical systems. For examples, coding and compression, denoising, signal enhancement, feature detection and extraction, amplitude and frequency demodulations, the Hilbert transform, analog-to-digital conversions, differentiation, accumulation or integration, etc., all involve digital filters. For some applications, nonlinearities are tailor-made to fit for a particular purpose.

Denoising application

Figure 1.5b shows an image corrupted by an additive white Gaussian noise. The mean square error of the noisy image is 1605.8382. Figure 1.5c shows a

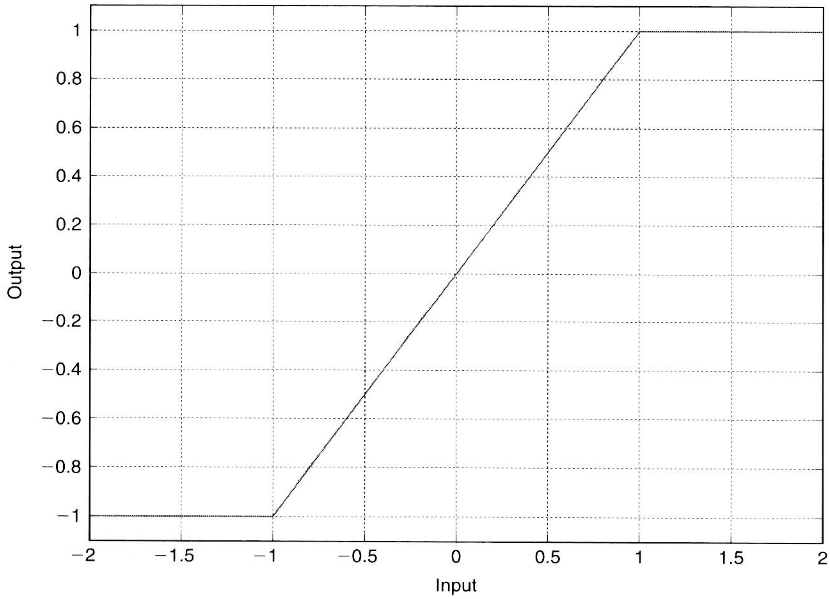


Figure 1.3 Input output relationship of a saturator with saturation level equal to 1.

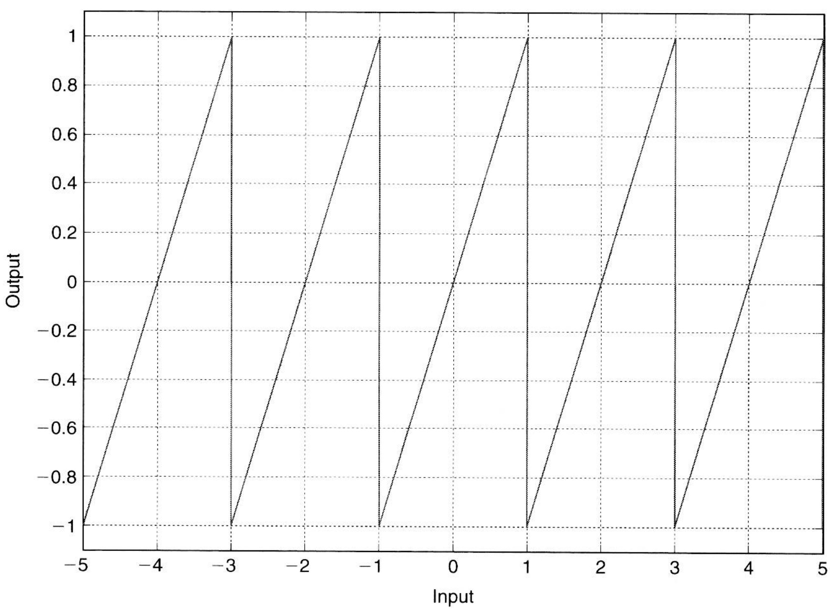


Figure 1.4 Input output relationship of a two's complement nonlinearity.

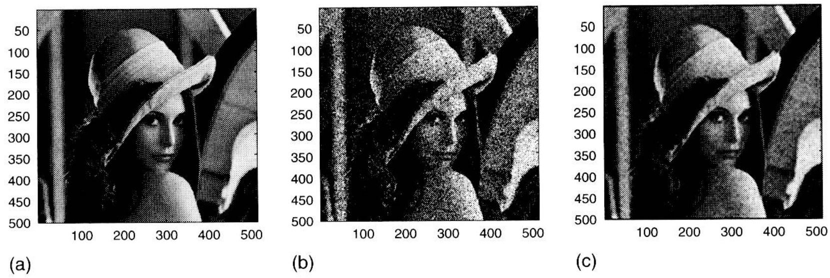


Figure 1.5 (a) Original image, (b) image corrupted by an additive white Gaussian noise and (c) image after lowpass filtering.

lowpass filtered image. The mean square error of the filtered image drops to 167.7439. This example illustrates that lowpass filtering can reduce an additive white Gaussian noise effectively. Another method for reducing additive white Gaussian noise is via the wavelet denoising approach. In this approach, signals are decomposed into different scales via a wavelet transform and wavelet coefficients are set to zero if their magnitudes are smaller than a certain threshold. It was found that this nonlinear technique can reduce additive white Gaussian noise effectively.

Coding application

Another application for imposing quantization and saturation intentionally in signal processing is coding and compression processes. Figure 1.6 shows

Figure 1.6 Quantized images.

some compressed images at different bit rates via quantization. After applying quantization, these images can be transmitted and stored efficiently.

CHALLENGES FOR THE ANALYSIS AND DESIGN OF DIGITAL FILTERS ASSOCIATED WITH NONLINEARITIES

A nonlinear system is said to be exhibiting:

- a limit cycle behavior if it exhibits a nontrivial periodic output behavior
- a fractal behavior if there is a self-similar geometric pattern exhibited on the phase plane and this self-similar geometric pattern is repeated at ever smaller scales to produce irregular shapes and surfaces that cannot be represented by classical geometry
- an irregular chaotic behavior if it is sensitive to its initial condition, a state trajectory is dense and consists of dense periodic orbits, but fractal patterns do not exhibit on the phase plane
- a nonlinear divergent behavior if some state variables tend to infinity but the corresponding linear part is strictly stable.

Although linear system theories are reasonably well-developed, these theories cannot be applied when trying to explain the above phenomena. This is because nonlinear systems are highly dependent on initial conditions and system parameters, while these properties are not found in linear systems. For nonlinear systems, it is useful to characterize the set of initial conditions and system parameters such that these phenomena would be utilized or avoided. For example, in audio applications, limit cycles correspond to annoying audio tones. Hence, it should be avoided. Moreover, nonlinear divergent behaviors should also be avoided because circuits may be damaged and serious disaster may occur. In secure communications, fractal and chaotic behaviors may be preferred because they correspond to a rich frequency spectrum.

Analyzing the stability property of nonlinear systems is also very challenging. Although Lyapunov stability theorem is powerful, it does not explain limit cycle, fractal and chaotic behaviors. Also, Lyapunov stability theorem requires a smooth Lyapunov candidate, which is difficult to find when the nonlinear function is discontinuous.

In sigma delta modulation, digital filters are usually designed in an unstable manner in order that a high signal-to-noise ratio can be achieved. In this case, it is very challenging to design a digital filter such that the stability of the system is guaranteed and limit cycle behavior is avoided.

AN OVERVIEW

This book is organized as follows. In Chapter 2, fundamentals of mathematics, digital signal processing and control theory, used throughout the book, are reviewed. These include linear algebra, fuzzy theory, sampling theorem, bifurcation theorem and absolute stability theorem.

From Chapters 3 to 9, digital filters associated with different nonlinearities are discussed. In Chapter 3, digital filters associated with the quantization nonlinearity are covered, and models for the quantization nonlinearity are introduced. Based on these quantization models, methods for improving the signal-to-noise ratios are presented. In Chapter 4, digital filters associated with the saturation nonlinearity are considered. Since oscillations may sometimes occur, the conditions for the occurrence of these oscillations and the stability conditions are presented, which is useful for both utilizing and avoiding the occurrence of these oscillations. From Chapters 5 to 8, digital filters associated with the two's complement arithmetic are discussed. Chapters 5, 6 and 7 cover autonomous, step and sinusoidal responses, respectively. In Chapter 8, complex digital filters associated with two's complement are considered, and in Chapter 9, digital filters associated with both the quantization and two's complement arithmetic are dealt with.

Finally, in Chapter 10, applications of digital filters associated with nonlinearities are presented; in particular, this chapter examines the applications on secure communications and computer cryptography.

2

REVIEWS

MATHEMATICAL PRELIMINARY

Eigen decomposition

Suppose an $n \times n$ matrix $\mathbf{A}$ has n linear independent eigenvectors, denoted as ξ_i for $I = 1, 2, \ldots, n$ and the corresponding eigenvalues are λ_i. Denote $\mathbf{T} \equiv [\xi_1, \xi_2, \ldots, \xi_n]$ and $\mathrm{D} \equiv diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$. Then $\mathbf{A}$ is diagonalizable and $\mathbf{A} = \mathbf{TDT}^{-1}$.

By employing the eigen decomposition, it can facilitate the evaluation of a power of a matrix. For example, if $\mathbf{A}$ has n linear independent eigenvectors, then $\mathbf{A}^k = \mathbf{TD}^k\mathbf{T}^{-1}$. If $\exists j \in \{1, 2, \ldots, n\}$ such that $|\lambda_j| > 1$, then some of the elements in $\lim_{k \to +\infty} \mathbf{A}^k$ would be unbounded. Hence, the BIBO stability condition of a discrete time linear system becomes all eigenvalues confined inside the unit circle.

Inverse of a map

A map $F : X \to Y$ is said to be injective if $\exists x_1, x_2 \in X$ and $y \in Y$ such that $F(x_1) = F(x_2) = y$, then $x_1 = x_2$. A map $F : X \to Y$ is said to be surjective if $\forall y \in Y$, $\exists x \in X$ such that $F(x) = y$. A map is said to be bijective if it is both injective and surjective. A map is invertible if and only if it is bijective.

Fuzzy theory

Fuzzy set

In traditional set theory, an element is either in or not in a set A, that is $x \in A$ or $x \notin A$. This kind of set is called a crisp set. A fuzzy set is a set that is characterized