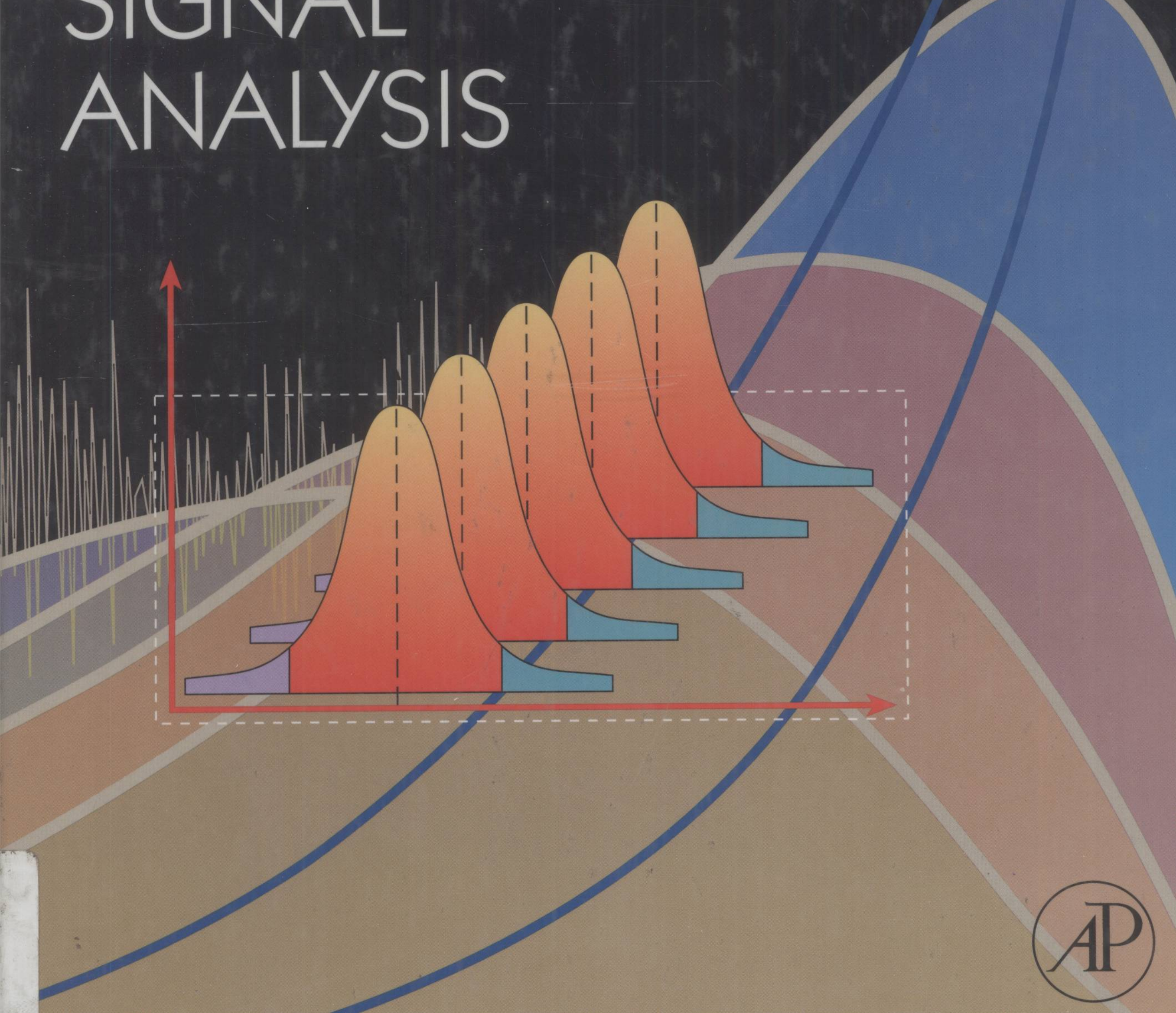
RICHARD SHIAVI

INTRODUCTION TO

APPLIED STATISTICAL SIGNAL ANALYSIS

Third Edition

Guide to Biomedical and Electrical Engineering Applications



INTRODUCTION TO APPLIED STATISTICAL SIGNAL ANALYSIS: GUIDE TO BIOMEDICAL AND ELECTRICAL ENGINEERING APPLICATIONS

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PREFACE

This book presents a practical introduction to signal analysis techniques that are commonly used in a broad range of engineering areas such as biomedical engineering, communications, geophysics, speech, etc. In order to emphasize the analytic approaches, a certain background is necessary. The book is designed for an individual who has a basic background in mathematics, science, and computer programming that is required in an undergraduate engineering curriculum. In addition one needs to have an introductory-level background in probability and statistics and discrete time systems.

The sequence of material begins with definitions of terms and symbols used for representing discrete data measurements and time series/signals and a presentation of techniques for polynomial modeling and data interpolation. Chapter 3 focuses on the windowing and the discrete Fourier transform. It is introduced by presenting first the various definitions of the Fourier transform and harmonic modeling using the Fourier series. The remainder of the book deals with signals having some random signal component and Chapter 4 reviews the aspects of probability theory and statistics needed for subsequent topics. In addition, histogram fitting, correlation, regression, and maximum likelihood estimation are presented. In the next chapter these concepts are extended to define random signals and introduce the estimation of correlation functions and tests of stationarity. Chapter 6 reviews linear systems and defines power spectra. Chapter 7 presents classical spectral analysis and its estimators. The periodogram and Blackman-Tukey methods are covered in detail. Chapter 8 covers autoregressive modeling of signals and parametric spectral estimation. Chapter 9 presents the classical uses of cross correlation and coherence functions. In particular, the practical techniques for estimating coherence function are presented in detail. Chapter 10 is a new chapter for the third edition and covers envelope estimation and kernel functions. Presentation of these topics is motivated by the growth in usage of these techniques. Envelope estimation is important not only for signals such as electromyograms but also when using high frequency carrier signals such as in ultrasound applications. The fundamentals of Hilbert transforms, analytic signals, and their estimation are presented. Kernel functions appear in the neuromuscular literature dealing with point processes such as action potentials. The main purpose is to create a continuous amplitude function from a point process. A summary of kernel functions and their implementation is presented.

The material in Chapter 10 is drawn with permission from the doctoral dissertation work of Robert Brychta and Melanie Bernard. They are both graduate students in Biomedical Engineering at Vanderbilt University. Robert's research is being done in the General Clinical Research Center and Melanie's is being done in the Visual System's laboratory.

The presentation style is designed for the individual who wants a theoretical introduction to the basic principles and then the knowledge necessary to implement them practically. The mode of presentation is to: define a theoretical concept, show areas of engineering in which these concepts are useful, define the algorithms and assumptions needed to implement them, and then present detailed examples that have been implemented using FORTRAN and more recently MATLAB. The exposure to engineering applications will hopefully develop an appreciation for the utility and necessity of signal processing methodologies.

The exercises at the end of the chapters are designed with several goals. Some focus directly on the material presented and some extend the material for applications that are less often encountered. The degree of difficulty ranges from simple pencil and paper problems to computer implementation of simulations and algorithms for analysis. For an introductory course, the environment and software recommended are those that are not overly sophisticated and complex so that the student cannot comprehend the code or script. When used as a course textbook, most of the material can be studied in one semester in a senior undergraduate or first year graduate course. The topic selection is obviously the instructor's choice.

Most of the examples and many of the exercises use measured signals, many from the biomedical domain. Copies of these are available from the publisher's Website.[3] Also available, for interactive learning, are a series of MATLAB notebooks that have been designed for interactive learning.[1,2] These notebooks are written in the integrated environment of Microsoft Word and MATLAB. Each notebook presents a principle and demonstrates its implementation via script in MATLAB. The student is then asked to exercise other aspects of the principle interactively by making simple changes in the script. The student then receives immediate feedback concerning what is happening and can relate theoretical concepts to real effects upon a signal. The final one or two questions in the notebooks are more comprehensive and ask the student to make a full implementation of the technique or principle being studied. This requires understanding all of the previous material and selecting, altering, and then integrating parts of the MATLAB script previously used.

[1] Shiavi, R., Learning Signal Processing Using Interactive Notebooks. *IEEE Transactions on Education*; 42:355-CD, 1999.
[2] Shiavi, R. "Teaching Signal Processing Using Notebooks". ASEE Annual Conference; Charlotte NC, June, 1999.
[3] Http://books.elsevier.com/companions/9780120885817

This book is dedicated to my wife, Gloria,
and to my parents who encouraged me and gave me
the opportunity to be where I am today.

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The author of a textbook is usually helped significantly by the institution by which he is employed and through surrounding circumstances. In particular I am indebted to the Department of Biomedical Engineering and the School of Engineering at Vanderbilt University for giving me some released time and for placing a high priority on writing this book for academic purposes. This being the third edition, There have been three sets of reviewers. I would like to thank them because they have contributed to the book through suggestions of new topics and constructive criticism of the initial drafts. In addition, I am very grateful to Robert Brychta and Melanie Bernard, both graduate students in Biomedical Engineering at Vanderbilt University. Their doctoral research provided the basis for the topics in Chapter 10.

LIST OF SYMBOLS

ENGLISH

$a(i), b(i)$	parameters of AR, MA, and ARMA models
A_m	polynomial coefficient
B	bandwidth
B_e	equivalent bandwidth
$c_x(k)$	sample covariance function
$c_{yx}(k)$	sample cross covariance function
C_n	coefficients of trigonometric Fourier series
$C_x(k)$	autocovariance function
$C_{xy}(k)$	cross covariance function
Cov[ ]	covariance operator
$d(n)$	data window
$D(f)$	data spectral window
e_i	error in polynomial curve fitting
$E[\ ]$	expectation operator
E_M	sum of squared errors
E_{tot}	total signal energy
f	cyclic frequency
f_d	frequency spacing
f_N	folding frequency, highest frequency component
f_s	sampling frequency
$f(t)$	scaler function of variable t

$f_x(\alpha)$, $f(x)$	probability density function
$f_{xy}(\alpha,\beta)$, $f(x, y)$	bivariate probability density function
$F_x(\alpha)$, $F(x)$	probability distribution function
$F_{xy}(\alpha,\beta)$, $F(x, y)$	bivariate probability distribution function
g	loss coefficient
g_1	sample coefficient of skewness
$h(t)$, $h(n)$	impulse response
$H(f)$, $H(\omega)$	transfer function
$I(f)$, $I(m)$	periodogram
Im( )	imaginary part of a complex function
$\Im[\ ]$	imaginary operator
$K^2(f)$, $K^2(m)$	magnitude squared coherence function
$L_i(x)$	Lagrange coefficient function
m	mean
N	number of points in a discrete time signal
p, q	order of AR, MA, and ARMA processes
P	signal power, or signal duration
$P[\ ]$	probability of [ ]
$P_m(x)$	polynomial function
q-q	quantile-quantile
r_Q	correlation coefficient for q-q plot
Re( )	real part of a complex function
$R_x(k)$	autocorrelation function
$R_{yx}(k)$	cross correlation function
$\Re[\ ]$	real operator
s_p^2	variance of linear prediction error
$S(f)$, $S(m)$	power spectral density function
$S_{yx}(f)$, $S_{yx}(m)$	cross spectral density function
T	sampling interval
$U(t)$	unit step function
Var[ ]	variance operator
$w(k)$	lag window
$W(f)$	lag spectral window
$x(t)$, $x(n)$	time function
$X(f)$, $X(m)$, $X(\omega)$	Fourier transform
z_m	coefficients of complex Fourier series

GREEK

α	significance level
$\gamma_x(t_0,t_1)$, $\gamma_{x(K)}$	ensemble autocovariance function
γ_1	coefficient of skewness
$\delta(t)$	impulse function, dirac delta function

$\delta(n)$	unit impulse, Kronecker delta function
$\varepsilon(n)$	linear prediction error
λ_i	energy in a function
$\Lambda_{yx}(f)$	co-spectrum
μ_k	kth central moment
$\eta(n)$	white noise process
$\xi(\tau)$	ensemble normalized autocovariance function
Ξ	Gaussian probability distribution function
ρ	correlation coefficient
$\rho_x(k)$	normalized autocovariance function
$\rho_{yx}(k)$	normalized cross covariance function
σ^2	variance
σ_e	standard error of estimate
σ_{xy}^2	covariance
$\phi(f)$	phase response
$\phi_{yx}(f)$, $\phi_{yx}(m)$	cross phase spectrum
$\Phi_{n(t)}$	orthogonal function set
$\varphi_x(t_0,t_1)$, $\varphi_x(k)$	ensemble autocorrelation function
$\Psi_{yx}(f)$	quadrature spectrum
ω	radian frequency
ω_d	radian frequency spacing

ACRONYMS

ACF	autocorrelation function
ACVF	autocovariance function
AIC	Akaike's information criterion
AR	autoregressive
ARMA	autoregressive-moving average
BT	Blackman-Tukey
CCF	cross correlation function
CCVF	cross covariance function
cdf	cumulative distribution function
CF	correlation function
CSD	cross spectral density
CTFT	continuous time Fourier transform
DFT	discrete Fourier transform
DTFT	discrete time Fourier transform
E	energy
erf	error function
FPE	final prediction error
FS	Fourier series

IDFT	inverse discrete Fourier transform
IDTFT	inverse discrete time Fourier transform
LPC	linear prediction coefficient
MA	moving average
MEM	maximum entropy method
MLE	maximum likelihood estimator
MSC	magnitude squared coherence
MSE	mean square error
NACF	normalized autocovariance function
NCCF	normalized cross covariance function
pdf	probability density function
PL	process loss
PSD	power spectral density
PW	power
TSE	total square error
VR	variance reduction
WN	white noise
YW	Yule-Walker

OPERATORS

$X(f)^*$	conjugation
$x(n) * y(n)$	convolution
$\hat{S}(m)$	sample estimate
$\widetilde{S}(m)$	smoothing
$\overline{x}(n)$	periodic repetition

FUNCTIONS

sgn(x)

$$\text{signum}(x) = 1, \quad x > 0$$
$$= -1, \quad x < 0$$

CONTENTS

2
EMPIRICAL MODELING AND APPROXIMATION

3
FOURIER ANALYSIS

4
PROBABILITY CONCEPTS AND SIGNAL CHARACTERISTICS

5
INTRODUCTION TO RANDOM PROCESSES AND SIGNAL PROPERTIES

6

RANDOM SIGNALS, LINEAR SYSTEMS, AND POWER SPECTRA

7
SPECTRAL ANALYSIS FOR RANDOM SIGNALS: NONPARAMETRIC METHODS

8
RANDOM SIGNAL MODELING AND PARAMETRIC SPECTRAL ESTIMATION

9
THEORY AND APPLICATION OF CROSS CORRELATION AND COHERENCE