CLASSICS IN MATHEMATICS

Richard Courant · Fritz John

Introduction to Calculus and Analysis I

微积分和数学分析引论

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Introduction to Calculus and Analysis

Volume I

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Preface

During the latter part of the seventeenth century the new mathematical analysis emerged as the dominating force in mathematics. It is characterized by the amazingly successful operation with infinite processes or limits. Two of these processes, differentiation and integration, became the core of the systematic Differential and Integral Calculus, often simply called "Calculus," basic for all of analysis.

The importance of the new discoveries and methods was immediately felt and caused profound intellectual excitement. Yet, to gain mastery of the powerful art appeared at first a formidable task, for the available publications were scanty, unsystematic, and often lacking in clarity. Thus, it was fortunate indeed for mathematics and science in general that leaders in the new movement soon recognized the vital need for writing textbooks aimed at making the subject accessible to a public much larger than the very small intellectual elite of the early days. One of the greatest mathematicians of modern times, Leonard Euler, established in introductory books a firm tradition and these books of the eighteenth century have remained sources of inspiration until today, even though much progress has been made in the clarification and simplification of the material.

After Euler, one author after the other adhered to the separation of differential calculus from integral calculus, thereby obscuring a key point, the reciprocity between differentiation and integration. Only in 1927 when the first edition of R. Courant's German *Vorlesungen über Differential und Integralrechnung*, appeared in the Springer-Verlag was this separation eliminated and the calculus presented as a unified subject.

From that German book and its subsequent editions the present work originated. With the cooperation of James and Virginia McShaue a greatly expanded and modified English edition of the "Calculus" wes prepared and published by Blackie and Sons in Glasgow since 1934, and distributed in the United States in numerous reprintings by Interscience-Wiley.

During the years it became apparent that the need of college and university instruction in the United States made a rewriting of this work desirable. Yet, it seemed unwise to tamper with the original versions which have remained and still are viable.

Instead of trying to remodel the existing work it seemed preferable to supplement it by an essentially new book in many ways related to the European originals but more specifically directed at the needs of the present and future students in the United States. Such a plan became feasible when Fritz John, who had already greatly helped in the preparation of the first English edition, agreed to write the new book together with R. Courant.

While it differs markedly in form and content from the original, it is animated by the same intention: To lead the student directly to the heart of the subject and to prepare him for active application of his knowledge. It avoids the dogmatic style which conceals the motivation and the roots of the calculus in intuitive reality. To exhibit the interaction between mathematical analysis and its various applications and to emphasize the role of intuition remains an important aim of this new book. Somewhat strengthened precision does not, as we hope, interfere with this aim.

Mathematics presented as a closed, linearly ordered, system of truths without reference to origin and purpose has its charm and satisfies a philosophical need. But the attitude of introverted science is unsuitable for students who seek intellectual independence rather than indoctrination; disregard for applications and intuition leads to isolation and atrophy of mathematics. It seems extremely important that students and instructors should be protected from smug purism.

The book is addressed to students on various levels, to mathematicians, scientists, engineers. It does not pretend to make the subject easy by glossing over difficulties, but rather tries to help the genuinely interested reader by throwing light on the interconnections and purposes of the whole.

Instead of obstructing the access to the wealth of facts by lengthy discussions of a fundamental nature we have sometimes postponed such discussions to appendices in the various chapters.

Numerous examples and problems are given at the end of various chapters. Some are challenging, some are even difficult; most of them supplement the material in the text. In an additional pamphlet more problems and exercises of a routine character will be collected, and moreover, answers or hints for the solutions will be given.

Many colleagues and friends have been helpful. Albert A. Blank not only greatly contributed incisive and constructive criticism, but he also played a major role in ordering, augmenting, and sifting of the problems and exercises, and moreover he assumed the main responsibility for the pamphlet. Alan Solomon helped most unselfishly and effectively in all phases of the preparation of the book. Thanks is also due to Charlotte John, Anneli Lax, R. Richtmyer, and other friends, including James and Virginia McShane.

The first volume is concerned primarily with functions of a single variable, whereas the second volume will discuss the more ramified theories of calculus for functions of several variables.

A final remark should be addressed to the student reader. It might prove frustrating to attempt mastery of the subject by studying such a book page by page following an even path. Only by selecting shortcuts first and returning time and again to the same questions and difficulties can one gradually attain a better understanding from a more elevated point.

An attempt was made to assist users of the book by marking with an asterisk some passages which might impede the reader at his first attempt. Also some of the more difficult problems are marked by an asterisk.

We hope that the work in the present new form will be useful to the young generation of scientists. We are aware of many imperfections and we sincerely invite critical comment which might be helpful for later improvements.

Richard Courant
Fritz John

June 1965

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