3rd Regional Conference on

Mathematical Physics

IIIrd Regional Conference on

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PREFACE

These are the proceedings of the IIIrd Regional Conference on Mathematical Physics held in Islamabad, Pakistan from 17 to 24 February, 1989. This series of Conferences was initiated by the Iranian, Pakistani and Turkish friends of ICTP Association formed in the Summer of 1986 at Trieste. However, the first in the series had already been planned before the Association was formed. The second Conference in this series was held in Adana, Turkey in September 1987. The region referred to originally included Iran, Turkey and Pakistan. The Conference covered a very broad range of topics in Mathematical Physics.

For the third Conference not only was the venue shifted but its character was also changed. The region was taken to include India and Bangladesh. Whereas the speakers in the previous Conferences had been mainly taken from the regic in the third Conference they came from all over the World. The topics of discussion were restricted practically entirely to Superstring Theory and General Relativity. Apart from these topics there were three talks on QCD and Yang-Mills theories, one on QED and one on computational techniques in Atomic Physics. Even of these the last named is not available in the proceedings. It is for this reason that we have entitled these proceedings 'Superstrings and Relativity'.

Despite the series of exciting lectures delivered at this Conference it was decided that the next Conference, scheduled to be held in Tehran, Iran in May 1990, should be more general in the topics discussed.

Proceedings of Conferences should appear promptly. On the other hand, they should include all the talks presented. It seems to be impossible to manage both the requirements. We have, therefore, made a compromise. We allowed six months for the respective authors to provide their papers and then went ahead and sent the manuscript to the press. Our arrangement of the material is simple. We present all papers on Superstrings in Part I, arranged in alphabetic order (by author's name). Next, in Part II, are the papers on Relativity, again in alphabetic order. Finally, in Part III are the rest of the papers on QCD (Yang-Mills) and QED.

In our opinion the Conference was a great success. Overall 85 people from 18 countries, of whom 35 were lecturers, participated. To make this possible financial and institutional support was necessary. We are most grateful to the Quaid-i-Azam University, particularly the Physics Department, for the institutional support. We also got institutional and financial support from the Pakistan Atomic Energy Commission and the Khan

Research Laboratories for which we are very grateful. We are also indebted to the University Grants Commission of Pakistan and the Pakistan Science Foundation for financial support. Support for foreign lecturers and participants was also provided by the International Centre for Theoretical Physics, Trieste, Italy, USAID Pakistan, the UNESCO Commission in India, ICESCO, World Laboratories, Geneva and the Ministry of Foreign Affairs, Government of France. We are indeed extremely grateful to them for their support. Ofcourse, no Conference can be run satisfactorily without some assistance from the Faculty and students. Our special thanks go to Dr. A. H. Nayyar, Rafia Ali and M. Nzar. Thanks are also due to all the other students and supporting staff, too numerous to mention.

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PART I: SUPERSTRINGS

TOROIDAL COMPACTIFICATION, MODULAR INVARIANTS, AND ORBIFOLDS

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Abstract

Modular invariants of simply laced groups are classified and modular invariant partition functionsfor the WZW models with $k = 1$ are enumerated.

In previous lectures conformal field theories have been presented in some detail and construction of partition function for these have been discussed . In particular there were discussions of the classification of various kinds of 2-dimensional conformal feild theories. One of these sets of theories is WZW models which correspends to motion of strings on group manifold G and the topological number, the so -called WZW coefficient k, which shows up as the central charge of the corresponding Kac-Moody algebra of the currents of the theory.

The classification of modular invariant partition functions of the $SU(2)$ WZW models for arbitrary level k has been successfully done and proved to be complete in a series of works by Gepner and Itzykson and Zuber and others, the so-called ADE classification. There are some partial results for other groups including $SU(3)$ and arbitrary k.

In this talk I would like to present a complete classification of the $k = 1$ case for the simply laced groups of interest in string theory *i.e.* for Rank $G \leq 24$.

The case $k = 1$ is of particular interest as it is known to be equivallent to the theory of string moving on a toroidally compatified space *i.e.* on the maximal torus of the group .

The partition function of the theory then would be of the form:

(1) $$Z(\tau, \bar{\tau}) = \Sigma_{i,j}\ N_{ij}\ \chi_i\ (\tau)\bar{\chi}_j(\bar{\tau}),$$

where χ_i are the characters of the Kac-Moody algebra corresponding to G at level $k = 1$ in the i-th representation. In the frame-work of toroidal cmpactification i lables the conjugacy class Λ_W/Λ_R, where Λ_R is the lattice on which compactification has been made. The sum is therefore a finite sum . N_{ij} are a set of non-negative integers. The condition of modular invariance of Z is :

under $T : \tau \longrightarrow \tau + 1$,

$$N_{ij} = 0 \text{ unless } W_i^2 - W_j^2 \epsilon 2Z \tag{2}$$

and under : $\tau \longrightarrow -\frac{1}{\tau}$

$$[S, N] = 0 \tag{3}$$

An obvious solution is $N_{ij} = \delta_{ij}$ which is the so called diagonal solution obtained by Gepner and Witten correspoding to a self-dual Lorentzian lattice in (Λ_W, Λ_W) .

Our main interest is in non-diagonal solutions. For this purpose we used the explicit form of the matrix S and solved eq.(1) and (2). Then we imposed the condition that the set of momenta should close under addition which in the language of conformal field theory is the condition of the closure of the operator algebra,

$$: e^{ip.X(z)} : : e^{ip'.Y(z')} := (z - z')^{p.p'} e^{i(p+p').X(z)} + \dots . \tag{4}$$

The Matreix S has the simple form

$$S_{kk'} = c \;\; e^{2\pi i w_k \cdot w_{k'}} \tag{5}$$

Thus we can readily find all the modular invariants of the group G, which taking into account the equality of characters of a representation and its conjugate reduces the dimension of the matrix $N_{ij}$and we get the only non-diagonal cases as : (we have written only N' where $N = N'\Delta$,

$$\Delta = \begin{pmatrix} 1 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \cdot \end{pmatrix},$$

$SU(8)$:

$$
\begin{array}{c} 0 \\ 4 \\ 2 \\ 1 \\ 3 \end{array}
\begin{pmatrix}
\alpha & \beta & & & \\
\beta & \alpha & & & \\
 & & \alpha+\beta & & \\
 & & & \alpha-\beta & \\
 & & & & \alpha-\beta
\end{pmatrix} \text{arbitrary}\alpha,\beta \tag{6}
$$

where the number on the left indicaties the $k-th$ representation of $SU(8)$.

$SU(9)$:

$$
\begin{array}{c} 0 \\ 3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & & \\
2\beta & \alpha+\beta & & & & \\
 & & \alpha-\beta & & & \\
 & & & \cdot & & \\
 & & & & \cdot & \\
 & & & & & \cdot
\end{pmatrix} \tag{7}
$$

$SU(12)$:

$$
\begin{array}{c} 1 \\ 5 \\ 0 \\ \cdot \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & \\
\beta & \alpha & & & \\
 & & \alpha+\beta & & \\
 & & & \cdot & \\
 & & & & \cdot \\
 & & & &
\end{pmatrix} \tag{8}
$$

$SU(15)$:

$$
\begin{array}{c} 1 \\ 4 \\ 2 \\ 7 \\ 0 \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & & \\
\beta & \alpha & & & & \\
 & & \alpha & \beta & & \\
 & & \beta & \alpha & & \\
 & & & & \alpha+\beta & \\
 & & & & & \cdot\ \cdot \\
 & & & & &
\end{pmatrix} \tag{9}
$$

$SU(16)$:

$$
(10) \qquad \begin{array}{c} 0 \\ 8 \\ 2 \\ 6 \\ 4 \\ 1 \\ \cdot \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & & & & & \\
\beta & \alpha & & & & & & & \\
 & & \alpha & \beta & & & & & \\
 & & \beta & \alpha & & & & & \\
 & & & & \alpha+\beta & & & & \\
 & & & & & \alpha-\beta & & & \\
 & & & & & & \cdot & & \\
 & & & & & & & \cdot & \\
 & & & & & & & & \cdot
\end{pmatrix}
$$

$SU(18)$:

$$
(11) \qquad \begin{array}{c} 0 \\ 6 \\ 9 \\ 3 \\ 1 \\ \cdot \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & & & & \\
2\beta & \alpha+\beta & & & & & & \\
 & & \alpha & \beta & & & & \\
 & & 2\beta & \alpha+\beta & & & & \\
 & & & & \alpha-\beta & & & \\
 & & & & & \cdot & & \\
 & & & & & & \cdot & \\
 & & & & & & & \cdot
\end{pmatrix}
$$

$SU(20)$:

$$
(12) \qquad \begin{array}{c} 1 \\ 9 \\ 3 \\ 7 \\ 0 \\ \cdot \\ \cdot \end{array}
\begin{pmatrix}
\alpha & \beta & & & & & \\
\beta & \alpha & & & & & \\
 & & \alpha & \beta & & & \\
 & & \beta & \alpha & & & \\
 & & & & \alpha+\beta & & \\
 & & & & & \cdot & \\
 & & & & & & \cdot
\end{pmatrix}
$$

$SU(21)$:

$$
(13) \qquad
\begin{array}{c}
1\\ 8\\ 2\\ 5\\ 4\\ 10\\ 0\\ \cdot\\ \cdot\\ \cdot
\end{array}
\begin{pmatrix}
\alpha & \beta & & & & & & & \\
\beta & \alpha & & & & & & & \\
& & \alpha & \beta & & & & & \\
& & \beta & \alpha & & & & & \\
& & & & \alpha & \beta & & & \\
& & & & \beta & \alpha & & & \\
& & & & & & \alpha+\beta & & \\
& & & & & & & \cdot & \\
& & & & & & & & \cdot \\
& & & & & & & \cdot &
\end{pmatrix}
$$

$SU(24)$:

$$
(14) \qquad
\begin{array}{c}
0\\ 1\\ 2\\ 4\\ 5\\ 6\\ 3\\ 9
\end{array}
\begin{pmatrix}
M_0 & & & & & & & \\
& M_1 & & & & & & \\
& & M_2 & & & & & \\
& & & M_4 & & & & \\
& & & & M_5 & & & \\
& & & & & \alpha_0+\beta_0 & & \\
& & & & & & \alpha_0-\beta_0 & \\
& & & & & & & \alpha_0-\beta_0
\end{pmatrix}.
$$

where

$$
M_0 = \begin{array}{c} 0 \\ 12 \end{array} \begin{pmatrix} \alpha_0 & \beta_0 \\ \beta_0 & \alpha_0 \end{pmatrix}, \; M_1 = \begin{array}{c} 1 \\ 7 \end{array} \begin{pmatrix} \alpha_1 & \beta_1 \\ \beta_1 & \alpha_1 \end{pmatrix}, \; M_2 = \begin{array}{c} 2 \\ 10 \end{array} \begin{pmatrix} \alpha_2 & \beta_2 \\ \beta_2 & \alpha_2 \end{pmatrix},
$$

and

$$
M_4 = \begin{array}{c} 4 \\ 8 \end{array} \begin{pmatrix} \alpha_0 & \beta_0 \\ \beta_0 & \alpha_0 \end{pmatrix}, M_5 = \begin{array}{c} 5 \\ 11 \end{array} \begin{pmatrix} \alpha_1 & \beta_1 \\ \beta_1 & \alpha_1 \end{pmatrix}, \left\{ \begin{array}{l} \alpha_0+\beta_0 = \alpha_2+\beta_2, \\ \alpha_0-\beta_1 = \alpha_1+\beta_1, \\ \alpha_1-\beta_1 = \alpha_2-\beta_2, \end{array} \right.
$$

$SU(25)$:

$$
(15)\qquad \begin{array}{c} 0 \\ 5 \\ 10 \\ \cdot \\ \cdot \\ \cdot \\ \end{array} \begin{pmatrix} \alpha & \beta & \beta & & & & \\ 2\beta & \alpha+\beta & 2\beta & & & & \\ 2\beta & 2\beta & \alpha+\beta & & & & \\ & & & \alpha-\beta & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & \cdot \end{pmatrix}, \ \alpha, \beta
$$

The groups $SO(2n)$ and E_6 and E_7 and E_8 have no non-diagonl N as soluttion .(Note that we have identical charactorsfor some representationshere and it is only in this sense that N is diagonal) .

The next step would be to impose the condition of positivity on N_{ij} which obviously reduces the possible solutions above .

Finally imposition of closure of operator algebra (The demand that the states close under addition) will reduce the set of solutions further . As an example take the first non-trivial $SU(n)$: $SU(8)$, then we get only one solution for N

$$
(16)\qquad N'_2 = \begin{array}{c} 0 \\ 4 \\ 2 \\ 1 \\ \cdot \\ \cdot \end{array} \begin{pmatrix} 1 & 1 & & & & \\ 1 & 1 & & & & \\ & & 2 & & & \\ & & & 0 & & \\ & & & & \cdot & \\ & & & & & \cdot \end{pmatrix}
$$

which corresponds to the Hilbert space,

$$
(17)\qquad \begin{aligned} H = (0,0) + (0,4) + (4,0) + (4,4) + (2,2) + (2,6) + (6,2) + (6,6) = \\ (0+4)^2 + (2+6)^2 \end{aligned}
$$

Note that the $N_{22} = 2$ corresponds to the appearanse of both states 2 and 6 which are cojugate. It in easily verified that this Hilbert space corresponds to a self-dual Lorentzian lattice of states .

The other cases could also be similarly analysed and we have in fact found all those possible Hilbert spaces of the conformal feild theory. We could list them just as in the case of $SU(8)$.