

Forces and fields



031
F1
V.1

8064946

Forces and fields

Advanced Physics Project for Independent Learning
Student's Guide



John Murray
in association with
Inner London Education Authority



E8064946

Acknowledgments

Thanks are due to the following who have kindly permitted the reproduction of copyright photographs: cover, B.B.C.; page 6, left, Colorsport, centre, Heinemann Educational Books Ltd., right, Photri; page 18, Lockyer Collection, Nat. Met. library; page 50, H. L. Collin; page 70, Jeol (UK) Ltd; figure 1.4, Philip Harris Ltd; figure 1.6, Mary Evans; figure 2.2, E. Leybold Nachfolger; figures 4.2a, 4.3, E.D.C.; figure 5.8b, Leybold-Heraeus.

The article on pages 36-7 is adapted from 'Modern Electrostatics' by A. W. Bright, *Physics Education*, Volume 6, 1974. The article on pages 66-7 is adapted from *Semiconductor Devices* by J. J. Brophy © 1977. Used with permission of McGraw-Hill Book Company. The questions on pages 67-8 are reproduced by permission of the Joint Matriculation Board.

Project team John Bausor (Director)
Leslie Beckett
Allan Covell
David Davies
Martin Hollins

© Inner London Education Authority 1980

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of John Murray (Publishers) Ltd., 50 Albemarle Street, London W1X 4BD

Printed in Great Britain by Martin's of Berwick

0 7195 3600 6 (Student's guide)
0 7195 3601 4 (Teacher's guide)

Summary of the features of this guide

This Unit is not one of the starting points of the APPIL course, so only a brief reminder of the purpose of the various features follows. For a fuller explanation of the way APPIL Units are designed you should read the Student's Handbook.

This guide is a programme for independent learning. It is not designed to be read through like a book, but is more like a set of instructions on how to proceed with your studies.

Objectives are listed at the beginning of each chapter: scan them before you start, and check them off after you finish.

Experiments are also listed at the start of a chapter with an indication of the time required and a note if the experiment has any special feature. You should take account of this in planning your work schedule. Full details of experiments are given at the end of the guide.

Self-assessment questions are to check your understanding of work covered, so you should be able to do most of them quite easily. Answers are given at the end of this guide.

Development questions involve you in a proof or idea which is being introduced. The answers are usually in the text or, for questions marked with an asterisk, at the end of the guide. Involving yourself in the development makes your learning more effective.

Study questions require resources apart from this guide, for example textbooks or experimental results. Books which are useful throughout are given at the start of each chapter, and other references are given where appropriate. Full answers to these questions are important as they will provide the basis for revision notes for the final examination.

Questions on objectives at the end of each chapter are used to check that the objectives for the chapter have been achieved.

Your teacher also has available for use at the appropriate time **an end-of-Unit test** and suggestions for **examination questions** taken from past A level papers.

Audio-visual aids. A film loop is required in chapter 1. Check with your teacher whether this is available.

Extensions provide a more detailed treatment of some topics than will be required by all students. There are a number of SYLLABUS EXTENSIONS for students following particular syllabuses. You should be advised by your teacher which you will need to complete.

Organising your time is important when there are activities like experiments which require special facilities. When you start a chapter scan through it first. Using the summary and the recommended study time make a work schedule for the chapter, to fit in with your timetable. The progress monitor, available from your teacher, should help you keep to your schedule.

Introduction to the Unit

In this Unit you will be studying an important concept in physics—a field of force—by considering gravitational, electric and magnetic fields. In most cases, the topics are covered to A level standard: others will be developed in a later Unit, and there is an indication at appropriate points in the text.

Chapter 1 opens with a consideration of gravitational forces and how they account for the observed motion of planets and satellites. Information from the Apollo 11 space flight is used to verify Newton's prediction of an inverse square law of gravitational force. Results from experiments with charged spheres lead to a similar relationship for electric forces.

Chapter 2 considers the way in which a scientist analyses the forces between masses and between like and unlike charges by introducing the idea of a field. The way in which a field can be described and its strength measured is studied in detail. The chapter concludes with a discussion of the electric field around different shaped conductors, considers some important industrial applications in electrostatics and the Van de Graaff generator.

Chapter 3 provides an introduction to capacitors, including their design and the derivation of formulae, which enables the energy stored in a capacitor to be calculated. An experiment with a large parallel-plate air capacitor and a reed switch should provide data to show the factors which determine its capacitance and to enable an important physical constant, called the permittivity constant, to be calculated.

Chapter 4 is concerned with magnetic fields. How such a field can be represented and its strength measured, and how it interacts with moving charges is studied in detail. The magnetic field due to different shapes of conductors is investigated using a Hall probe and equations which relate the magnetic field to the current and geometry of the conductors are derived. The chapter concludes with a development of the principles of an important measuring instrument, the moving-coil galvanometer.

Chapter 5 is about motion in gravitational, electric and magnetic fields. In section 5.1 the motion of satellites in a circular orbit and the conditions under which a body can escape from a gravitational field are studied. In section 5.2 the forces acting on charged particles in electric and magnetic fields are studied and used to explain how electron beams are accelerated, focused and deflected in an oscilloscope and how high energy charged particles are produced in a cyclotron.

Recommended study times

You should spend about 6 to 7 weeks on this Unit, as follows:

Chapter 1	1 week	Chapter 4	2 weeks
Chapter 2	2 weeks	Chapter 5	1 week
Chapter 3	1 week		

Contents

Summary of the features of this guide 3

Introduction to the Unit 4

Chapter 1 Gravitational and electric forces 6

- 1.1 Gravitational forces 8
- 1.2 Newton's law of gravitation 8
- 1.3 Universal gravitational constant 10
- 1.4 Electric forces 12
- 1.5 Coulomb's law 14
- Questions on objectives 17

Chapter 2 Field and potential 18

- 2.1 Fields of force 20
- 2.2 Radial field 22
- 2.3 Potential 25
- 2.4 Potential in a radial field 29
- 2.5 Fields near conductors 34
- Questions on objectives 37

Chapter 3 Capacitance and capacitors 38

- 3.1 Introduction 40
- 3.2 Capacitance 40
- 3.3 The parallel-plate capacitor 42
- 3.4 Capacitor networks 45
- 3.5 Energy stored in a capacitor 46
- Questions on objectives 47

Chapter 4 Magnetic forces and fields 50

- 4.1 Magnetic fields 52
- 4.2 Forces in magnetic fields 54
- 4.3 Magnetic flux density 56
- 4.4 Force on a moving charge 57
- 4.5 Measurements of magnetic flux density 59
- 4.6 Measurement of current 63
- Questions on objectives 68

Chapter 5 Motion in fields 70

- 5.1 Motion in a gravitational field 72
- 5.2 Motion in electric and magnetic fields 75
- Questions on objectives 79

Experiments

- FF1: Investigation of the force between two charged spheres 80
- FF2: Electric field patterns 81
- FF3: The electric field between two parallel plates 82
- FF4: Investigation of the potential around a sphere 83
- FF5: Electrostatic investigations 84
- FF6: The parallel-plate capacitor 86
- FF7: Energy stored in a capacitor 88
- FF8: Force on a current in a magnetic field 89
- FF9: Investigation of magnetic flux density using a Hall probe 91

Answers

- Chapter 1 93
- Chapter 2 94
- Chapter 3 97
- Chapter 4 99
- Chapter 5 102

Abbreviations used in the text 103

Standard symbols used in this Unit 104

Values of physical constants 104

Chapter

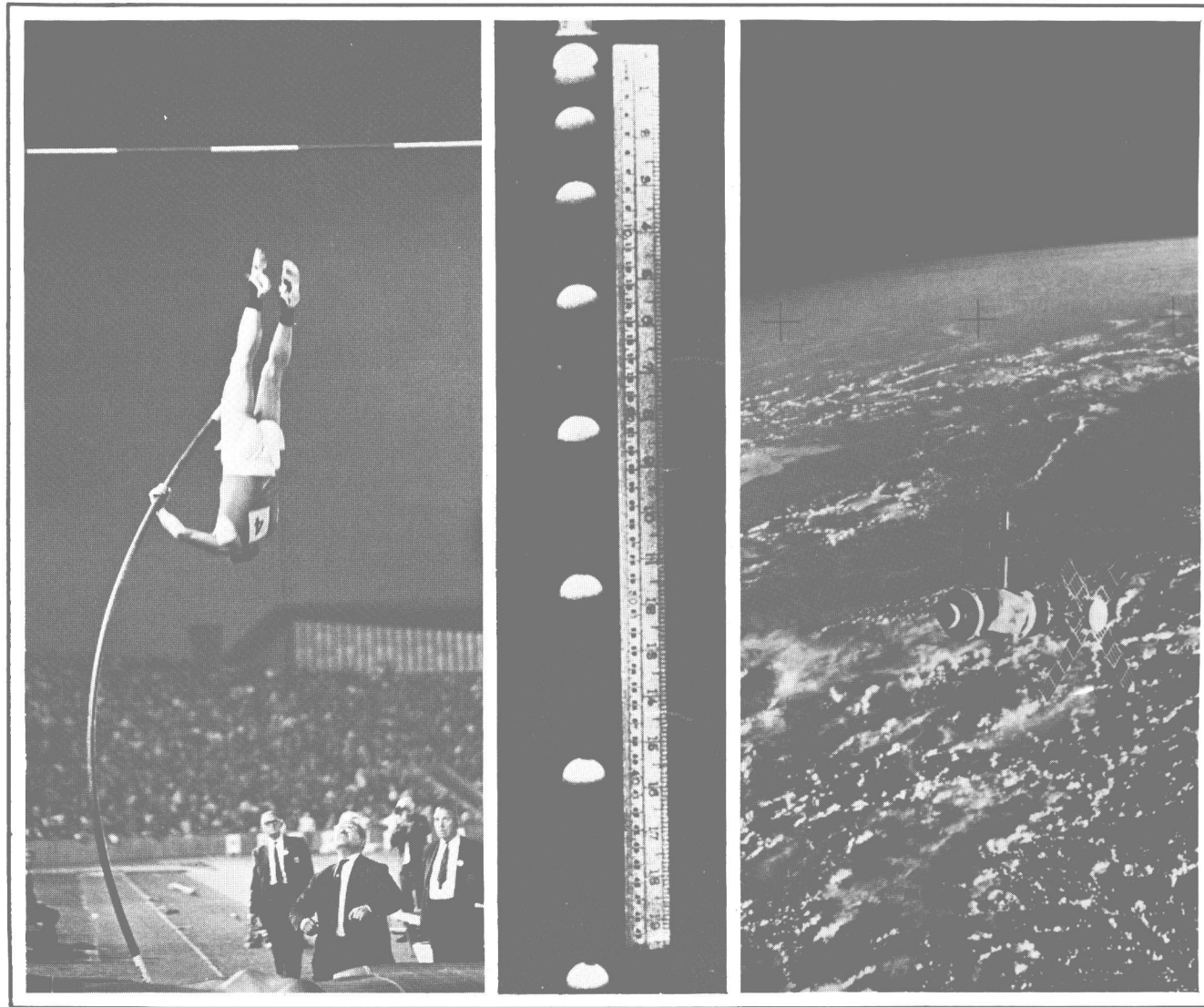
1

Gravitational and electric forces

Aim

The aim of this chapter is to help you to develop an understanding of two sorts of forces which act at a distance; the gravitational forces between particles, and between larger bodies such as the Earth and a satellite; the electric forces between point charges and bodies such as charged spheres.

You will do experiments and use data to show that both gravitational and electric forces follow an inverse square law.



Chapter

1

Study time: 1 week

Objectives

When you have completed the work in this chapter you should be able to:

- 1 Use correctly the following scientific terms: conductor, insulator, unlike and like charges, coulomb, permittivity, electrostatic induction.
- 2 Define and derive the dimensions for the universal gravitational constant, (G).
- 3 Recall the standard symbols and units for: charge, gravitational constant, permittivity constant (ϵ_0).
- 4 State how the force of gravitational attraction between two particles depends upon the mass of the particles and their distance apart.
- 5 State how the force of electric attraction or repulsion between two charged particles in a vacuum depends upon the magnitude of the charges and their distance apart.
- 6 Solve problems involving gravitational and electric forces.
- 7 SYLLABUS EXTENSION
Describe a laboratory experiment to determine the gravitational constant.
- 8 SYLLABUS EXTENSION
State and explain Kepler's laws of planetary motion.

Experiments in chapter 1

FF1 Investigation of the force between two charged spheres
(1 hour)

References

Akrill	Chapter 14
Bennet	Chapter 11
Duncan FWA	Chapter 1
Duncan MM	Chapter 7
Nelkon	Chapters 2, 31
Wenham	Chapters 34, 35, 36
Whelan	Chapters 20, 42
Williams	Chapter 20

1.1 Gravitational forces

Some effects of gravitational forces are familiar to us, for example the opening photographs of this chapter show that: the Earth exerts a pull on ourselves; an object when released and allowed to fall freely will accelerate towards the centre of the Earth; an artificial satellite can orbit the Earth.

The central figure shows a multiframe photograph of a ball falling freely. The Earth exerts a pull on the ball. There is a mutual attraction between the Earth and the ball.

Q 1.1 Self-assessment question

- At the Earth's surface freely falling bodies have an acceleration of 10 m s^{-2} . What is the gravitational force per kilogram?
- Calculate the gravitational force which is exerted on a ball of mass $5 \times 10^{-2} \text{ kg}$.
- What is the force which the ball exerts on the Earth?
- Why doesn't the Earth appear to accelerate towards the ball? Explain your answer. ■

This attractive force is called a *gravitational* force or the force of gravity. It also accounts for the observed motion of the planets and the behaviour of satellites. The force of attraction exerted by the Earth on the satellite provides the necessary centripetal force to keep it in its orbit. This explanation was first put forward by Sir Isaac Newton in a book called *Principia Mathematica*—the mathematical principles of natural philosophy. In it he proposed a precise law of attraction to explain the motion of the Moon around the Earth and the planets around the Sun (figure 1.1), and suggested that the same law applies also to objects near the Earth. At the time,

this was a strange idea: people thought 'earthly' and 'heavenly' objects were quite different.

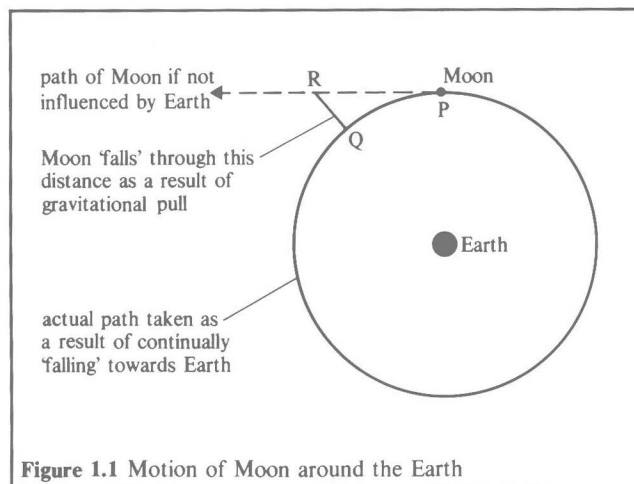


Figure 1.1 Motion of Moon around the Earth

1.2 Newton's law of universal gravitation

Newton stated that all particles of matter attract each other, and that the force between them depends on their masses and their distance apart. Consider two particles A and B, masses m_1 and m_2 respectively, separated by a distance r (figure 1.2). A particle is a convenient imaginary object, which has mass but no size. Matter can be thought of as made of particles at a suitable distance apart.

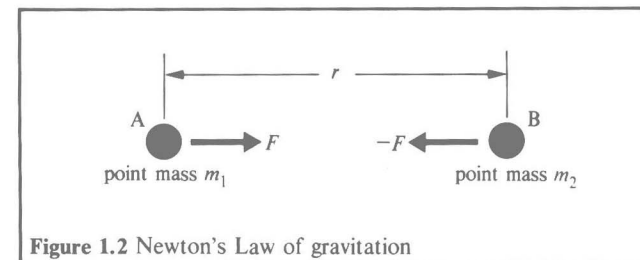


Figure 1.2 Newton's Law of gravitation

Particle A exerts a force F on particle B, and B exerts an equal and opposite force on A. According to Newton, the magnitude of this force is

- directly proportional to the product of the masses of the two particles

$$F \propto m_1 m_2 \quad (r \text{ constant})$$

- inversely proportional to the square of their distance apart.

$$F \propto \frac{1}{r^2} \quad (m_1 \text{ and } m_2 \text{ constant})$$

Combining these statements, the gravitational attraction

F between two particles of masses m_1 and m_2 separated by a distance r is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

or \rightarrow

$$F = G \frac{m_1 m_2}{r^2}$$

where G is a universal constant whose value must be found experimentally.

Note: Newton showed that from a point outside a sphere the sphere appears to behave as if all its mass is concentrated at its centre. He delayed the publication of his work on gravitation for over twenty years until he had proved this. Thus we can apply his law of gravitation to bodies such as the Earth and the Moon, and r will be the distance between the centres of the masses.

Q 1.2 Self-assessment question

Suggest why Newton assumed that the gravitational force was

- (a) proportional to the mass of the attracted body,
- (b) proportional to the mass of the attracting body. ■

An inverse square law

About a century before Newton, over a period of twenty years the Danish astronomer Tycho Brahe made accurate observations on the motion of the planets. This data was studied by Kepler, who put forward a description of planetary motion which is summarised in three laws. These laws describe how the planets behave in the Solar System. They do not indicate why a planet moves in an elliptical orbit, or what provides the central force, that is, the force towards a focus of the ellipse.

Kepler's laws provided Newton with a summary of the observed motion of the planets. From these he was able to show that the force on the planet must be directed towards the Sun and that it must be inversely proportional to the square of the distance between the Sun and the planet (assuming a circular orbit).

Q 1.3 Development question

For a circular orbit we can express Kepler's third law as

$$T^2 \propto r^3$$

where T is the period of revolution and r is the radius of the circle. We will now show how this leads to an inverse square law of attraction.

Suppose that a planet of mass m moves with a speed v in a circular orbit of radius r , figure 1.3.

(a) The acceleration of the planet towards the centre of the orbit is v^2/r . Show that the force which is necessary to cause the planet to move in this orbit is given by

$$F = \frac{m v^2}{r}$$

(b) If T is the time taken for the planet to make one complete revolution show that

$$v = \frac{2\pi r}{T}$$

(c) Hence show that

$$F \propto \frac{1}{r^2} \quad \blacksquare$$

The law of gravitation which Newton developed to explain the motion of the planets has in recent times been used to calculate and predict the paths of satellites and lunar probes. The information in table 1.1 is from the

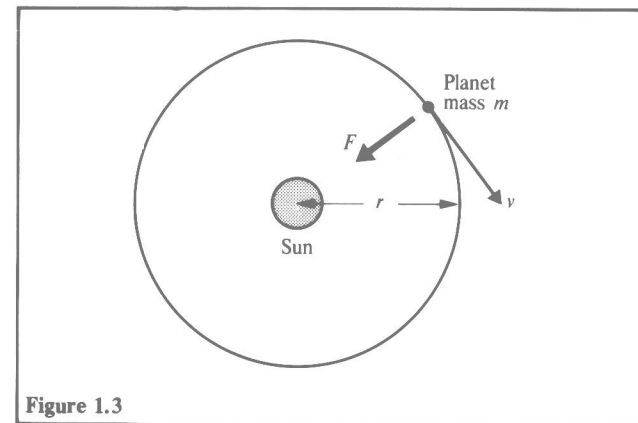


Figure 1.3

Apollo 11 space flight, which covers a period when the spacecraft was travelling more or less directly away from the centre of the Earth, with the motors shut down. This data can be used to support the inverse square law of attraction.

Table 1.1

Time from launch/ h: min: s:	Distance from Earth's centre $r/10^6$ m	Speed $v/\text{m s}^{-1}$
03:58:00 (2)	26.3	5374
04:08:00 (2A)	29.0	5102
05:58:00 (3)	54.4	3633
06:08:00 (3A)	56.4	3560
09:58:00 (4)	95.7	2619
10:08:00 (4A)	97.2	2594
19:58:00 (5)	169.9	1796
20:08:00 (5A)	170.9	1788

Q 1.4 Self-assessment question

(a) In table 1.1 the pair of values, (2) and (2A), are only ten minutes apart in time. In this time the speed of the spacecraft decreased by 272 m s^{-1} . At what mean rate was the speed changing?

(b) Write down (without further calculation) an estimate for the gravitational pull of the Earth on each kilogram of the spacecraft at a distance of $27.7 \times 10^6 \text{ m}$ from the Earth, this being the average of the distances at points (2) and (2A).

(c) Use the other pairs of data points to plot a graph which tests whether the gravitational force on one kilogram varies as the inverse square of the distance.

(d) Plot a second graph to check by means of a possible straight line whether the data corresponds to an inverse square law relation. ■

EXTENSION

Q 1.5 Study question

Explain with the aid of figure 1.1 how Newton tested the universality of the law of gravitation by comparing the gravitational effect of the Earth on the Moon with the effect of a body falling near the Earth's surface. ■

SYLLABUS EXTENSION

Kepler's laws

Q 1.6 Study question

State and explain Kepler's laws of planetary motion. ■

1.3 Universal gravitational constant

The constant G is called the universal constant of gravitation; it has a magnitude of $6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Rearranging the equation $F = G m_1 m_2 / r^2$ we have

$$G = \frac{F r^2}{m_1 m_2}$$

From this relationship we can see that G is numerically equal to F when r , m_1 and m_2 are all equal to unity. G is numerically equal to the gravitational attraction between two point or spherical 1 kg masses when they are 1 m apart. This is a very small force indeed!

Q 1.7 Self-assessment question

How does this force compare with the Earth's gravitational pull on a mass of 1 kg? ■

Q 1.8 Self-assessment question

(a) Show that the dimensions of G are $[\text{M}^{-1} \text{L}^3 \text{T}^{-2}]$

(b) What do you understand by the term a universal constant? ■

Q 1.9 Self-assessment question

(a) A communications satellite has a weight W when on the surface of the Earth. What will be the Earth's gravitational pull on the satellite when it is in a circular orbit of radius twice the radius of the Earth?

(b) It is more realistic to consider the satellite in an orbit of radius 1.1 times the Earth's radius (i.e. about 640 km above the Earth). What is the gravitational pull in this case? ■

Q 1.10 Self-assessment question

Two spherical bodies A and B of the same uniform density, situated in outer space, attract each other with a force F . What is the attraction between two similar bodies of the same density but each of twice the diameter and situated three times as far apart as A and B? ■

Q 1.11 Self-assessment question

Estimate the force of gravitational attraction between two adult human beings when standing side by side. ■

SYLLABUS EXTENSION

Determination of G

There have been many experiments to determine G . Early methods were concerned with measuring the deviation of a pendulum bob line from the vertical due to the presence of a large mountain. The first laboratory method was carried out by Cavendish in 1798 who used a torsion balance to measure the force of attraction between two small masses. His apparatus was modified by Boys in 1895 who used a quartz fibre suspension system. The quartz fibre was capable of supporting a considerable mass and also offered less resistance to twist than a wire of the same strength. This enabled him to reduce the scale of the apparatus and consequently he avoided trouble due to draughts.

Figure 1.4 shows a modern simple version of Boys' apparatus.

Q 1.12 Study question

Make detailed notes on a method of determining the gravitational constant G , paying attention to

- (a) a description of the apparatus, including a diagram,
- (b) the factors which determine the deflecting and restoring torques, and
- (c) how a value for G can be calculated. ■

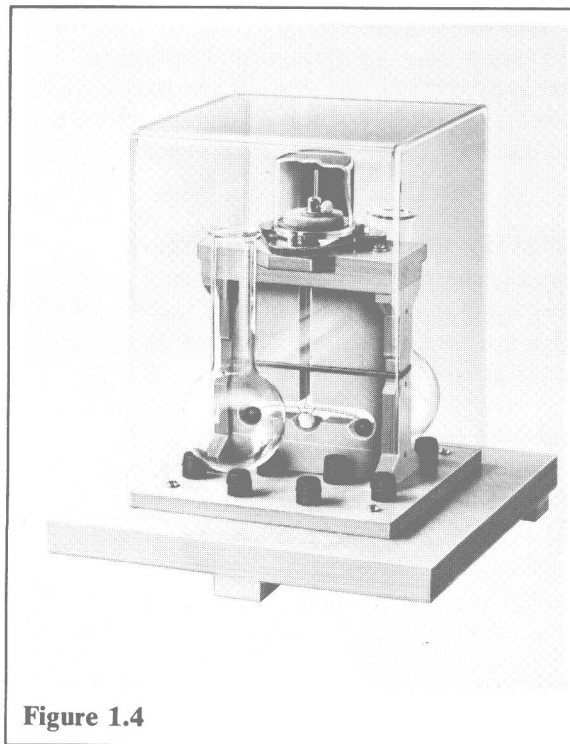


Figure 1.4

AV FF 1 Filmloop Measurement of G —the Cavendish experiment

This shows the principle of the torsion balance method for determining G , using film of an experiment in progress.

EXTENSION

Development of theory

The history of the development of the theory of planetary motion provides an excellent example of the growth and use of theory in science.

Background reading

You should consult one or more of the following references:

Rogers, E. M. *Physics for the inquiring mind*. Part 2 Astronomy: a history of theory. OUP, 1960.

Gamow, G. *Gravity*, Heinemann Science Study Series no. 17, 1961.

Wenham, E. J. *Planetary astronomy*. Longmans Physics Topics, 1969.

PSSC. *Physics*. Chapter 22 Universal gravitation and the solar system. Heath, 1971.

Feynman, R. *The character of physical law*. Chapter 1 The law of gravitation, an example of a physical law. BBC, 1965.

Q 1.13 Study question

Discuss the contributions of Copernicus, Kepler and Newton to the study of gravitation, and explain how the theory of universal gravitation was established. ■

Is gravity getting weaker?

Einstein's general theory of relativity, and theories which predict that the force of gravity diminishes as the universe expands, are discussed in a recent article in *Scientific American*.

Background reading

Van Flandern, T. C. 'Is gravity getting weaker?' *Scientific American*, February 1976.

1.4 Electric forces

In the previous section you learned that there is a force of gravitational attraction between all bodies. This has practical consequences only if at least one of the attracting bodies has enormous mass, as has the Earth. Gravitational forces are not the only forces acting at a distance between material bodies. You have no doubt discovered that when two strips of polythene are rubbed with a cloth they exert forces on each other. These forces are called *electrostatic* or *electric* forces. To explain this we say that the materials have acquired a charge. Experiments show that there are two types of charge, one positive and the other negative, and that like charges repel and unlike charges attract.

Q 1.14 Study question

What is the evidence that there are *only two* types of charge? ■

The electron model

All materials are atomic in structure. Each atom can be considered as a neutral, uncharged body, but it consists of a positively charged nucleus surrounded by negatively charged electrons. The electrons are held near the nucleus by electrostatic attraction. In certain substances it is possible to remove some of the surface electrons, leaving a positively charged surface on the material and taking away an equal negative charge.

Q 1.15 Study question

How does the electron model explain
(a) the fact that when a polythene rod is rubbed with a cloth it acquires a negative charge,
(b) the fact that when cellulose acetate is rubbed with a cloth it acquires a positive charge? ■

Conductors and insulators

Experiments show that we can distinguish between two types of materials

- (i) those that retain an electric charge on the surface—called insulators or non-conductors (such as polythene, glass, etc.),
- (ii) those that allow charges that are placed on the surface, which when isolated repel each other, to redistribute themselves over the surface—called conductors (such as metals or carbon).

We can also distinguish between conductors and insulators by their ability to conduct an electric current. (This is considered in the Unit *Electrical properties*.)

Q 1.16 Study question

Explain how the electron model can account for the difference between conductors and insulators. ■

Charging by induction

Figure 1.5 illustrates how a metal sphere can be charged by a process known as induction. The sequence of events is as follows:

- (a) When the rod of polythene is rubbed with a cloth, the cloth loses electrons to the polythene; the cloth is charged positively, the rod negatively.
- (b) When the cloth is removed the rod remains charged.
- (c) When a conducting metal sphere on an insulating stand is brought near the rod, some of the conduction electrons are repelled to one side leaving the sphere with different charges on the two sides. This separation of charges by an electric force is known as *induction*.
- (d) The sphere is now connected to earth by touching it.
- (e) Finally, the polythene rod is removed and the metal sphere has a positive charge.

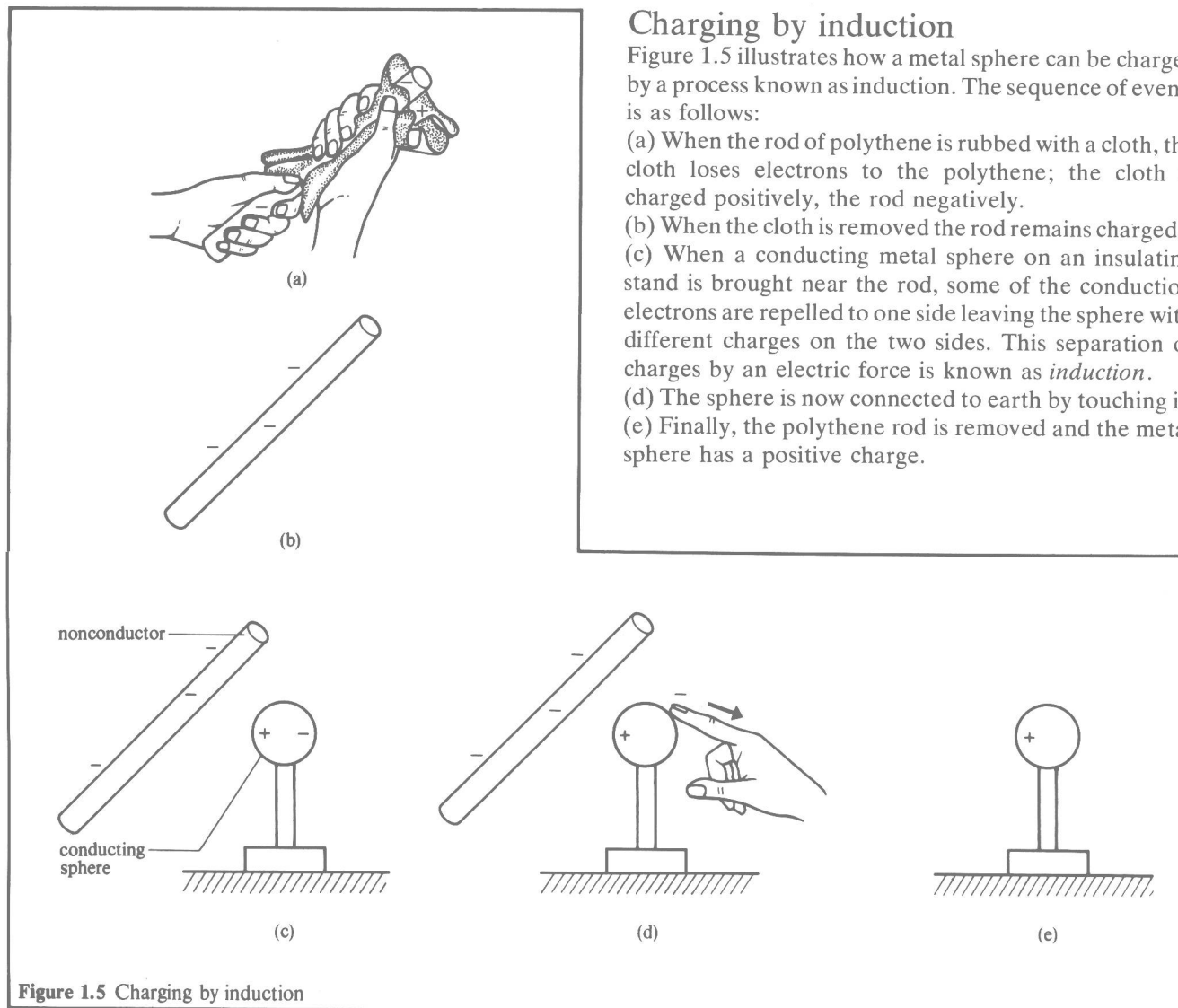


Figure 1.5 Charging by induction

Q 1.17 Study question

- (a) How does the electron model explain the fact that the metal sphere has a positive charge?
(b) Draw a series of diagrams to illustrate how, starting with a charged acetate rod, the sphere is left with a negative charge. ■

The charge that is left is called the induced charge and it has the *opposite* charge to the inducing charge.

Q 1.18 Self-assessment question

- How do you account for the following observations?
(a) When somebody walks across a carpet and then touches a radiator a spark may be produced.
(b) When a plastic comb or ruler is rubbed it can attract small pieces of paper. ■

The leaf electroscope

How can we tell whether a body has a positive or negative charge? We certainly can't decide by just looking at it! (Or by weighing—the mass of the displaced electrons is infinitesimally small compared with the body itself!) One possible way is to explore the region around a charged body with a small charged sphere, e.g. a pithball or polystyrene sphere suspended by nylon or silk thread.

Q 1.19 Self-assessment question

- What observations would indicate the presence of
(a) a positively charged body, and
(b) a negatively charged body? ■

This however, is not a very convenient method; we need a better instrument — the leaf electroscope is one.

Q 1.20 Study question

- (a) Describe with the aid of a diagram the construction of the leaf electroscope.
(b) Draw a sequence of labelled diagrams to explain how you would charge the electroscope positively by induction.
(c) Explain how you would use an electroscope to test whether a body has a positive or negative charge. ■

A good electroscope, once charged, should hold its charge for a considerable period of time. (There must be good insulation between the metal plate and the casing and no traces of moisture.) If, however, the surrounding air is heated or made conducting in another way, e.g. by means of ionising radiation, the charge on the electroscope decreases and the leaf will collapse. The rate at which it falls can be used to measure the strength of, say, a source of X-rays. One of the observations that led to the discovery of cosmic rays was the fact that a charged electroscope gradually lost its charge.

1.5 Coulomb's law

The magnitude of the force between two 'point' charges was investigated in 1785 by the French physicist Coulomb. He used a torsion balance that he had invented (similar to the one shown in figure 1.6). An

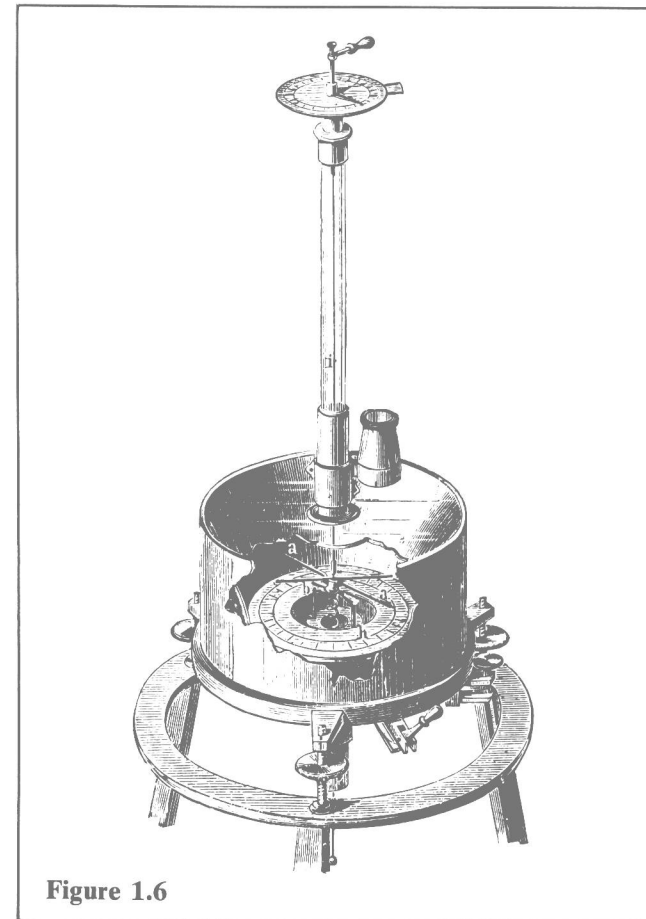


Figure 1.6

insulated rod which was suspended by means of silver wire twisted when a force was exerted on one end. This twisting effect was used to measure the force between a charged body attached to one end of the rod and another charged body placed near it. (Compare this apparatus with that used by Cavendish for the determination of G .)

In the following experiment you will set up a simplified version of Coulomb's apparatus.

Experiment FF1 Investigation of the force between two charged spheres

The aim of this experiment is to investigate how the force between two charged spheres depends upon the magnitude of the charges and their distance apart.

An inverse square law

Suppose there are two point charges A and B, charges Q_1 and Q_2 respectively, separated by a distance r . (The letter Q is used to symbolise the quantity of electricity, i.e. the charge.)

Consider the force F on B (figure 1.7). If both charges are positive (or both negative) there will be a repulsive force on B. (B also exerts an equal and opposite force on A.)

If one of the positive charges is replaced by a negative charge, there is an attractive force between the two charges (figure 1.8).

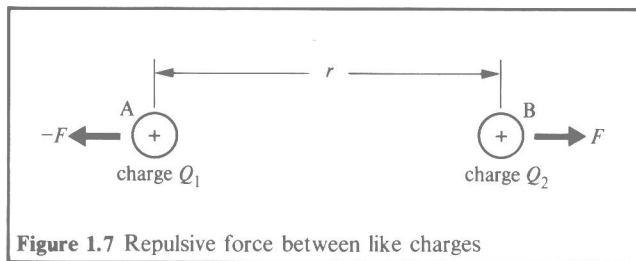


Figure 1.7 Repulsive force between like charges

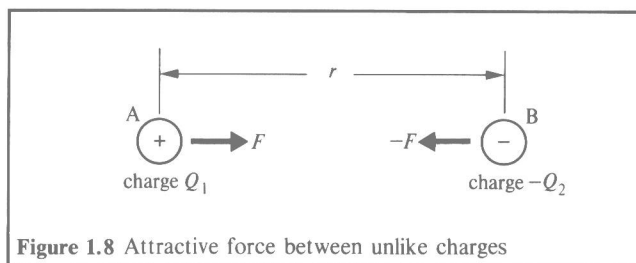


Figure 1.8 Attractive force between unlike charges

The electric force of repulsion between two like charges, Q_1 and Q_2 , separated by a distance r in a vacuum is given by

$$F \propto \frac{Q_1 Q_2}{r^2}$$

or
$$F = k \frac{Q_1 Q_2}{r^2}$$

where k is a constant whose value is determined by experiment.

The magnitude of this force is

(i) directly proportional to the product of the point charges

$$F \propto Q_1 Q_2 \quad (r \text{ constant})$$

(ii) inversely proportional to the square of their distance apart

$$F \propto \frac{1}{r^2} \quad (Q_1 \text{ and } Q_2 \text{ constant})$$

(iii) dependent upon the insulating medium which separates them.

Q 1.21 Self-assessment question

The information in table 1.2 was obtained from an experiment with charged spheres. The experimental arrangement was similar to that in experiment FF1. The deflection of the suspended sphere from its mean position was x , and the separation of the spheres was d . The suspended sphere hung 0.5 m below its support.

Table 1.2

Deflection $x/10^{-2}$ m	Separation $d/10^{-2}$ m	(Separation) ² $d^2/10^{-4}$ m ²	1/(Separation) ² $d^{-2}/10^4$ m ⁻²
1.0	11.0	121.00	0.008
3.5	6.5	42.25	0.024
5.5	5.5	30.25	0.033
7.5	4.5	20.25	0.049
9.0	4.0	16.00	0.062
11.0	3.5	12.25	0.072

- (a) Use this information to deduce a relationship between the force and the distance between the centres of the spheres.
 (b) The charge on each sphere was 5×10^{-9} C and the mass of the suspended sphere was 1.1×10^{-4} kg.

Estimate a value for the force constant k in the equation

$$F = k \frac{Q_1 Q_2}{r^2}$$

Note: See experiment FF1 for details of how to calculate the force on the suspended sphere. ■

The unit of charge

Coulomb's law can be used to define a unit of charge. We could decide to make the magnitude of k exactly 1, and define a unit of charge so that two unit charges separated by a unit distance in vacuo exert unit force on each other. There exists a unit of charge based on this choice; it is called the electrostatic unit of charge or statcoulomb. However, in the International System of units (SI) that we use, the unit of charge is derived from the unit of current, the ampere. The unit of charge is the coulomb and is defined as the charge that flows past a given cross-section in a conductor in one second when the current is equal to one ampere. If the coulomb is adopted as the unit of charge, then the constant k in Coulomb's law must be found by experiment, e.g. by measuring the force between known point (or spherical) charges separated by a known distance. (Compare this with the determination of G .) This is not the method that is adopted in practice (see chapter 3).

The permittivity constant

The value of the force constant k for a vacuum is approximately 9.0×10^9 N m² C⁻².

It is, however, more usual to write the constant in the form

$$k = \frac{1}{4 \pi \epsilon_0}$$

where ϵ_0 (pronounced epsilon nought) is called the *permittivity constant* or the *permittivity of free space*.

Thus, for a vacuum, we can write Coulomb's law as

$$\rightarrow F = \frac{1}{4 \pi \epsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$$

Notes: (i) The reason for introducing the constant $1/4 \pi$ is not immediately obvious but, by doing this, in equations which refer to situations where there is no spherical symmetry (for example, a uniform field) it does not appear. Equations which are written in this way are said to be rationalised.

(ii) It is usual to assume that the permittivity of air is the same as that of a vacuum.

(iii) The usual unit for permittivity is the farad per metre, F m⁻¹. This will be introduced in a later part of the Unit.

Q 1.22 Self-assessment question

- (a) Calculate the force of attraction between an electron and a proton which are 10^{-8} m apart.
 (b) If the electron and proton were free to move towards each other, what would be the acceleration of each?
 (charge on an electron = -1.6×10^{-19} C
 mass of electron = 9.1×10^{-31} kg
 charge on a proton = 1.6×10^{-19} C
 mass of proton = 1.7×10^{-27} kg
 $\frac{1}{4 \pi \epsilon_0} = 9.0 \times 10^9$ N m² C⁻²) ■