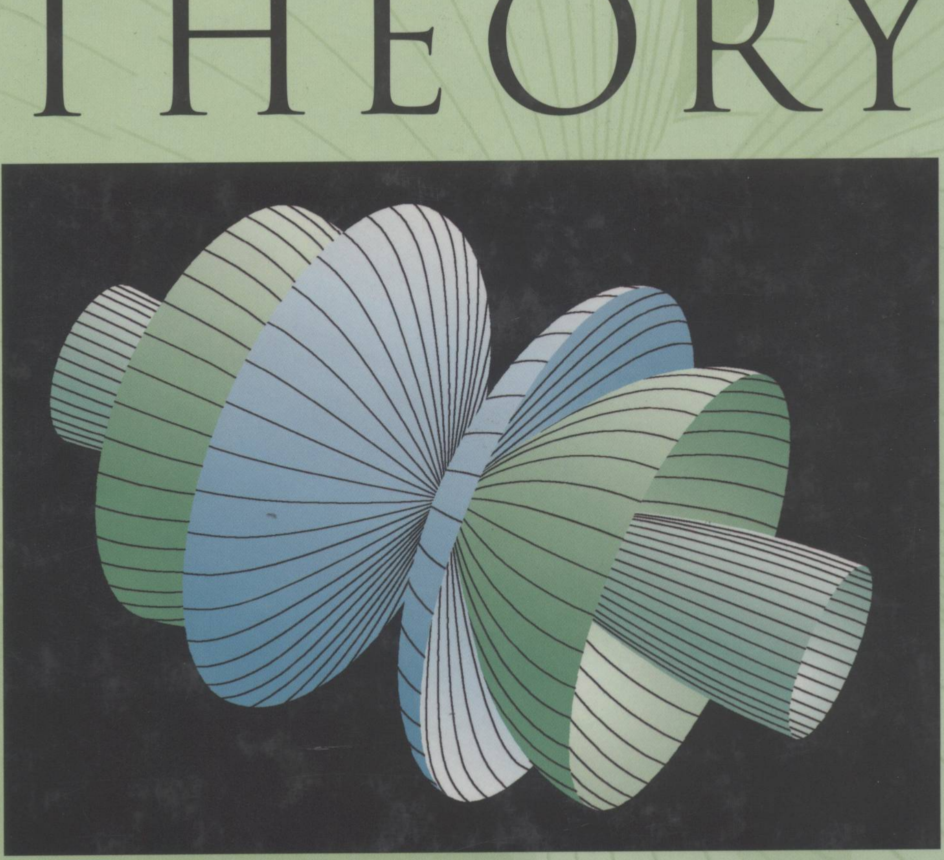
Second Edition

A First Course in STRING THEORY



Barton Zwiebach

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A First Course in String Theory
Second Edition

Barton Zwiebach is once again faithful to his goal of making string theory accessible to undergraduates. He presents the main concepts of string theory in a concrete and physical way to develop intuition before formalism, often through simplified and illustrative examples. Complete and thorough in its coverage, this new edition now includes the AdS/CFT correspondence and introduces superstrings. It is perfectly suited to introductory courses in string theory for students with a background in mathematics and physics.

This new edition contains completely new chapters on the AdS/CFT correspondence, an introduction to superstrings, and new sections covering strings on orbifolds, cosmic strings, moduli stabilization, and the string theory landscape. There are almost 300 problems and exercises, with password protected solutions available to instructors at www.cambridge.org/zwiebach.

Barton Zwiebach is Professor of Physics at the Massachusetts Institute of Technology. His central contributions have been in the area of string field theory, where he did the early work on the construction of the field theory of open strings and then developed the field theory of closed strings. He has also made important contributions to the subjects of D-branes with exceptional symmetry and tachyon condensation.

From the first edition

'A refreshingly different approach to string theory that requires remarkably little previous knowledge of quantum theory or relativity. This highlights fundamental features of the theory that make it so radically different from theories based on point-like particles. This book makes the subject amenable to undergraduates but it will also appeal greatly to beginning researchers who may be overwhelmed by the standard textbooks.'

Professor Michael Green, University of Cambridge

'Barton Zwiebach has written a careful and thorough introduction to string theory that is suitable for a full-year course at the advanced undergraduate level. There has been much demand for a book about string theory at this level, and this one should go a long way towards meeting that demand.'

Professor John Schwarz, California Institute of Technology

'There is a great curiosity about string theory, not only among physics undergraduates but also among professional scientists outside of the field. This audience needs a text that goes much further than the popular accounts but without the full technical detail of a graduate text. Zwiebach's book meets this need in a clear and accessible manner. It is well-grounded in familiar physical concepts, and proceeds through some of the most timely and exciting aspects of the subject.'

Professor Joseph Polchinski, University of California, Santa Barbara

'Zwiebach, a respected researcher in the field and a much beloved teacher at MIT, is truly faithful to his goal of making string theory accessible to advanced undergraduates – the test develops intuition before formalism, usually through simplified and illustrative examples . . . Zwiebach avoids the temptation of including topics that would weigh the book down and make many students rush it back to the shelf and quit the course.'

Marcelo Gleiser, Physics Today

'. . . well-written . . . takes us through the hottest topics in string theory research, requiring only a solid background in mechanics and some basic quantum mechanics . . . This is not just one more text in the ever-growing canon of popular books on string theory . . . '

Andreas Karch, Times Higher Education Supplement

'. . . the book provides an excellent basis for an introductory course on string theory and is well-suited for self-study by graduate students or any physicist who wants to learn the basics of string theory'.

Zentralblatt MATH

'. . . excellent introduction by Zwiebach . . . aimed at advanced undergraduates who have some background in quantum mechanics and special relativity, but have not necessarily mastered quantum field theory and general relativity yet . . . the book . . . is a very thorough introduction to the subject . . . Equipped with this background, the reader can safely start to tackle the books by Green, Schwarz and Witten and by Polchinski.'

Marcel L. Vonk, Mathematical Reviews Clippings

Cover illustration: a composite illustrating open string motion as we vary the strength of an electric field that points along the rotational axis of symmetry. There are three surfaces, each composed of two lobes joined at the origin and shown with the same color. Each surface is traced by a rotating open string that, at various times, appears as a line stretching from the boundary of a lobe down to the origin and then out to the boundary of the opposite lobe. The inner, middle, and elongated lobes arise as the magnitude of the electric field is increased. For further details, see Problem 19.2.

Contents

PART I

BASICS

1 A brief introduction

Here we meet string theory for the first time. We see how it fits into the historical development of physics, and how it aims to provide a unified description of all fundamental interactions.

1.1 The road to unification

Over the course of time, the development of physics has been marked by unifications: events when different phenomena were recognized to be related and theories were adjusted to reflect such recognition. One of the most significant of these unifications occurred in the nineteenth century.

For a while, electricity and magnetism had appeared to be unrelated physical phenomena. Electricity was studied first. The remarkable experiments of Henry Cavendish were performed in the period from 1771 to 1773. They were followed by the investigations of Charles Augustin de Coulomb, which were completed in 1785. These works provided a theory of static electricity, or electrostatics. Subsequent research into magnetism, however, began to reveal connections with electricity. In 1819 Hans Christian Oersted discovered that the electric current on a wire can deflect the needle of a compass placed nearby. Shortly thereafter, Jean-Baptiste Biot and Felix Savart (1820) and André-Marie Ampère (1820–1825) established the rules by which electric currents produce magnetic fields. A crucial step was taken by Michael Faraday (1831), who showed that changing magnetic fields generate electric fields. Equations that described all of these results became available, but they were, in fact, inconsistent. It was James Clerk Maxwell (1865) who constructed a consistent set of equations by adding a new term to one of the equations. Not only did this term remove the inconsistencies, but it also resulted in the prediction of electromagnetic waves. For this great insight, the equations of *electromagnetism* (or electrodynamics) are now called "Maxwell's equations." These equations unify electricity and magnetism into a consistent whole. This elegant and aesthetically pleasing unification was not optional. Separate theories of electricity and magnetism would be inconsistent.

Another fundamental unification of two types of phenomena occurred in the late 1960s, about one-hundred years after the work of Maxwell. This unification revealed the deep relationship between electromagnetic forces and the forces responsible for weak interactions. To appreciate the significance of this unification it is necessary first to review the main developments that occurred in physics since the time of Maxwell.

An important change of paradigm was triggered by Albert Einstein's special theory of relativity. In this theory one finds a striking conceptual unification of the separate notions of space and time. Different from a unification of forces, the merging of space and time into a spacetime continuum represented a new recognition of the nature of the *arena* where physical phenomena take place. Newtonian mechanics was replaced by relativistic mechanics, and older ideas of absolute time were abandoned. Mass and energy were shown to be interchangeable.

Another change of paradigm, perhaps an even more dramatic one, was brought forth by the discovery of quantum mechanics. Developed by Erwin Schrödinger, Werner Heisenberg, Paul Dirac and others, quantum theory was verified to be the correct framework to describe microscopic phenomena. In quantum mechanics classical observables become operators. If two operators fail to commute, the corresponding observables cannot be measured simultaneously. Quantum mechanics is a framework, more than a theory. It gives the rules by which theories must be used to extract physical predictions.

In addition to these developments, four fundamental forces had been recognized to exist in nature. Let us have a brief look at them.

One of them is the force of gravity. This force has been known since antiquity, but it was first described accurately by Isaac Newton. Gravity underwent a profound reformulation in Albert Einstein's theory of general relativity. In this theory, the spacetime arena of special relativity acquires a life of its own, and gravitational forces arise from the curvature of this dynamical spacetime. Einstein's general relativity is a classical theory of gravitation. It is not formulated as a quantum theory.

The second fundamental force is the electromagnetic force. As we discussed above, the electromagnetic force is well described by Maxwell's equations. Electromagnetism, or Maxwell theory, is formulated as a classical theory of electromagnetic fields. As opposed to Newtonian mechanics, which was modified by special relativity, Maxwell theory is fully consistent with special relativity.

The third fundamental force is the weak force. This force is responsible for the process of nuclear beta decay, in which a neutron decays into a proton, an electron, and an antineutrino. In general, processes that involve neutrinos are mediated by weak forces. While nuclear beta decay had been known since the end of the nineteenth century, the recognition that a new force was at play did not take hold until the middle of the twentieth century. The strength of this force is measured by the Fermi constant. Weak interactions are much weaker than electromagnetic interactions.

Finally, the fourth force is the strong force, nowadays called the color force. This force is at play in holding together the constituents of the neutron, the proton, the pions, and many other subnuclear particles. These constituents, called quarks, are held so tightly by the color force that they cannot be seen in isolation.

We are now in a position to return to the subject of unification. In the late 1960s the Weinberg–Salam model of *electroweak* interactions put together electromagnetism and the weak force into a unified framework. This unified model was neither dictated nor justified only by considerations of simplicity or elegance. It was necessary for a predictive and consistent theory of the weak interactions. The theory is initially formulated with four massless particles that carry the forces. A process of symmetry breaking gives mass to three of these particles: the W^+, the W^-, and the Z^0. These particles are the carriers of the weak force. The particle that remains massless is the photon, which is the carrier of the electromagnetic force.

Maxwell's equations, as we discussed before, are equations of classical electromagnetism. They do not provide a quantum theory. Physicists have discovered quantization methods, which can be used to turn a classical theory into a quantum theory – a theory that can be calculated using the principles of quantum mechanics. While classical electrodynamics can be used confidently to calculate the transmission of energy in power lines and the radiation patterns of radio antennas, it is neither an accurate nor a correct theory for microscopic phenomena. Quantum electrodynamics (QED), the quantum version of classical electrodynamics, is required for correct computations in this arena. In QED, the photon appears as the quantum of the electromagnetic field. The theory of weak interactions is also a quantum theory of particles, so the correct, unified theory is the quantum electroweak theory.

The quantization procedure is also successful in the case of the strong color force, and the resulting theory has been called quantum chromodynamics (QCD). The carriers of the color force are eight massless particles. These are colored gluons, and just like the quarks, they cannot be observed in isolation. The quarks respond to the gluons because they carry color. Quarks can come in three colors.

The electroweak theory together with QCD form the Standard Model of particle physics. In the Standard Model there is some interplay between the electroweak sector and the QCD sector because some particles feel both types of forces. But there is no real and deep unification of the weak force and the color force. The Standard Model summarizes completely the present knowledge of particle physics. So, in fact, we are not certain about any possible further unification.

In the Standard Model there are twelve force carriers: the eight gluons, the W^+, the W^-, the Z^0, and the photon. All of these are bosons. There are also many matter particles, all of which are fermions. The matter particles are of two types: leptons and quarks. The leptons include the electron e^-, the muon μ^-, the tau τ^-, and the associated neutrinos ν_e, ν_μ, and ν_τ. We can list them as

$$\text{leptons}: \; e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau .$$

Since we must include their antiparticles, this adds up to a total of twelve leptons. The quarks carry color charge, electric charge, and can respond to the weak force as well. There are six different types of quarks. Poetically called flavors, these types are: up (u), down (d), charm (c), strange (s), top (t), and bottom (b). We can list them as

$$\text{quarks}: \; u, d, c, s, t, b .$$

The u and d quarks, for example, carry different electric charges and respond differently to the weak force. Each of the six quark flavors listed above comes in three colors, so this gives $6 \times 3 = 18$ particles. Including the antiparticles, we get a total of 36 quarks. Adding leptons and quarks together we have a grand total of 48 matter particles. Adding matter particles and force carriers together we have a total of 60 particles in the Standard Model.

Despite the large number of particles it describes, the Standard Model is reasonably elegant and very powerful. As a complete theory of physics, however, it has two significant shortcomings. The first one is that it does not include gravity. The second one is that it has about twenty parameters that cannot be calculated within its framework. Perhaps the simplest example of such a parameter is the dimensionless (or unit-less) ratio of the mass of the muon to the mass of the electron. The value of this ratio is about 207, and it must be put into the model by hand.

Most physicists believe that the Standard Model is only a step towards the formulation of a complete theory of physics. A large number of physicists also suspect that some unification of the electroweak and strong forces into a Grand Unified Theory (GUT) will prove to be correct. At present, however, the unification of these two forces appears to be optional.

Another attractive possibility is that a more complete version of the Standard Model includes supersymmetry. Supersymmetry is a symmetry that relates bosons to fermions. Since all matter particles are fermions and all force carriers are bosons, this remarkable symmetry unifies matter and forces. In a theory with supersymmetry, bosons and fermions appear in pairs of equal mass. The particles of the Standard Model do not have this property, so supersymmetry, if it exists in nature, must be spontaneously broken. Supersymmetry is such an appealing symmetry that many physicists believe that it will eventually be discovered.

While the above extensions of the Standard Model may or may not occur, it is clear that the inclusion of gravity into the particle physics framework is not optional. Gravity must be included, with or without unification, if one is to have a complete theory. The effects of the gravitational force are presently quite negligible at the microscopic level, but they are crucial in studies of cosmology of the early universe.

There is, however, a major problem when one attempts to incorporate gravitational physics into the Standard Model. The Standard Model is a quantum theory, while Einstein's general relativity is a classical theory. It seems very difficult, if not altogether impossible, to have a consistent theory that is partly quantum and partly classical. Given the successes of quantum theory, it is widely believed that gravity must be turned into a quantum theory. The procedures of quantization, however, encounter profound difficulties in the case of gravity. The resulting theory of quantum gravity appears to be ill-defined. As a practical matter, in many circumstances one can work confidently with classical gravity coupled to the Standard Model. For example, this is done routinely in present-day descriptions of the universe. A theory of quantum gravity is necessary, however, to study physics at times very near to the Big Bang, and to study certain properties of black holes. Formulating a quantum theory that includes both gravity and the other forces seems fundamentally necessary. A *unification* of gravity with the other forces might be required to construct this complete theory.

1.2 String theory as a unified theory of physics

String theory is an excellent candidate for a unified theory of all forces in nature. It is also a rather impressive prototype of a complete theory of physics. In string theory all forces are truly unified in a deep and significant way. In fact, all the particles are unified. String