Boundary Elements X

Vol. 1 Mathematical and Computational Aspects

Editor: C. A. Brebbia

CMP Computational Mechanics Publications

Springer-Verlag

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Computational Mechanics Publications
Southampton Boston

Springer-Verlag Berlin Heidelberg New York
London Paris Tokyo

C.A. BREBBIA

Computational Mechanics Institute
Ashurst Lodge
Ashurst, Southampton
S04 2AA
UK

British Library Cataloguing in Publication Data
Boundary elements X.
1. Engineering. Mathematics. Boundary element methods
I. Brebbia, C.A. (Carlos Alberto),
620'.001'515353
ISBN 1-85312-017-0 Set
ISBN 1-85312-013-8 V.1
ISBN 1-85312-014-6 V.2
ISBN 1-85312-015-4 V.3
ISBN 1-85312-016-2 V.4

Library of Congress Catalog Card Number 88-71494

ISBN 1-85312-013-8 Computational Mechanics Publications Southampton
ISBN 0-945824-00-9 Computational Mechanics Publications Boston, USA
ISBN 3-540-50091-X Springer-Verlag Berlin Heidelberg New York London Paris Tokyo
ISBN 0-387-50091-X Springer-Verlag New York Heidelberg Berlin London Paris Tokyo
ISBN 3-540-50095-2
ISBN 0-387-50095-2 4 volume set

Printed in Great Britain by Unwin Brothers Limited, Old Woking, Surrey

C M P

OTHER VOLUMES AVAILABLE

Boundary Elements X
Vol.2 Heat Transfer, Fluid Flow and Electrical Applications
Editor: C.A. Brebbia

This volume contains the edited versions of some of the papers presented at the tenth International Conference on Boundary Element Methods held at Southampton in September 1988. The book deals with heat transfer, including diffusion and convection problems and general potential problems. Other sections are dedicated to papers on fluid mechanics and fluid dynamics followed by a substantial number of papers on electrostatics and electromagnetics applications.

Boundary Elements X
Vol.3 Stress Analysis
Editor: C.A. Brebbia

This book contains the edited versions of some of the papers presented at the tenth International Conference on Boundary Element Methods held at Southampton in September 1988. The volume covers stress analysis topics of current interest including inelastic problems, fracture mechanics, contact problems, plate bending, design sensitivity and optimization, industrial applications and several papers on computer codes.

Boundary Elements X
Vol.4 Geomechanics, Wave Propagation and Vibrations
Editor: C.A. Brebbia

This book contains some of the edited papers presented at the tenth International Conference on Boundary Element Methods held at Southampton in September 1988 and dealing with geomechanics, wave propagation and vibrations. Special sections are dedicated to papers on rock and soil mechanics and soil dynamics, acoustics, elastodyamics and plate vibrations and dynamics coupling.

FOREWORD

The success of the Boundary Element Method in the last ten years has been mainly due to the development of the technique as a practical engineering tool. The year 1978 was a milestone in this regard, as it was then that classical boundary integral equations were interpreted in a different way. The emphasis then on the use of quasi-variational concepts and transformations of the type already known in finite elements gave origin to a new methodology which was well adapted to the direct boundary integral formulation. In spite of the rapid advances in boundary element research in subsequent years, these definitions are still accepted as the basis of the method.

One of the most versatile interpretations of the technique is provided by the use of Lagrangian multipliers, which permits a simple deduction of the boundary integrals starting with the differential equations governing the problem. Unfortunately this concept, which is not only mathematically correct but very elegant, has been misunderstood, perhaps because classical boundary integral scientists were not familiar with variational calculus. This branch of mathematics is one of the most difficult to comprehend, possibly because it entails understanding a series of philosophical concepts rather than purely algebraic expressions.

Another area of confusion has been the relationship between boundary elements and other numerical techniques. Erroneous concepts about symmetry and the correct interpretation of the integral equations in terms of energy in the nineteen seventies were followed by a better understanding of the basic principles in the nineteen eighties. Many of the original problems seem to have been simply due to attempts to force the boundary element method to conform to finite element concepts.

The most important development in the last 10 years has been the awareness of the engineering and scientific community that boundary elements is a new and more powerful technique than finite elements, and that the latter can be seen as a particular case of BEM rather than the other way round.

From the history of science point of view it is interesting to point out that while finite elements was a method predominantly based on approximations, boundary elements combine them with powerful analytical solutions. This combination, which in a way was a revaluation of past work is the more powerful aspect of boundary elements and the one that gives the method great accuracy of results and versatility.

From the engineering point of view, boundary elements can be seen in many cases as a computational technique which is better conditioned than finite elements for analysis and design. While FEM analysis demands an unne essary discretization of the domain, boundary elements is a function of tl surface configuration only. Further advances are still required to elimina the need to discretize surfaces by treating each of them as one large eleme instead. The concept of elements is in itself an FEM idea and the next sta should be to develop boundary surfaces or patches.

The work carried out in non-linear and time dependent problems has up now also suffered from the application of old FE concepts. Undue emphas is put on the subdivision of the continuum into a fixed grid consisting the so-called cells. With the exception of some papers such as those dealir with moving internal boundaries and others related to the dual reciprocit method, most applications of BEM in non-linear and time dependent prol lems are too closely related to similar work previously done using finit elements. In this regard it is necessary for BEM scientists to learn to b more audacious and innovative in their thinking in order to realize the fu potentialities of the new method.

These ten years of BEM research have been particularly rewarding for thi editor who has seen the technique developing from its humble beginning into a powerful engineering method. What in 1978 was an eccentric ob session which was deemed to be unnecessary, has now become not only a established research technique but a powerful tool for engineering analysis

Carlos Brebbia

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SECTION 1 - BASIC FORMULATIONS

INVITED PAPER

The Generalized Boundary Element Method for Nonlinear Problems

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ABSTRACT

The present work represents an attempt to apply extensions of the boundary element method recently being used in the boundary-value problems and the initial boundary value problems to nonlinear problems. Our new method based on the set of boundary integral equations derived for each subdomain by using the fundamental solution for the linearized nonlinear differential operator of the problems is proposed. The resulting system of quasi-nonlinear equations is solved by using of the simple iterative procedure. In order to show the workability and the validity of the proposed method, numerical results of the one-dimensional boundary-value problems are presented.

INTRODUCTION

The boundary element method has developed in many fields in natural science and engineering (see e.g. References 1,2,3). It is found that the boundary element method is a powerful numerical solution procedure for both the linear boundary-value problems and initial boundary value problems in various mathematical models. However, applications of the boundary element method to nonlinear problems are a few in comparison with linear problems. We can find many important and interest problems in nonlinear continuum mechanics.

In order to assert the dominant position of the boundary element method we must show the effectiveness and adaptability of the method through applications to many kinds of nonlinear problems. From this point of view, we have been challenging the integral equation analyses of geometrically nonlinear problems of thin elastic body (e.g.,Tosaka et.al.,4,5) and nonlinear problems of viscous fluid flow (e.g.,Tosaka et.al.,6-9). In these studies we proposed the hybrid-type integral equation method (or the boundary-domain integral equation method) in which the fundamental integral equation possesed not only the boundary integral terms but also the volume integral terms including the unknown functions. The system of nonlinear integral equations was derived from the weighted residual expression by making use of the fundamental solution for a linear differential operator which was only a part of the original nonlinear differential operator. This system was discretized with not only boundary elements but also internal cells. The final nonlinear set of matrix equations with a full coefficient matrix was solved effectively by the Newton-Raphson iterative procedure instead of the simple iterative procedure.

In this paper, the new boundary element method is presented for nonlinear problems. This method has the properties as some generalization of a standard boundary element method. The basic idea of this method is to derive a boundary integral equation in subdomain by using a fundamental solution for the linearized differential operator of the nonlinear governing equation. We take into consideration of some nonlinear effect in the used fundamental solution. The derived boundary integral equation is discretized with only the boundary element. The final system of matrix equations for the problems is constructed by combinating the equations of each subdomain in consideration of connectibity at the interface. Consequently, we have the added advantage that the final system to be solved becomes to banded matrix equations. An effective procedure to solve this quasi-nonlinear system is developed. A simple iteration algorithm is proposed instead of the Newton-Raphson iterative method which had been used in our previous studies on nonlinear problems. We may regard the above-mentioned method as the generalized boundary element method in a sense that our method is based on the boundary integral equation in each subdomain instead of the nonlinear integral equation in the whole given domain.

The new method is applied to one-dimensional nonlinear boundary-value problems. We adopt two nonlinear differential equations as a sample problem in this study. Especially, the Burgers' equation which has the similarity to the well-known Navier-Stokes equations in incompressible viscous fluid flow problems is considered as a mathematical modelling to examine applicability of the proposed method. Numerical results to demonstrate the versatility and accuracy of the method are presented for boundary-value problems of the nonlinear equation with high nonlinear coefficient.

GENERALIZED BOUNDARY ELEMENT METHOD

Nonlinear problems

Let us consider the boundary-value problem of the following nonlinear differential equation:

$$A(u) = f \qquad in\ \Omega \tag{1}$$

In this equation (1), we assume that the nonlinear differential operator A is given by the following sum of the linear operator L and the nonlinear one N :

$$A(u) = Lu + N(u) \tag{2}$$

Now, we propose the generalized boundary element method for solving approximately the boundary-value problem of the nonlinear equation (1) with (2). First of all we construct the given domain Ω as the union of the subdomains Ω_i as follows:

$$\Omega = \bigcup_{i=1}^{N} \Omega_i \tag{3}$$

In each subdomain Ω_i , it is assumed that the nonlinear operator $N(u)$ can be linearized as:

$$N(u) \doteqdot v\bar{L}\ u \tag{4}$$

where the function v is considered as the known value of the unknown function u and the operator $\bar{L}$ is a linear one.

With the above linearization (4), the nonlinear differential equatioin (1) turns out the following linear one, which is the fundamental equation for our method:

$$A(u) \doteqdot Lu + v\bar{L}u = f \qquad in\ \Omega \tag{5}$$

Boundary integral equation

Let us derive the boundary integral equation formulation of (5) by using the standard manner in the direct boundary element method 1). We start from the following weighted residual expression of (5) for the weighting function w^* :

$$\int_{\Omega_i} (Lu + v\bar{L}u - f)w^*\, d\Omega = 0 \tag{6}$$

Integrating this expression by the divergence theorem, we can obtain the inverse form such that

$$\int_{\Omega_i} u(L^* + v\bar{L}^*)w^* d\Omega = \int_{\Gamma_i} \{u(Bw^*) - (Bu)w^*\} d\Gamma$$

$$+ \int_{\Gamma_i} v\{u(\bar{B}w^*) - (\bar{B}u)w^*\} d\Gamma$$

$$+ \int_{\Omega_i} fw^* d\Omega \tag{7}$$