OPTIMAL CONTROL AND STOCHASTIC ESTIMATION

Theory and Applications

Volume 1

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and
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Optimal Control and Stochastic Estimation: Theory and Applications Volume 1

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Optimal Control and
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Volume 1

Dedication

To my wife Wendy, and my children
Andrew and Claire

Michael J. Grimble

To my children Eleanor and Laurence,
and my mother and father

Michael A. Johnson

Foreword

The development and application of linear optimal control theory are the central themes of this book. As is well known, the LQP/LQG theory has provided a paradigm for control system design whose potential applications stretch far and wide, and the authors include new and unusual applications of the theory to challenging industrial design problems. Also it is clear that many of the theoretical developments they describe are strongly motivated by their experiences with serious applications, and by the demands of 'real' problems.

I first met the authors in the early 'seventies when they joined my group to undertake postgraduate research into the advanced problems of automation in the metal industries. Our aim was the application of rigorous intellectual approaches to the applied research problems of modelling and control, and the use whenever sensible of advanced control-theoretical ideas and techniques. In those early days as on many subsequent occasions, we were both surprised and delighted by the theoretical and conceptual challenges which emerged naturally during the pursuit of solutions to the 'real' and apparently banal problems of industry which at first sight appeared little connected with 'pure' research.

My work with Mike Grimble and the late and very gifted Martin Fuller was concerned with obtaining solutions to the roll gap equations for tinplate rolling. Here 'limiting' reductions are in prospect, the physical phenomena encountered are somewhat surprising and the process is not straightforward to control. The solution procedures we developed utilized function space concepts and iterative techniques developed originally for the solution of two-point boundary value problems arising in control optimizations. We succeeded in computing what are, as far as I know, the only published solutions to these equations and obtained considerable insight into tinplate phenomena.

Optimization also played a key role in the work I undertook with Mike Johnson. We set ourselves the very considerable task of exploiting the properties of optimal controllers to isolate 'good' multivariable control 'structures' for a tandem mill. We successfully produced new-bound formulae for linear optimal control systems and plausibly managed to isolate the main structures for the control of gauge and tension in a tandem mill.

Since those early days these researches have matured and resulted in the design of a whole range of advanced mill computer control schemes which have proved both incisive and effective in industrial terms. This much I perhaps expected. What I did not at the time anticipate was the Odyssey on which my former associates had been inadvertently launched as they pursued and developed their initial interests in optimal control theory and its application. Apart from this book, these now long-standing interests have led them to establish the Industrial Control Unit at the University of Strathclyde, Glasgow, Scotland.

The IEEE Special Issue on *LQG/LQP and Kalman–Bucy Filtering,* published in the 'seventies, represented a milestone in technical publication in this area, containing over three hundred pages of new technical material and of the order of a thousand key references. It serves to reflect how the subject had even then attracted wide interest and acquired a rich theoretical structure. This initial interest and activity seems to have continued unabated over the last decade as widely differing concepts and results have emerged. In seeking to make this material fall into place and to establish a fresh conceptual perspective in the whole area the authors have undertaken a task which is both daunting and worthwhile, and perhaps overdue! As is widely accepted, publications in this area are generally 'heavy' on theory and 'light' on significant applications outside the aerospace industry. By the inclusion of non-trivial application studies in refreshingly new areas the authors have further enhanced the book. I wish them well.

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Authors' Preface

Our interest in optimal control and stochastic estimation theory and its applications developed from our experience on the Industrial Automation Group at Imperial College, London and our industrial experience with GEC Electrical Projects Ltd., Rugby, and the London Research Station of the British Gas Corporation.

The Industrial Automation Group at Imperial College provided a stimulating research environment which we have endeavoured to emulate at the Industrial Control Unit of the University of Strathclyde. Professor Bryant's group at Imperial college was mainly concerned with Steel Industry Applications. Optimal control and optimization theory found their way into many areas of modelling and control not usually found in textbook theory. The first motivation for the present text was therefore to record some of the many useful areas of optimal control which are of value in applications.

The authors greatly admired the 1972 text of Kwakernaak and Sivan (*Linear Optimal Control Systems*) in producing a very accessible but comprehensive collection of work on linear optimal control and filtering systems. However, in many of the areas considered by Kwakernaak and Sivan, advances have now been made which are not covered in any follow-on text. This was the second motivation for this book and it is hoped that this text fills this gap and extends the work of these authors, so that the combined volumes present a modern and comprehensive overview of the field.

Chapter 1 includes a review of basic systems theoretic results needed in the following chapters. It also introduces the Hilbert space-gradient approach to the solution of optimal control problems which is an underlying theme of much of the analysis.

Very few recent texts include the Wiener–Hopf optimization theory which provides a useful transfer-function approach to solving linear optimal control and estimation problems. This topic is introduced in the Chapter 2 and the simplicity of the basic ideas are illustrated. However, recent theoretical analyses in the s- or z-domains, have been based upon a polynomial systems approach. Hence, in Chapter 11, these two different methods are brought together and shown to be parallel approaches to the same problem. Indeed the insight gained by looking at the two techniques enables the connection between Diophantine equations and partial fraction expansions, for example, to be explained. Chapter 3 introduces our approach to the multivariable root-loci control problem. That is, an analysis is presented for describing the locations of and properties of the closed-loop poles of an optimal system as the control weighting becomes vanishingly small. The approach taken in this chapter has not been described in previous text books.

The exponents of non-optimal multivariable design methods often point to a weakness of optimal control as being the problem of selecting the cost-weightings matrices. There are, of course, some problems such as ship-steering control systems design which give rise to natural cost functions to be minimized. However, if optimal control is only to be used as a design procedure, where optimality is not important, then other methods must be chosen for selecting these matrices. In Chapter 4, a collection of different techniques is described which, for example, enable the eigenstructure in the optimal system to be predetermined.

Some of the Q and R optimal cost-weighting selection procedures are based upon the asymptotic properties of optimal systems. Chapter 5 includes a discussion of the maximal accuracy which can be achieved with optimal regulators as the control weighting tends to zero. This topic gives some insight into possible control difficulties which may be present in a system. For example, the presence of non-minimum phase zeros in a plant will mean the error component of the cost function cannot be driven to zero, however large the control signals become. The maximal accuracy results are used later in the chapter to enable the best set of input actuators to be selected. In chemical plants and large systems there is often a difficulty of making a choice between many different possible combinations of input actuators or output measurements. The maximal accuracy results throw some light on the best set of input actuators to employ.

The subject of system structure assessment is considered further in Chapter 6. An optimal control problem can be solved for each possible system structure and the relative merits of each design can be judged based upon the minimum costs which can be achieved and the number of inputs and outputs employed. In practice, because of the large number of possible combinations of inputs and outputs which may be tried, it would be computationally very expensive to solve a large number of optimal control problems and hence the subject dealt with in this chapter is that of establishing easily computable bounds on the optimal cost. Given such bounds it is then possible to make a rapid assessment of different system structures based upon the estimated cost values. The middle section of this chapter concentrates on the subject of fixed structure optimal control problems. There has been a lot of interest over the last few years from the academic community in the use of constant output feedback optimal control laws and the many results in this area are surveyed and the properties discussed.

The Kalman filter has been one of the most successful developments of the modern control theory. There are numerous applications in many industries including the aerospace and marine industries. The Kalman filter and the time domain aspects of filtering theory are considered in Chapter 8.

The first part of the chapter concentrates on a solution of the continuous-time filtering problem. Most modern books on filtering theory use the Wiener process rather than the older white-noise models. From a practical point of view, the white-noise system representation is to be preferred since it simplifies the notation and analysis. However, since many books include chapters of this type the former approach is taken in Chapter 8. The latter parts of the chapter discuss the discrete-time Kalman filtering problem. A great strength of the Kalman filter is that it is so appropriate for discrete-time implementation. The numerical problems in implementing the algorithm are considered and prediction and smoothing problems are discussed.

The frequency-domain properties of Kalman filters are considered in Chapter 9. These properties provide insight into the behaviour of the filter and also show the relationship of the filter to more classical frequency-domain-based techniques. The chapter also includes an introduction to finite-time filtering, prediction and smoothing problems. Finite impulse response filters find wide application in signal processing but they are not so common in the control field. The approach taken in this chapter is novel and has not been published before.

The main control system application for the Kalman filter is as part of a state estimate feedback control loop, as described in Chapter 10. The separation principle of stochastic optimal control theory is introduced in this chapter using the Hilbert space techniques once again. As in the filtering chapters both continuous-time and discrete-time problems are considered. The digital version of the LQG controller is used extensively in applications and is probably the only controller for multivariable systems which adequately caters for the different characteristics of the noise sources and disturbances. It is interesting that the multivariable frequency-domain design methods developed by MacFarlane, Rosenbrock and Mayne seem particularly suited to deterministic systems design but the need to include so many design objectives in stochastic applications leads naturally to the use of an LQG design framework.

It would have been difficult to imagine a few years ago that any new methods of representing systems would have a very large impact on control design techniques and analysis. However, the polynomial systems approach pioneered by Kučera has resulted in a much greater insight into the frequency-domain properties of optimal systems and has also influenced applications areas such as multivariable self-tuning control systems design. There are now basically three approaches to the s- or z-domain solution of optimal control problems. The first approach considered in Chapter 2 reflects the Wiener transfer function based solution of optimal control and filtering problems. The second is the fractional system description used by workers such as Desoer and Vidyasagar which is not considered in this text. The third method is the polynomial systems approach which provides much insight and also has the tremendous advantage of simplifying the computational algorithms which are needed. Hence there are several objectives for Chapter 11:

1. To present what is becoming the standard approach to estimation z-domain optimal control and filtering problem solutions: the polynomial systems approach.
2. To show the relationship between the Wiener and polynomial systems methods.

The idea of a stabilizing controller parameterization is also introduced in this chapter. This avoids difficulties with unstable pole-zero cancellations which can occur in the older Wiener–Hopf approaches.

The area of adaptive and self-tuning control is discussed in Chapter 12. Although the self-tuning controller was first developed in the early 1970's, it is only recently that it has now found its way into real applications in any substantial numbers. The number of adaptive controllers on the market is increasing rapidly and many different types of devices are being produced for different areas of application and duties. The majority of adaptive controllers are based on either optimal control concepts or on PID controller structures. Chapter 12 on self-tuning control algorithms therefore illustrates an excellent applications area for optimal control. The chapter also details many of the different control laws which may be employed for this type of design. Both implicit and explicit self-tuning schemes are discussed and the practical problems of implementation are considered.

It is hoped that the applications Chapters 7 and 13 will illustrate the power of the optimal control techniques in several industrial applications. The design of deterministic optimal multivariable systems is considered in Chapter 7 and stochastic systems are considered in Chapter 13. Most of the material in Chapter 7 is concerned with the design of a shape-control system for a cold-rolling mill. The control of shape or flatness in mills involves the measurement of a large number of variables typically over 30 and the control of 8 to 10 actuators. This chapter is based on experience gained on the design on the shape-control system for the Sendzimir mills of the British Steel Corporation at Sheffield. In fact, the actual control law which was developed by BSC and Unit engineers and which was implemented, was not based on optimal control theory since is was necessary to have particularly simple control structure.

However, the details of the modelling and control problem presented in Chapter 7 are based on the study. It is often the case that optimal and non-optimal designs are completed for particular projects and the design which is most appropriate is chosen for implementation.

By way of contrast, the optimal control approach has particular advantages in stochastic systems as illustrated in the ship-positioning design procedure described in Chapter 13. In this case, optimal and non-optimal designs were completed for the control problem but the optimal control approach was selected for implementation by the GEC Electrical Projects Co. Ltd. The stochastic optimal control solution to the ship positioning control problem was first introduced by Jens Balchens' Group at Trondheim. It is now the accepted standard solution to the problem and the Kalman filter design which was used has demonstrated its flexibility by being used in various roles without requiring major changes to the scheme. Chapter 13 also includes a description of the application of the Kalman filter to soaking pit temperature estimation problems. There are many industrial problems which fall into the category of estimating temperatures in an environment where direct measurement is difficult or impossible. The chapter illustrate one of the key requirements of a Kalman filter, that of producing a good model on which to base the filter. The art of good Kalman filter design lies in the expertise of the systems modeller.

It is hoped that this text will be of value to engineers in industry by providing an indication of the power and scope of the optimal control and estimation approach. It should also be clear that there are many areas which are suitable for further research. Several of the topics discussed are open research questions which should be of value to postgraduate research engineers. The work of Kwakernaak and Sivan has been widely adopted for undergraduate teaching and we hope the book will complement this volume and will be of value to students.

There are many individuals we should thank for contributions to the book. In particular we are indebted to Professor Mark Davis (Imperial College), Professor Martins de Carvalho (Instituto de Engeharia de Sistemas e Computadores, Portugal), Professor Adrian Roberts (Queens University, Belfast) and Dr. William Leithead (Industrial Control Unit) for their detailed comments on Chapter 8, 10 and 11. Many of the research engineers of the Industrial Control Unit have provided simulation results and we should like to thank our former colleagues: Dr. Patrick Fung (Spar Aero-Space, Canada), Dr. Tom Moir (Paisley College of Technology, Scotland), Dr. Munie Gunawardene (Robert Gordons Institute of Technology, Aberdeen, Scotland), Dr. John Ringwood (NIHE, Dublin), Dr. John Fotakis (Public Petroleum Corporation, Greece), Dr. Ken Dutton (formerly British Steel Corporation, Sheffield) and Dr. Malcolm Daniels (University of Dayton, Illinois, USA).

Similarly, there are many individuals whose lectures and talks have

influenced our thinking on the subject of Control Engineering and we should particularly like to note the work of Professors Åström, MacFarlane, Rosenbrock, and Professor Greyham Bryant. Indeed, it was our time spent at Imperial College, London which awakened and stimulated our interest in this subject. There are many members of the present Industrial Control Unit who are introducing new ideas and who will no doubt be mentioned in future publications. We highly value this interaction with the members of our group.

Most of the diagrams were drawn by Wendy Grimble and the manuscript was typed by Ann Taylor and Sheena Dinwoodie; their unstinting and enthusiastic help is gratefully acknowleged. We must record also our thanks to Ian McIntosh (John Wiley and Sons) for his encouragement and advice.

One final point before leaving you to your studies. Students often believe that their lecturers or professors know all about their subjects and have infinite wisdom in the topic of their choice. This is, of course, far from the truth but we hope it will not be too evident from the text that you are about to enjoy. We wish you good reading.

MICHAEL J. GRIMBLE AND MICHAEL A. JOHNSON

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