BASIC MATHEMATICS FOR RADIO AND ELECTRONICS

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FOREWORD

IT is a pleasure to me to contribute a foreword to this book written by an old student of mine. It is well entitled "basic mathematics" for one of the outstanding features of the book is its fundamental character. The laying of sure foundations is very essential in mathematics. As the author says in the introduction, a great many people have been compelled by force of circumstances to take up the study of radio with a very scanty mathematical background, and to these the book should make a very special appeal. In no branch of engineering is a thorough knowledge of basic mathematics more essential than in radio. By its very nature radio involves the study of rapid variations of current and voltage of various degrees of complexity, and a student cannot hope to get a clear understanding of even the simplest problems unless he has a thorough grasp of the fundamentals of differential and integral calculus and of vector algebra. A valuable feature of the book is the gradual development of the subject step by step, and the pains taken at every step to endow the symbols with definite meaning. It is all too easy for a student to attain some facility in the manipulation of symbols without having any real understanding of the physical realities involved. It is important for the student to clothe his symbols with physical reality and to appreciate the implications of each step of the mathematical development, and this the author has done his utmost to encourage.

Although written primarily for radio engineers it would be a great mistake to assume that it is not suited to students of other subjects. The first six of the seven chapters into which the book is divided are quite general, and it is only in the final chapter that the mathematical methods developed in the earlier chapters are applied to radio problems and even these are largely of a general character that will interest students of any branch of physics or engineering.

G. W. O. Howe

Glasgow University

INTRODUCTION

A VERY long while ago—longer than I care to remember—I was invited to write a series of articles for the *Wireless Engineer*, or rather for *Experimental Wireless* as it was called in those days, on " Mathematics for Wireless Amateurs ". I gladly accepted the invitation because I had already discovered that the best way of learning a subject is to write a book about it. So it proved in this case, and I have ever since been grateful that I was thus constrained to seek out and describe in simple language the true inwardness of the mathematical methods that I had for years been using in my daily work. So also it seems were some readers of the series, judging by kind comments and letters received then and since. I take this opportunity of thanking them, for it is largely due to this encouragement that I have been given the opportunity of turning the series into a book.

The original articles were written for those whose understanding of " wireless ", as we called it then, was limited by a lack of knowledge of mathematics. Hence the "radio" in the title, and those sections which deal particularly with the analysis of alternating-current circuits ; but the really basic ideas of mathematics are common to all its applications, and I therefore hope that the work may be of some use to an even larger class of students—and perhaps even to some teachers. (If this sounds presumptuous, I can say in excuse that there were teachers among those who commended the original series.)

May I anticipate certain criticisms by stating briefly the considerations that have guided both the selection of the material and the manner of its treatment.

Since it was intended to include in a single work all the main parts of elementary mathematics, each of which has usually a textbook all to itself, the choice lay between, on the one hand, a fairly thorough discussion of the basic ideas, with little room left for the detail of their development and application, and, on the other, a condensed statement of rules and formulæ for use in a more or less rule-of-thumb way; but this last is surely a second-class way, suitable only for second-class minds. Even if space had been no object I would still have chosen to emphasise the basic ideas rather than their detailed development and application. In mathematics, as in many other activities, it is the first step which counts. If the basic ideas are well and truly grasped, their application is a happy hunting ground for the adventurous mind, which will indeed learn more by going astray on its own than by being carefully guided along well-worn tracks. (The important thing is, not to avoid mistakes, but never to make the same mistake twice.) What I have done therefore is to select in each branch of the subject those elementary but fundamental ideas which I have found, in many years of practical experience in radio work, to be definitely necessary or specially useful. Further, I have tried at each stage to link up these basic ideas with the real world of sensory experience from which they were, of course, originally derived, however abstract they may appear to be. In this connection I am glad to acknowledge my great indebtedness to Professor G. W. O. Howe, whose early teaching inculcated in me this realistic and critical habit of mind.

Since the original series was written, Mr. Lancelot Hogben has shown how completely an apparently academic, not to say esoteric, subject can be vitalised and humanised by a natural and evolutionary method of description. I mention his work, not to challenge comparison, which would be foolish, but rather to claim a modest kinship, at least in respect of attitude and intention. I commend it to future writers of textbooks, in the spirit of the man who hung up in his chicken-run an ostrich egg marked with the words "Keep your eye on this and do your best".

Teddington,
Middlesex

F. M. COLEBROOK

REVISER'S PREFACE

WHEN I was asked by the publishers to undertake the revision of this book by the late Mr. Colebrook, under whom I once worked at the National Physical Laboratory, Teddington, I very soon became aware that any alterations (except the correction of a few trivial misprints) would merely spoil the original. The first seven chapters have therefore been preserved in their entirety; Section 95 has been somewhat expanded.

Chapters 8 and 9 contain new matter not in the original text for which I alone am responsible. The elements of operational calculus and matrices (with not more than two rows and columns) have been discussed, together with a number of miscellaneous topics, such as numerical computation and normal distributions. The object has been to make the reader's first encounter with these subjects sufficiently encouraging to enable him to face without fear textbooks which deal more adequately with operational calculus, matrices etc. As far as possible, I have tried to make the symbols and notation used consistent with that of the first seven chapters.

Mathematics when unknown and unfamiliar is rather a frightening and unwelcome subject to most engineers. The chief aim of both the original and the revised parts of this book is to remove mathematical inhibitions, and to reveal the power of mathematics to clarify and simplify practical work.

Teddington,
Middlesex

J. W. HEAD

Chapter 1

ELEMENTARY ALGEBRA: THE FUNDAMENTAL IDEAS

1. Symbols

ONE of the most characteristic things about algebra is its appearance, and to a beginner it is certainly not prepossessing. In place of concrete and understandable numbers, to which we have learned to attach definite meanings, we are confronted with such things as

$$a + b = c.$$

The immediate and natural reaction is like that of the little girl who knew her tables up to twelve times and was asked what was three times thirteen—"Don't be silly. It doesn't exist". The writer's earliest recollection of algebra was precisely like that, which shows that the matter was not clearly explained to him, or at least not clearly enough. The whole point is that the letters of ordinary algebra are not really being used as letters at all. They are just symbols which stand for numbers, and it so happens that the letters of the alphabet are very convenient symbols to use because they have an agreed shape and known names. In addition, certain other symbols are used which are either a short way of making statements (for instance, the symbol "=" is only a short way of writing "is the same as" or "is equal to") or else instructions to do certain things with the numbers represented by the letters.

2. Algebra As A Generalisation of Arithmetic

Bearing in mind the real character of the letter symbols used in algebra, we can understand this statement taken from Chrystal's textbook. "Ordinary algebra is simply the general theory of those operations with quantity of which the operations of ordinary arithmetic are a particular case. The fundamental laws of this algebra are therefore to be sought for in ordinary arithmetic." This, then, let us proceed to do.

3. Addition

What do we really mean by the addition of two numbers in ordinary arithmetic? Briefly, a number is a group of ones that we know by name. Adding two numbers means finding the name of the group which contains as many ones as the two groups put together. Thus we know (by memory now, but originally by trial with fingers or beans) that the group two combined with the group three has the same number of ones as the group that we have agreed to call five.

The above example is an ideal case, concerned with pure numbers. In practice, however, we shall not be concerned with pure numbers but with numbers of things—volts, amperes, pounds, shillings and pence, cabbages or kings—and here we come to one of the most important rules in the whole of mathematics.

Things can only be added together in the arithmetical sense if they are things of the same kind.

For instance, three apples can be added to two apples, and the resulting group can be called five apples. But can three apples be added to two oranges? Yes, in the sense that they can all be put into the same dish, but the number five cannot be attached to the group—unless indeed you call them five fruits, but then you are obeying the general rule, for you have obliterated the distinction between the two kinds of things, and have really added three fruits to two fruits; in other words, the things have been regarded as of the same kind. But obviously this can only be done if the distinction between the two kinds is unimportant for the purpose in mind, and in radio problems this never happens.

The above is the essence of what is known by the rather impressive title of "The Theory of Dimensions". It will be considered more fully later on, but for the present it will be enough to realise that if the working out of a given problem in wireless leads to the conclusion

$$L+R=10 \text{ ohms},$$

where L means some number of microhenries and R means some number of ohms, then the result is wrong without any further consideration, because it adds together two numbers of things of different kinds and calls the result a single number of one of the kinds.

In the ordinary arithmetic series of numbers that we know by simple names, one, two, three, four, five, and so on, the successive members get larger and larger in an orderly, uniform, and simple way. For this reason, numbers are a very convenient scale or "yard-stick" for describing size or quantity. This may appear to be an extension of the simple idea of pure number, but in fact it is a direct application of it, the only difference being that each kind of quantity has its own kind of "one" or "unit". Thus a current of 5 amperes means a current having 5 units or "ones" of current, the unit for this kind of quantity being called the ampere. Moreover, just as in pure numbers we have a simple way of describing large numbers in terms of certain special groups of ones—tens, hundreds, thousands and so on, so also in some kinds of quantity we have a series of special names for larger and larger groups of the unit of that quantity—inches, feet, yards, for example; and it is just too bad that our "scales of notation" in quantities of various kinds are not the "decimal" scale that we use for numbers. Perhaps the less said about this the better.

Now we can go on to the finding out of the generalised rules of arithmetical addition, and the easiest way will be to fix on some one particular kind of thing that we can make a picture of either mentally or actually. Then, on the understanding that all our numbers, or symbols standing for numbers, mean numbers of this particular thing, we shall be obeying the fundamental rule about addition, and the conclusions arrived at will apply generally to any form of arithmetical addition. A convenient thing will be a travel or a journey of, say, one inch in some definite direction, this direction being from left to right

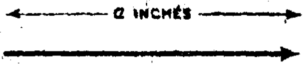

Fig. 1.—Representing the number *a* by a journey of *a* inches in a horizontal direction to the *right*

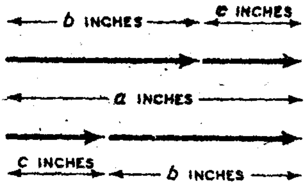

Fig. 2.—Showing that a journey *a* can be made up of journey *b* and then journey *c*, or journey *c* first and then journey *b*

parallel to the bottom edge of the page. This may seem a curious thing to choose, but it will be found later that it is very suitable for finding out the rules about subtraction also, which are not so easy to understand as those of addition.

Any number, say three, of these things will mean three journeys of one inch added together, and since the adding of two journeys means starting the second from the finishing point of the first, this will be the same as a journey of three inches in the given direction. In general any number a will mean a journey of a inches in the given direction (Fig. 1). Suppose now that the journey a is carried out in two stages, first a journey b and then a journey c, as shown in the upper part of Fig. 2. Then a can be described as the result of adding the journey c to the journey b, that is,

$$a = b + c.$$

Further, it is clear that it does not matter which of the two journeys b or c is made first (lower part of Fig. 2), so that

$$a = b + c = c + b.$$

In the same way it will be just as easy to show that

$$\begin{aligned} b + c + d &= b + d + c \\ &= c + b + d \\ &= d + c + b, \text{ etc., etc.,} \end{aligned}$$

d being another journey of d inches added to the other two. The same idea could be extended to any number of journeys without alteration and we thus arrive at the second general rule about addition.

A succession of additions will lead to the same result whatever the order in which they are carried out.

4. The Use of Brackets

Returning to the statement

$$a = b + c,$$

this expresses the idea that the two journeys b and c can be considered as a single journey. Similarly, any number of journeys b, c, d, e, etc. can, if desired, be associated together and considered to be a single journey $b + c + d + e +$ etc. Or, again, two of these journeys can be considered as a single journey, if that is convenient for some particular purpose, leaving the others as separate journeys. When it is desired to consider any particular group of journeys (or numbers) as a single thing, that group can be enclosed between two brackets thus $(c+d)$, and that means that we will take this as a whole, without regard to the fact that it is actually made up of two parts. Thus the compound group of numbers a, b, c, d can be written in the form

$$a + b + c + d$$

or in the form

$$a + b + (c + d),$$

the c and d numbers being, so to speak, wrapped up into a single brown paper parcel. This is called the Law of Association for addition, though it is certainly very difficult to see why it should be called a law at all, any more than the wrapping up of things in a brown paper parcel should be called the Law of Brown Paper. However, the idea of associating certain sets of numbers together by means of brackets is a useful one in practice.*

5. The Addition of Double Groups

Before leaving this subject of addition it may be as well to return for a moment to our three apples and two oranges,

* The associative law for addition is usually expressed: $a + (b + c) = (a + b) + c$.

because they can be used to illustrate a very important extension of the idea of addition, one which will prove useful in connection with alternating current circuits.

It has been shown that the group three apples plus two oranges cannot be expressed in any simpler form, since the two parts of the group cannot be combined in the sense of arithmetical addition. Two such double groups can be so combined, however. For instance, three apples plus two oranges combined with four apples plus six oranges can obviously be expressed as one double group, seven apples plus eight oranges, that is, the two sets of apples can be added and the two sets of oranges can be added. To generalise this idea, suppose the letters a, b, c, etc., to represent numbers of apples, and the Greek letters α, β, γ, etc., to represent numbers of oranges, and suppose that the working out of a problem concerned with these double groups of apples and oranges leads to the statement

$$a + \alpha + b + c + \beta + \gamma = d + \delta.$$

Then the number d on the right-hand side must be the result of adding together all the apples on the left-hand side, and similarly the number δ must represent all the oranges on the left-hand side. The statement is therefore equivalent to two separate statements

$$a + b + c = d$$
$$\alpha + \beta + \gamma = \delta.$$

If this simple idea is thoroughly assimilated, the reader will find that he has got a firm footing in the "complex number" or "operator" method of working out alternating current circuits.

6. Subtraction

The idea of subtraction in the ordinary arithmetical sense is one with which life makes us almost painfully familiar. Returning to the ideal case, we know (by memory) that if two ones are subtracted or removed from a group of five ones, then what remains is the group that we have agreed to call three, and if the ones are, say, pounds, then our understanding of the process is intensified