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propagation of transient elastic waves in stratified anisotropic media

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PROPAGATION OF TRANSIENT ELASTIC WAVES IN STRATIFIED ANISOTROPIC MEDIA

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Contents

Chapter 1

INTRODUCTION

Summary

Seismic waves are one of the standard diagnostic tools that are used to determine the mechanical parameters (volume density of mass, compressibility, elastic stiffness) in the interior of the earth and the geometry of subsurface structures. There is increasing evidence that in the interpretation of seismic data—especially shear-wave data—the influence of anisotropy must be taken into account. With this in mind, we present a method to compute the seismic waves that are generated by an impulsive source in a stratified anisotropic medium.

1.1 Statement of the problem

Although the present monograph has been written with the seismic applications in mind, the method that is developed is not limited to solid-earth geophysics. The classical example of waves in anisotropic media are the elastic waves in crystals (Musgrave 1970). Furthermore, many ultrasonic devices are constructed by using layers of piezoelectric (and, hence, anisotropic) materials; often these devices, too, are appropriately described by the idealized geometries discussed in this book. Electromagnetic waves in the ionosphere (Budden 1961, Felsen and Marcuvitz 1973, p.740) and in integrated optical devices (crystal optics) (Born and Wolf 1980, p.665, Yariv and Yeh 1984) are further examples where anisotropy of the medium must be included in the description. In short, the methods discussed in this monograph are applicable wherever waves propagate in stratified, anisotropic media.

Seismic anisotropy is very common in the earth. There are several causes of anisotropy: the interleaving of thin sedimentary beds, the presence of preferentially oriented cracks, the occurrence of stress-induced effects, and the alignment of crystals or grains. Until recently, interpretation of seismic data focused mainly on compressional waves. In the latter, anisotropy is expected to be of little importance; it is often simply ignored. But, with the increasing resolution of seismic observations, the influence of anisotropy is often noticeable, and of particular importance when shear waves are analyzed. In fact, one can safely state that in the real earth there exists no isotropic rock. How strong is the anisotropy and how strong does it manifest itself in elastic wave propagation? To answer this question quantitatively, we need a good understanding of how anisotropy affects the observed seismic wave fields. Such an understanding can be gained by building appropriate computer models and calculating, in some idealized geometries, the acoustic wave fields and their accompanying seismic records. The present monograph describes one idealized model, viz., the stratified medium. In this model, the influence of anisotropy in each layer can be studied separately.

There are fundamental differences between acoustic wave propagation in isotropic and anisotropic elastic media. These differences are already manifest in the propagation of uniform plane waves in a homogeneous region with a horizontal boundary. In an *isotropic* medium, one can distinguish between the compressional (P) and the, vertically or horizontally polarized, shear waves (SV and SH, respectively). The decomposition into these three eigenwaves is based on the polarization of their particle displacement, or their particle velocity, with respect to the horizontal plane, so that "V" means "vertical" and "H" means "horizontal." The particle velocity of the P waves is curl-free, while the particle velocities of the SV and the SH waves are divergence-free.

In a *weakly anisotropic* medium (weak in the sense that the medium is characterized by constitutive coefficients that are only small perturbations of the ones pertaining to some isotropic medium), the plane waves in a certain direction of propagation can still be labeled as quasi-P (qP) waves, with approximately longitudinal polarization of the particle velocity, and quasi-SV (qSV) and quasi-SH (qSH) waves, with approximately transverse polarization of the particle velocity (Keith and Crampin 1977b, Aki and Richards 1980, p.188). The decomposition into these three modes is again carried out with respect to the horizontal plane, so that "V" means "mostly vertical" and "H" means "mostly horizontal." In a *strongly anisotropic* medium, there are three plane waves with mutually orthogonal particle velocity polarizations in every direction of propagation. The wave speeds of these waves are different and vary with direction. Identification of the waves according to specific dominant particle velocity polarizations is now meaningless. Except in certain symmetry directions, none of the three waves is either curl-free or divergence-free in its particle velocity.

In this monograph, we investigate features of the wave field radiated by a concentrated source in a medium that is horizontally stratified, and where each layer is a homogeneous, and arbitrarily anisotropic, solid. The configuration serves as a canonical problem. Two sources, which are of seismic interest, are considered in detail, viz. a point source of expansion (model for an explosive source) and a point force (model for a mechanical vibrator). The theory also applies to point sources of deformation rate, which are adequate models for earthquake generation. The results can guide the interpretation of experimental data acquired in the more complicated situations met in practice, where the layers may not all be parallel.

1.2 The method of solution employed

The standard approach to the problem stated in Section 1.1 is to employ a Fourier transformation with respect to time and Fourier transformations with respect to the horizontal spatial coordinates. To obtain numerical results, the relevant inverse transformations have to be evaluated numerically, possibly using asymptotic methods in certain regions of space-time. This approach is similar to the frequency-wavenumber integration method used by Booth and Crampin (1983) and Fryer and Frazer (1984).

We solve the problem by a different method, viz., by applying the

Cagniard-de Hoop method (de Hoop 1960, 1961, see also Miklowitz 1978, p.302, and Aki and Richards 1980, p.224), which, in general, requires considerably less computation. In multilayered media the wave field is represented as a sum of generalized rays (Spencer 1960). Each of these is a wave constituent with a unique trajectory determined by interactions at interfaces and propagation through layers. The Cagniard-de Hoop method is then applied to each generalized ray individually. With this method, the computational results can be obtained relatively easily with any degree of accuracy; they can thus be used as a check on the accuracy of the numerical procedures that are employed to evaluate the inversion integrals in the standard treatment of the problem, i.e. the frequency-wavenumber integration method. The latter numerical technique seems to be the only available procedure that can be used when the materials have an arbitrary loss mechanism, or when the geometry involves curved surfaces.

We shall compare our method to various alternative methods at appropriate places in this monograph. Some of these alternative methods that can deal with anisotropy in layered media are: the frequency-wavenumber integration method mentioned above, ray-tracing methods for asymptotic high-frequency results and first-arrival analysis, and finite-element and finite-difference methods. The latter numerical methods can deal with arbitrarily inhomogeneous media, but require enormous computational effort. In contrast, the Cagniard-de Hoop method and the frequency-wavenumber integration method are both integral-transformation methods that are restricted to a special geometry, such as horizontal stratification and time invariance of the configuration.

The monograph consists of two parts: acoustic waves in isotropic media and acoustic waves in anisotropic media. The separate formulation for isotropic media has been included for didactical purposes. Although the general method is the same, the resulting expressions are simpler than the ones in anisotropic media. By offering the opportunity to look back at the analogous expression for isotropic media, the physical interpretation of the results pertaining to anisotropic media becomes much easier.

There is, however, another reason to write down explicitly, in Chapters 3 to 5, the analysis of acoustic wave propagation in stratified isotropic media with the aid of the Cagniard-de Hoop method. This reason is that nowhere in the literature the solution of this problem has been written up completely in its simplest form. Many publications on this problem can be found, but we feel there is always something missing. First, Cagniard's (1939) book and the translation by Flinn and Dix (1962) contain the original, intricate version of the transformation back to the time domain with the intermediate complex time variable. The latter difficulty was circumvented by de Hoop (1960), who simplified the transformation scheme such that the time variable remains real all the way through, but de Hoop's (1960) paper deals only with the case of a source in infinite space. Meanwhile, many authors used the method to solve a variety of specific problems. We mention the solution of Lamb's problem, where the elastic half space is considered (de Hoop 1961, Gakenheimer 1969). Some authors used only the two-dimensional version of the method (Achenbach 1973, p.298). Others studied multilayered structures, but either missed de Hoop's modification (Pao and Gajewski 1977), solved only the scalar acoustic case (Aki and Richards 1980), or introduced approximations to the method (Wiggins and Helmberger 1974). Of course, there are many positive aspects to these papers; in fact, we have used them as much as possible. We mention great educational clarity (Aki and Richards 1980), elegant notation (Pao and Gajewski 1977) and a very clear and detailed statement of the results (Johnson 1974, Wiggins and Helmberger 1974). In particular, we shall present the solution to the problem in its simplest form and state its concise solution for the all-encompassing case of an arbitrarily layered isotropic solid.

A final reason to include the analysis for isotropic media is that we want to present a formalism for wave propagation in stratified media that is generally applicable, i.e. to anisotropic media as well. Therefore, we have avoided concepts that are only advantageous in isotropic media, such as circularly cylindrical coordinates and the wave equations for the scalar and the one-component vector potentials. In this book, it becomes clear that Cartesian coordinates and the wave equations for the particle velocities and stresses are the more general ingredients since they are the keystones for the analysis of wave propagation in anisotropic media. The absence of directional independence in anisotropic media, removes the advantage of circularly cylindrical coordinates (and the corresponding separation of variables) in analyzing the acoustic waves radiated by a concentrated source in a stratified medium. Furthermore, the properties of anisotropic media are most easily expressed through Cartesian tensors, so a Cartesian reference frame is the simplest setting for the wave phenomena. (This does not withstand the fact that when studying wave propagation generated by a source in a geometry with circularly cylindrical interfaces, it may still be necessary to employ circularly cylindrical coordinates to satisfy the boundary conditions at the interfaces; the latter type of problem is beyond the scope of our present analysis.)

For the same reason of general applicability, we have used in Chapters 2, 6, and 7 a formalism that applies to arbitrarily anisotropic media. Several authors who have discussed the influence of anisotropy have limited themselves to special cases of symmetry, like transversely isotropic media (Payton 1983), or to weakly anisotropic media (Booth and Crampin 1983). The range of anisotropy in geophysical applications, however, is not limited to these special cases.

1.3 Numerical considerations

In the numerical treatment of the problem the following steps can be distinguished: (1) Selection of the generalized rays that have to be included in the calculation, (2) Calculation of the Cagniard-de Hoop contour for each generalized ray, (3) Its use to construct, by inspection, the time-domain Green's function, (4) Convolution of the Green's function with the source pulse to arrive at the complete waveform at the receiver position. The numerical methods consist of simple algorithms: an eigenvalue procedure to obtain the wave speeds (in anisotropic media), an iterative root-finding procedure to get the Cagniard-de Hoop contour, and the evaluation of a finite-range convolutional integral.

The standard objection to the generalized-ray/Cagniard-de Hoop method is that: "In a many-layered model there are far too many rays for efficient computation" (Chin, Hedstrom, and Thigpen 1984). This objection can be overcome by using an appropriate (energy-based) criterion in selecting rays.

In fact, selection of the generalized rays is crucial to the success of the method. First, we note that all generalized rays are causal functions of time; hence, they arrive, one after another, at an observation point. In practice, one is only interested in a synthetic seismogram