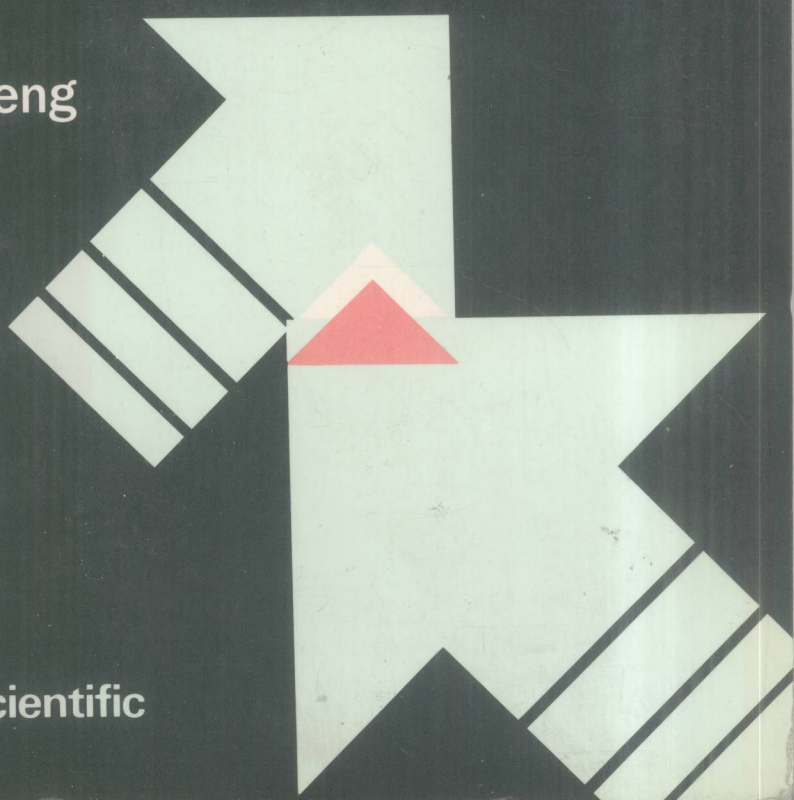


World Scientific Series on Directions in Condensed Matter Physics — Vol. 8

# **SCATTERING AND LOCALIZATION OF CLASSICAL WAVES IN RANDOM MEDIA**

Edited by  
**Ping Sheng**

**World Scientific**

An abstract geometric design in the lower right quadrant of the cover. It features a large, light green arrow pointing towards the top right. Within the arrow's shaft, there are three parallel diagonal lines. A small red triangle is positioned at the junction where the arrow's shaft meets its head. The background of the entire cover is dark, with some lighter green and black geometric shapes forming a larger, less distinct pattern.

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## PREFACE

The study of scattering and propagation of classical waves, i.e. electromagnetic and elastic waves, is a topic familiar to a broad cross section of scientists and engineers. This is so because waves are the principal means of communication, detection, and measurement. As a condensed-matter physicist, I am no exception to having a general knowledge base about wave scattering. However, it was only during the past decade, through the consideration of problems in oil exploration and subsurface imaging, that I learned about the fascinating aspects of multiply-scattered wavefield. The timing of this learning experience was especially fortunate, since it coincided with breakthroughs in the understanding of the localization phenomenon, a concept originally proposed by P. W. Anderson in 1958 in the context of electron diffusion in random potentials. These insights, achieved through a combination of phenomenological theories, experiments, developments in mathematical techniques, and large-scale numerical simulations, were soon realized to imply important predictions about the statistical characteristics of multiply-scattered classical waves. It was during my participation in the subsequent flurry of research activities that I became keenly aware of the need for a central reference where these exciting new developments can be quickly assimilated by a beginner as well as by experts in related fields. This is the source of my primary motivation that gives rise to the present volume.

The collection of papers contained in this volume is roughly balanced between theory and experiment. They have the common

theme of being related, in one way or the other, to the characteristics of classical waves under the condition of strong multiple scattering. The phenomenon of localization, being the single most prominent theoretical prediction for the strong scattering regime, is examined from different perspectives by a number of authors. It should be noted that each article is self-contained and therefore can be read independently. In that sense the ordering of the chapters is immaterial. However, I have grouped papers on electromagnetic waves or general properties of scalar waves towards the front of the volume and those dealing with acoustic waves or specific topics towards the rear of the volume.

I wish to thank every contributing author for the effort in writing an up-to-date and pedagogical account of his/her topic. I also very much appreciate the interest and technical help of the World Scientific staff. In particular, the care and attention of Miss P. H. Tham is essential in catching many mistypes I would had overlooked. Finally I wish to thank my wife, Deborah, for the support and understanding of a husband who had on many nights stayed at home but away from the family.

Ping Sheng  
*Annandale, NJ*  
1990

**SCATTERING AND LOCALIZATION  
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# THE LOCALIZATION OF WAVES IN DISORDERED MEDIA\*

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## 1. INTRODUCTION

Wave propagation in ordered and weakly disordered media is a subject with a long and rich history. It is a phenomenon which is common to everyday experience and as such would at first glance appear as an unlikely source for fundamentally new and unexplored physics. It is precisely such an unexplored regime which I will discuss in this article. The existence of propagating electromagnetic and thermal modes plays an important role in nearly all condensed matter systems. It is precisely this ubiquitous aspect of classical wave propagation which can lead to far reaching consequences if for some reason such modes were either absent or failed to propagate. The localization of classical waves and in particular light poses entirely new experimental and theoretical challenges, some unencountered and some overlooked in its electronic counterpart. It touches the very heart of our understanding of transport of wave-like phenomenon in disordered media whether quantum mechanical or classical, while at the same time offering the possibility of the most direct experimental test of these ideas. It is my aim in this article to describe some of the basic motivations for pursuing this goal, and to review in simple physical terms the underlying concepts. In addition I will present a pedagogical introduction to the formal mathematical description of localization and discuss some of the outstanding experimental and theoretical challenges, which have emerged in this field.

The theory of localization dates back to the classic paper of Anderson.<sup>1</sup> For a recent review of the rather extensive literature on electronic localization, the reader may wish to consult the article of Lee and Ramakrishnan.<sup>2</sup> The study of classical localization focussed initially on elastic waves in disordered solids. A first principles theory of localization of phonons in a disordered elastic medium in  $2+\epsilon$  dimensions was introduced by John, Sompolinsky and Stephen<sup>3</sup> based on the field theoretical formulation of electron localization by Wegner.<sup>4</sup> This theory has been recovered and extended by diagrammatic perturbation methods.<sup>5,6</sup>

In the case of electronic localization, nature provides a variety of readily available materials. It is convenient to describe these materials

by simple model Hamiltonians such as disordered tight binding models and continuum white noise models.<sup>7,8</sup> Such an approach to classical localization, can in some instances “discard the baby with the bath water” as will be discussed at length in Sec. 2 of this article and can lead to quite misleading conclusions about the observability of the effect. This is particularly evident in the case of electromagnetic wave localization. In this regime, the precursor effect to localization has been experimentally observed<sup>9–13</sup> and quantitative agreement between theory<sup>14–19</sup> and experiment has been established. The precursor effect, referred to as coherent backscattering is illustrated in Fig. 6 and will be discussed at length in Sec. 3. Extrapolation of this weak scattering behavior also leads to the highly plausible prediction that an actual photon mobility edge separating extended states from localized states may be observable<sup>20–24</sup> for sufficiently high dielectric contrast materials in an intermediate frequency window separating extended states at both higher and lower frequencies. The approach to this issue has been largely divided into two fundamentally different, although in some ways complementary, lines of reasoning. These are *microscopic* or single particle scattering resonance models<sup>22–24</sup> on the one hand, and on the other hand *macroscopic* or Bragg-like resonance mechanisms.<sup>25</sup> Before discussing the relative merits of these two approaches I will review some of the basic motivations for studying classical localization and in particular optical localization:

(i) It provides for the first time a complete test of the scaling theory of localization.<sup>26</sup> Optical propagation in lossless dielectric media provide the ideal realization of a single excitation in a static random medium at room temperature and is not hampered by the inevitable presence of electron-electron interactions and electron-phonon interactions which occur in the study of electron localization. High resolution optical techniques also provide for the first time a detailed study of the angular, spatial and temporal dependence of the specific intensity of light and not simply the average diffusion coefficient. This will provide a more complete understanding of localization as a critical phenomenon.

(ii) Photon localization will provide a more detailed understanding of the role of different types of disorder, spatial correlations and short range order in the determination of transport properties of waves. As will be discussed, the details of the structure factor and in particular, short range order play a vital role in determining the nature of transport and the very existence of localization. This is in contrast to the simple view put forward by Ioffe and Regel<sup>27</sup> which suggests that localization will occur whenever the elastic mean free path becomes comparable to the inverse wave vector of the wave. Optical studies in systems with well characterized forms of disorder and short or long range order will strengthen our understanding of the existence of pseudogaps in the density of electronic states in amorphous semiconductors and the resulting transport properties. High dielectric contrast colloidal suspensions may be useful in this regard. In systems in which there is an interplay between order and disorder, the Ioffe-Regel condition<sup>27</sup> for localization depends sensitively on the underlying structure factor, as I will describe, and must accordingly be reformulated.

(iii) Optical propagation in disordered layered dielectric media provides valuable laboratory models of geophysical wave propagation. Such materials may be fabricated by molecular beam epitaxy. These studies are valuable in seismology and oil exploration. In the strong scattering limit they pose new theoretical problems such as the incorporation of macroscopic anisotropies into the theory of localization.

(iv) Optical propagation in the diffusive scattering regime has already proved to be a valuable new spectroscopic tool for studying the hydrodynamics of complex fluids. This has been termed "Diffusing Wave Spectroscopy".<sup>28-30</sup> The time autocorrelation of light intensity in the laser speckle pattern is a measure of the velocity autocorrelation function of scattering particles. This technique has already been applied successfully to weak scattering systems such as polyballs in suspension. Static and dynamic structure of complex fluids especially in the vicinity of phase transformations may be studied even in the absence of wave localization. If higher dielectric contrast materials are prepared this could in turn yield new insights concerning the effect of



static and dynamic structure on localization.

(v) Periodic dielectric superlattice structures possessing photonic band gaps or pseudogaps in the optical density of states have recently been suggested<sup>25</sup> as candidates for strong localization of light. This also has important implications for the interaction of light with atoms and molecules. For instance it leads to the inhibition of spontaneous emission which is of importance in the design of new and more efficient lasers.<sup>31</sup> It will likewise affect photochemical rates and radiative transfer rates of molecules and radicals in solution.

(vi) Photon localization and band gaps also provide an important new avenue for the study of nonlinear optical effects and optical bistability. As in the case of polarons in a disordered medium<sup>32,33</sup> there is a substantial synergetic interplay between localization and self focussing nonlinearities. This is of importance for future optically based technologies such as bistable switching devices, optical transistors<sup>34</sup> and neural optical computers.<sup>35</sup>

The choice of photons as opposed to other classical waves as the focus of this discussion is largely motivated by this potential for important applications as well as the obvious availability of high resolution measurement techniques in optics. However, it should be mentioned that many of the ideas to be discussed apply equally well to the other systems. The most notable of these in terms of experimental realization are third sound localization in  $^4\text{He}$  films on rough substrates,<sup>36,37</sup> phonon localization in disordered solids as evidenced by the low temperature thermal conductivity plateau<sup>38-40</sup> and also surface plasmon localization on rough metal surfaces.<sup>41,42</sup> Under the more general heading of non-electronic localization I also mention the intriguing experiments by Tawel and Canter<sup>43</sup> on positron annihilation in dense helium gases at low temperatures. Here an anomalous peak in the annihilation rate is observed when the positron mean free path becomes comparable to the position de Broglie wavelength suggesting the possibility of a localization related phenomenon.

I conclude this section with an overview of the remaining sections in this article. A relatively complete qualitative picture of classical wave