CONTRIBUTIONS TO MATHEMATICAL ECONOMICS

In Honor of Gérard Debreu

Edited by

WERNER HILDENBRAND, *University of Bonn*
ANDREU MAS-COLELL, *Harvard University*

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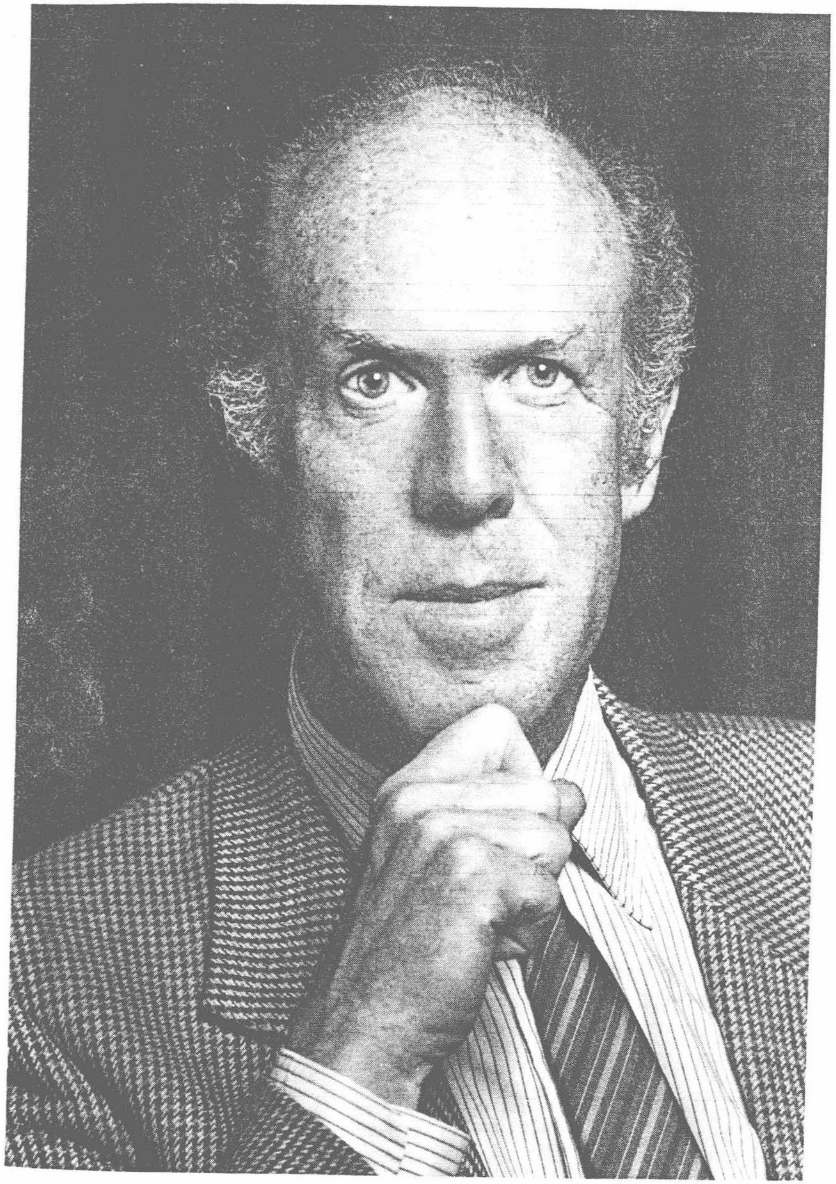

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PREFACE

This book is dedicated to Gérard Debreu on the occasion of his 65th birthday.

Gérard Debreu's scientific opus is widely known and has had a broad and deep influence on economic theorizing. It suffices to mention *Theory of Value* and the compilation *Mathematical Economics: Twenty Papers of Gérard Debreu*. The introduction to the latter volume assesses Debreu's contribution to economic theory and explains the part played by these papers in the development of mathematical economics. There would be no point in repeating that analysis here.

The group of contributors to this volume is distinguished by one form or another of close association with Gérard Debreu; we have known him as student, colleague, co-author, disciple, and friend. Most of us have spent substantial time at Berkeley and have benefited, there and elsewhere, from his silent encouragement, from his discreet criticism, and perhaps most importantly, from his clear setting of scientific standards. We may not always have heeded his advice or, hélas!, have reached the standards – yet we have tried.

We owe much to Gérard Debreu. The present collection of papers is a modest sign of gratitude.

Cher Gérard, bon anniversaire!

W. H.
A. M-C.
Berkeley, MSRI, October 1985

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Chapter 1

GENERAL EQUILIBRIUM WITH RATIONAL EXPECTATIONS*

BETH ALLEN

University of Pennsylvania, Philadelphia, PA, USA

1. Introduction

Surely an extremely important aspect of an economy is uncertainty, which thus should be reflected in our economic models. Asymmetric information constitutes an additional natural feature of such microeconomic models. In general, the phenomenon of different agents possessing different initial information leads to individual economic choices (or demands) which depend on the private information held by these agents. This typically results in equilibrium relations which reflect the information. Hence, economic equilibria may convey some information to agents who do not initially possess such information. Perhaps this transmission of information can be justified best as part of a long-run static equilibrium. The transfer of information by equilibrium prices forms the key innovation defining the concept of rational expectations, which was first introduced by Muth (1961). The idea has been incorporated into much recent research in microeconomics, including not only pure theory, but also such diverse applied areas as industrial organization and finance. The same term is utilized to describe related concepts in the recent macroeconomic literature.[1] My current opinion points to some version of rational expectations equilibrium as our best equilibrium concept for perfectly competitive economies under uncertainty and with asymmetric information.

Yet rational expectations raise a number of problems for microeconomic theory. Perhaps the most basic (and certainly the most extensively studied to date) of these is that there may not exist any rational expectations equilibrium; the potential failure of this primary consistency criterion for a microeconomic model with rational expectations features is quite troublesome, as it indicates

*This paper was prepared during the author's visit to CORE. Research support from the National Science Foundation and from a NATO Research Fellowship is gratefully acknowledged.

[1] See, for instance, Shiller (1978) for a survey.

that the further analysis of rational expectations equilibrium may be totally invalid. This possible non-existence problem – its nature, causes, examples, and especially positive results – forms the focus of this paper, which is organized as follows: The relation between this research area and the scientific contributions of Gérard Debreu are explained in the next section. A basic general equilibrium model with asymmetric information and rational expectations is presented in Section 3. Section 4 is devoted to explaining the concept of rational expectations equilibrium. Counterexamples to the existence of such equilibria appear in Section 5. More positively, existence results for revealing rational expectations equilibria are analyzed in Section 6. The following section proposes a few modifications of the equilibrium concept to deal with the non-existence problem in situations in which there cannot be revealing rational expectations equilibria. Then some more recent existence results, for both rational expectations equilibria and related concepts, are surveyed in Section 8. Finally, my own perspective on this research area and my views on a number of related open problems are discussed briefly in the concluding section.

2. *Relation to the research of Gérard Debreu*[2]

The obvious substantive link between the study of rational expectations and Debreu's contributions to economics lies in the formalization of uncertainty in economic theory, as exemplified in Chapter 7 of *Theory of Value* [Debreu (1959)], but which derived its roots from some of Gérard's earlier work [i.e., Debreu (1960)]. In the elegant and thoughtful concluding chapter of his monograph, Debreu (1959) sets out a basic model for evolving uncertainty which is patterned on the notion of a decision tree. Both preferences and commodities are state-dependent, so that, in particular, the description of an economic good includes all of its physical characteristics, its location, its delivery date, and the event (or subset of states of the world) in which it is available. Since markets are complete, all economic decisions can be made at the beginning of time and implemented through a system of contingent contracts. Even as consumers gain information, they do not wish to alter their

[2] Much of the research discussed here is related to Debreu's work in the more personal sense that he provided encouragement, advice, and above all an example of the scientific standards to which we should aspire. In particular, Gérard stimulated my interest in this topic and supervised my Ph.D. dissertation, which resulted in the publications cited here as Allen (1981a, 1981b, 1982b), plus a portion of Allen (1984).

previous (contingent) commitments, so that markets need never reopen for additional trading. Although this feature can be criticized for its unrealism in particular economic situations – it may require the presence of an inordinate number of markets – and has subsequently been generalized by others, the fact remains that the concept of state-dependent preferences, accompanied by possibly state-dependent demands, is absolutely basic to the theoretical analysis of economies under uncertainty.

In a more general way, as with virtually all recent work in microeconomic theory, the rational expectations research area owes much to Gérard's continual professional insistence on rigorous proofs and explicit statements of assumptions. His advocacy of axiomatic methods in economic analysis has imprinted its style on this research area too, despite the fact that the economic motivations for this work tend to be derived from somewhat more applied topics, such as questions in finance, industrial organization, and macroeconomics.

One by-product of the rigorous axiomatic approach to theoretical economics is the viewpoint that the existence of economic equilibrium should be considered to be an extremely important consistency condition or minimal requirement for the analysis of a particular economic model. This strand of thought, which runs through much of Gérard's early work and cumulates in *Theory of Value* [see also the recent survey paper, Debreu (1982), and the references listed there], is reflected in the search for existence results for rational expectations equilibrium. Gérard's uncompromising emphasis on the importance of existence results for economic equilibrium is also partly responsible for the justified discomfort with which economists have reacted to counterexamples to the existence of rational expectations equilibrium.

These counterexamples have led to a search for generic results. Initiated by Debreu (1970), the generic approach consists of the quest for economic results which are satisfied by typical rather than rare cases, or alternatively, for assumptions which are typically satisfied if one cannot find very reasonable and economically interpretable sufficient hypotheses for a desired result. For the generic rational expectations results discussed here, as in that article, regularity plays a key role; the mathematical techniques are those of differential topology, especially Sard's Theorem (on the regular values of a smooth map) and its generalization to transversality theory. Because of this, as is true for most generic results in economic theory, one must work with a "smooth" economic model, rather than with a continuous model of the set theoretic genre as studied in *Theory of Value*. Once again, we are indebted to Gérard Debreu for his revival of the differential viewpoint in economics and for his formulation of the rigorous foundations of this approach. See Debreu (1972, 1976a) and also his survey papers on this topic, Debreu (1974, 1976b).

3. The basic model

To some extent, the axiomatic model proposed here is derived from Debreu (1959, ch. 7) and its extension by Radner (1968) to situations encompassing asymmetric information. For simplicity, I examine a static pure exchange economy with an arbitrary finite number l of commodities. Because rational expectations models with incomplete markets are more interesting, I shall assume, to avoid complications, that state-dependent contingent commodity contracts are not available; in particular, financial assets are not present in my model. Production is not considered because the fundamental problems of rational expectations arise in pure exchange economies, so that no essential richness or interesting economic phenomena are excluded by this approach. To the extent possible, I wish to use only assumptions which are consistent with general equilibrium theory, which are commonly and innocuously assumed in general equilibrium theory, or which can be demonstrated to be generic. In particular, I am not especially interested in results that are specialized to a single market or which depend crucially on particular functional forms for consumers' utility functions (i.e., constant absolute risk aversion) or for the distributions of random variables (i.e., normality).[3]

To describe the uncertainty in the economy, let $(\Omega, \mathscr{F}, \mu)$ be an abstract probability triple. This means that Ω is a set, $\mathscr{F}$ is a σ-field of measurable subsets of Ω, and μ is a (countably additive) probability measure defined on $(\Omega, \mathscr{F})$. Interpret Ω as the set of states of the world and suppose that Ω is large enough to include all details which are payoff-relevant and which are known by any (non-null subset of) agents (using all of their pooled information) when markets are open.[4] Let $\omega \in \Omega$ denote a typical state of the world. Members of the distinguished class $\mathscr{F}$ of measurable subsets of Ω are called events; they include all subsets of states of the world upon which agents can potentially condition their acts. The probability μ describes the likelihood of various events; think of it as an ex ante objective or subjective probability. For most of the results discussed in this paper, different agents may have different subjective probabilities defined on $(\Omega, \mathscr{F})$ providing that they're not too different – everyone must agree about which events occur with probability zero.[5]

[3] I won't attempt to include a discussion here of the enormous literature concerning rational expectations results for such cases; see Grossman's (1981) survey article.

[4] Descriptions distinguishing states of the world which do not influence any agent's utility, or states which cannot ever be separated based on the pooled information of all agents (until after the market meets) are irrelevant for rational expectations equilibrium and can therefore be ignored.

[5] Notice that, in the definition of rational expectations equilibrium in the next section, an exceptional null set arises; if agents disagree about the definition of subsets of states of the world which occur with probability zero, rational expectations equilibrium need not be well defined.

The set of agents in the economy is denoted A, which may be taken to be a finite set except for some of the results discussed in Sections 7 and 8, where a non-atomic continuum of (uninformed) agents is needed. For the large economies results, I may without loss of generality use a standard representation so that $A = [0,1]$, the measurable coalitions are the Borel subsets $\mathscr{B}([0,1])$, and the measure describing the "sizes" of various agents (or frequencies of various types of agents) is the 1-dimensional Lebesgue measure λ on $([0,1], \mathscr{B}([0,1]))$. All agents are assumed to take prices as given and act competitively. Rational expectations with strategic behavior by traders give rise to some very important and interesting open questions.

Each agent $\alpha \in A$ is described by an initial endowment vector[6] $e_\alpha \in \mathbb{R}^l_{++}$ of commodities, a state-dependent (cardinal) utility function $u_\alpha(\cdot\,;\cdot)$ defined on $\mathbb{R}^l_{++} \times \Omega$, and a private information sub-σ-field $\mathscr{F}_\alpha \subseteq \mathscr{F}$. Assume that any event in $\mathscr{F}$ can be verified by the pooled information of all agents – in symbols, $\mathscr{F} = \sigma(\bigcup_{\alpha \in A} \mathscr{F}_\alpha)$, where $\sigma(\mathscr{G})$ denotes the smallest σ-field generated by the class $\mathscr{G}$ of subsets of Ω. Traders desire to maximize conditional expected utility, where the ex post state-dependent preferences and attitudes toward risk are specified by the utilities $u_\alpha\colon \Omega \to \mathscr{K}_\alpha$, where $\mathscr{K}_\alpha$ is a compact subset of $C^r(\mathbb{R}^l_{++}, \mathbb{R})$, $r \geq 2$, endowed with the (weak) C^r compact-open topology.[7] Assume that each u_α is measurable for the Borel subsets of $\mathscr{K}_\alpha$. Interpret $u_\alpha(x;\omega)$ as the utility to agent α of consuming $x \in \mathbb{R}^l_{++}$ when the state of the world is known to be $\omega \in \Omega$. For (almost) every $\alpha \in A$, assume that every $u_\alpha(\cdot\,;\omega) \in \mathscr{K}_\alpha$ satisfies the following three properties:

(i) strict (differentiable) monotonicity: $D_x u_\alpha(x;\omega) \gg 0$ for all $x \in \mathbb{R}^l_{++}$.
(ii) strict (differentiable) concavity: $D_{xx} u_\alpha(x;\omega)$ is negative definite for all $x \in \mathbb{R}^l_{++}$, and
(iii) boundary condition: the closures in $\mathbb{R}^l$ of the indifference surfaces $u_\alpha(\cdot\,;\omega)^{-1}(c)$ are disjoint from the boundary $\partial\mathbb{R}^l_{++}$ of the positive orthant for all $c \in \mathbb{R}$.

Observe that these three conditions imply that (almost) every agent's state-dependent preferences are "smooth" in the sense of Debreu (1972, 1976a) so that, conditional on any measurable event $S \in \mathscr{F}$, demands are well defined

[6] Smooth $\mathscr{F}_\alpha$-measurable state-dependent endowments can be permitted; for details, see the particular articles cited later in this paper. However, to avoid potential bankruptcy problems, each agent must know his own endowment with certainty.

[7] The (weak) C^r compact-open topology is the topology of C^r uniform convergence on compact subsets, as opposed to the (strong) C^r Whitney topology on a space of C^r functions. See Hirsch (1976) for mathematical details.

C^{r-1} functions.[8] Finally, if A is infinite, let $e\colon A \to \mathbb{R}^l_{++}$ denote the (Borel) measurable initial endowment assignment map, and assume that the endowment distribution $\lambda \circ e^{-1}$ has compact support K contained in $\mathbb{R}^l_{++}$ and that $\mathscr{K} = \bigcup_{\alpha \in A} \mathscr{K}_\alpha$ is a compact subset of $C^r(\mathbb{R}^l_{++}, \mathbb{R})$ satisfying conditions (i), (ii), and (iii).[9] Assume that the utility map $u\colon A \times \Omega \to C^r(\mathbb{R}^l_{++}, \mathbb{R})$ is jointly measurable, which automatically implies that (with an abuse of notation) $u\colon A \times \Omega \times \mathbb{R}^l_{++} \to \mathbb{R}$ is jointly measurable. The mild compact support assumptions have the economic interpretation that different agents in different states of the world do not have vastly different state-dependent preferences and that there are uniform upper and lower bounds (which are automatic whenever A is a finite set) for initial endowment vectors.

4. *Rational expectations equilibrium*

The key idea of rational expectations is that agents associate subsets of states of the world with particular equilibrium price vectors that they observe. This occurs, for instance, because traders know (or have already learned) the relationship between states and prices in a long-run stationary equilibrium. Rationality then requires that traders utilize all available information, including in particular the information conveyed by equilibrium prices, when they attempt to maximize expected utility. Information is contained in equilibrium prices because individual demands reflect agents' initial private information. Neither others' private information nor their individual demands are hypothesized to be observable, but the prevailing price corresponding to the true state of the world and the correct equilibrium mapping from states to prices must be known by all agents, except possibly those with complete initial private information $\mathscr{F}_\alpha = \mathscr{F}$. Notice that the notion of rational expectations equilibrium defined here applies to perfectly competitive economies. Agents are not allowed to act strategically in choosing their demand functions so as to avoid potentially disadvantageous equilibrium allocations arising from the information that they give to other traders through the resulting rational expectations equilibrium price functions.

In order to permit information transmission, prices must be allowed to differ across states of the world in the model. By definition, a *price function* is a

[8] See, for example, lemma 5.1 and proposition 5.6 in Allen (1985c).

[9] This uniformity condition can be weakened somewhat at the expense of more notation: see the articles cited in Sections 7 and 8 for details.

measurable[10] mapping $p\colon \Omega \to \Delta$, where price vectors for the l commodities are normalized to lie in the $(l-1)$-dimensional open unit simplex

$$\Delta = \left\{ q \in \mathbb{R}^l_{++} \,\middle|\, \sum_{j=1}^{l} q_j = 1 \right\} \subset \mathbb{R}^l.$$

To avoid confusion, $p(\cdot)$ or simply p always denotes an entire price function (or relation between states of the world and prices), while q denotes a price vector specifying one normalized price for each (non-state-contingent) physical commodity.

Given a price function $p\colon \Omega \to \Delta$, each agent $\alpha \in A$ chooses a demand (which is a function under my assumptions) $x_\alpha\colon \Omega \to \mathbb{R}^l_{++}$ depending only on his available information so as to maximize conditional expected utility, given that information, subject to his budget constraint for each state of the world. Note that budget constraints (and also market clearing requirements) are required to hold separately for almost every state of the world, rather than in expectation. Hence, $x_\alpha(\cdot)$ is a $\mathscr{F}_\alpha \vee \sigma(p)$-measurable $\mathbb{R}^l_{++}$-valued function such that for μ-almost all $\omega \in \Omega$,

$$x_\alpha(\omega) \in \arg\max \{ \mathrm{E}(u_\alpha(x_\alpha; \cdot) | \mathscr{F}_\alpha \vee \sigma(p))(\omega)\colon \\ p(\omega) \cdot x_\alpha(\omega) \le p(\omega) \cdot e_\alpha \};$$

i.e., for every $x'_\alpha\colon \Omega \to \mathbb{R}^l_{++}$ satisfying $\mathscr{F}_\alpha \vee \sigma(p)$-measurability and the budget constraint $p(\omega) \cdot x'_\alpha(\omega) \le p(\omega) \cdot e_\alpha$ for μ-almost every $\omega \in \Omega$, I have

$$\int_\Omega u_\alpha(x'_\alpha; \omega)\, \mathrm{d}\mu\, (\omega | \mathscr{F}_\alpha \vee \sigma(p)) \le \int_\Omega u_\alpha(x_\alpha; \omega)\, \mathrm{d}\mu\, (\omega | \mathscr{F}_\alpha \vee \sigma(p)),$$

where $\mu(\cdot | \mathscr{G})$ denotes a (fixed) version of proper regular conditional μ-probability given the sub-σ-field $\mathscr{G} \subset \mathscr{F}$. For technical reasons, the conditional expected utility operations are actually carried out on the compact subset $\mathscr{K}_\alpha$ of the Frechet space $C^r(\mathbb{R}^l_{++}, \mathbb{R})$; see Allen (1981b) for details.

If $p\colon \Omega \to \Delta$ is such that every market clears almost surely when agents condition on the information transmitted by p, then the price function $p\colon \Omega \to \Delta$ and the allocation functions $x_\alpha\colon \Omega \to \mathbb{R}^l_{++}$, $\alpha \in A$, form a *rational*

[10] For subsets of $\mathbb{R}^l$, measurability is with respect to the Borel subsets $\mathscr{B}(\mathbb{R}^l)$ when no other σ-field is specified.

expectations equilibrium. More formally, this equilibrium concept is defined as follows:

Definition. A *rational expectations equilibrium* for an economy with finitely many agents is defined to be an equivalence class of $\mathscr{F} = \sigma(\bigcup_{\alpha \in A} \mathscr{F}_\alpha)$-measurable price functions[11] $p^*\colon \Omega \to \Delta$, and for each agent $\alpha \in A$, an equivalence class of $\mathscr{F}_\alpha \vee \sigma(p^*)$-measurable allocation functions $x_\alpha^*\colon \Omega \to \mathbb{R}^l_{++}$ such that

(i) for every $\alpha \in A$, $p^*(\omega) \cdot x_\alpha^*(\omega) \le p^*(\omega) \cdot e_\alpha$ and if $x'_\alpha\colon \Omega \to \mathbb{R}^l_{++}$ satisfies the information constraint that $x'_\alpha(\cdot)$ be $\mathscr{F}_\alpha \vee \sigma(p^*)$-measurable and the budget constraint that $p^*(\omega) \cdot x'_\alpha(\omega) \le p^*(\omega) \cdot e_\alpha$ for μ-almost every $\omega \in \Omega$, then

$$\int_\Omega u_\alpha(x'_\alpha(\omega); \omega)\, \mathrm{d}\mu\, (\omega | \mathscr{F}_\alpha \vee \sigma(p^*))$$

$$\le \int_\Omega u_\alpha(x^*_\alpha(\omega); \omega)\, \mathrm{d}\mu\, (\omega | \mathscr{F}_\alpha \vee \sigma(p^*)),$$

(ii) $\sum_{\alpha \in A} x_\alpha^*(\omega) \le \sum_{\alpha \in A} e_\alpha$ for μ-almost every $\omega \in \Omega$.

Remarks

(1) If the economy has infinitely many agents, the allocations $x_\alpha^*(\cdot)$ are required to be jointly measurable in $\alpha \in A$ and $\omega \in \Omega$ [in addition to the information constraint that each x_α^* be $\mathscr{F}_\alpha \vee \sigma(p^*)$-measurable] and the sum in (ii) is replaced by an integral over $\alpha \in A$ with respect to Lebesgue measure λ on the set of agents in standard representation.

(2) The possibility that markets may fail to clear for an event which occurs with probability zero is quite natural – recall that regular conditional probabilities are only well defined up to a null subset of states of the world.

(3) To ease exposition, I refer to price and allocation functions as rational expectations equilibria, rather than insisting on the formalism that an equilibrium consists of an entire equivalence class (modulo the equivalence relation of almost sure equality) of measurable functions; no confusion should result.

(4) If $(p^*, x_\alpha^*\ (\alpha \in A))$ is a rational expectations equilibrium, say that p^* is a *rational expectations equilibrium price function*. Because utilities are strictly concave, the price function p^* uniquely determines (up to the equivalence class) equilibrium allocation functions for the rational expectations equilibrium. Hence I may identify rational expectations equilibria with rational expectations equilibrium price functions and ignore the derivation of their corresponding allocation functions.

[11]Alternatively, I could define a rational expectations equilibrium to be a measurable price correspondence for which every measurable selection has these properties.

Observe that, in a rational expectations model, agents' demands are defined only for equilibrium prices.[12] Given a measurable price function $p\colon \Omega \to \Delta$ which agent α believes to be the correct relationship between states of the world and prices, if α observes some price vector $q \in A$ which is "impossible" [meaning that for every $\omega \in \Omega$, $q \ne p(\omega)$], then α's conditional expected utility is undefined. The trader cannot proceed to act when his information about the model and his observations are contradictory.

A further criticism of rational expectations as an economic equilibrium concept is provided by Beja (1976) and Grossman and Stiglitz (1976) who observe that agents have no incentive to gather and use information which will be reflected in rational expectations equilibrium prices whenever doing so is inconvenient or costly. Beja (1976) is concerned with the process by which such information can enter prices, while Grossman and Stiglitz (1976) focus on the situation in which information is costly to purchase. In my opinion, these objections lose their force whenever prices fail to convey *all* information. This suggests that research should concentrate on cases in which rational expectations equilibrium prices cannot carry all relevant information. Not only are such situations more realistic, but they also admit a much wider range of economically interesting behaviors, including possibly the non-trivial optimal endogenous purchase of information and its strategic utilization.

5. *Some counterexamples*

Although Radner (1967) recognized the possible non-existence problem for rational expectations equilibria much earlier, Kreps (1977) provided the first explicit counterexample. As Radner (1967) observed, discontinuities in the dependence of expectations on prices can invalidate the use of a fixed point argument to demonstrate that rational expectations equilibria exist. For general equilibrium theorists, this phenomenon is troublesome because it suggests some potentially very serious inconsistencies in an otherwise attractive model of economic behavior with asymmetric information.

To illustrate the non-existence problem, I present a very simple counterexample which is explained in more detail in Allen (1984). The idea is

[12]I recall Frank Hahn to be the source of this objection (in the oral tradition) to rational expectations, but I know of no written reference for this point.

essentially the same as in the Kreps (1977) example, but the verifications and calculations are a bit easier for this one.

Example. Suppose that there are two consumers, two commodities, and two states of the world. Call the commodities x and y, with (normalized) prices p and $1 - p$, respectively. Each consumer (indicated by subscripts 1 and 2) has a log-linear utility function in each state of the world, heads (H) and tails (T), which are known to be equally probable.

The first agent knows the state of the world before trading. His state-dependent preferences are representable by the following utility functions:

$$u_1(x, y; H) = \tfrac{1}{3}\log x + \tfrac{2}{3}\log y,$$
$$u_1(x, y; T) = \tfrac{2}{3}\log x + \tfrac{1}{3}\log y.$$

The second agent is initially uninformed, but he learns the state of the world after trading. His preferences are also state-dependent:

$$u_2(x, y; H) = \tfrac{2}{3}\log x + \tfrac{1}{3}\log y,$$
$$u_2(x, y; T) = \tfrac{1}{3}\log x + \tfrac{2}{3}\log y.$$

In the absence of further information, he maximizes (unconditional) expected utility

$$\mathrm{E}u_2(x, y) = \tfrac{1}{2}\log x + \tfrac{1}{2}\log y.$$

The second agent cannot observe the demand of the first agent. However, he can see the prevailing market clearing prices and he is assumed to know, in equilibrium, the relationship between states of the world and prices.

Each agent has a non-random initial endowment $e_1 = e_2 = (1, 1)$ of one unit of each commodity, so that wealths equal unity at all (normalized) prices. The first agent demands $\frac{1}{3}/p$ units of x and $\frac{2}{3}/(1-p)$ units of y in state H, and $\frac{2}{3}/p$ units of x and $\frac{1}{3}/(1-p)$ units of y in state T. If the second agent uses only his private information, his demands are independent of the (unknown to him) state of the world and equal $\frac{1}{2}/p$ units of x and $\frac{1}{2}/(1-p)$ units of y. Market clearing requires

$$\tfrac{1}{3}/p(H) + \tfrac{1}{2}/p(H) = 2 \Rightarrow p(H) = \tfrac{5}{12}$$

in state H, and

$$\tfrac{2}{3}/p(T) + \tfrac{1}{2}/p(T) = 2 \Rightarrow p(T) = \tfrac{7}{12}$$

in state T. Notice that these prices are different in different states: $p(H) \neq p(T)$. Thus, when the uninformed agent knows the function $p(\cdot)$ and observes a prevailing price of either $\frac{5}{12}$ or $\frac{7}{12}$ for the first commodity, he is able to discern the correct state of the world. Rather than maximizing expected utility, he can do better by maximizing his state-dependent utility function corresponding to the true state of the world. Of course this alters his demand function to one which is state-dependent. The market clearing conditions are changed to the following when both agents can recognize the true state of the world:

$$\tfrac{1}{3}/p(H) + \tfrac{2}{3}/p(H) = 2 \Rightarrow p(H) = \tfrac{1}{2},$$
$$\tfrac{2}{3}/p(T) + \tfrac{1}{3}/p(T) = 2 \Rightarrow p(T) = \tfrac{1}{2}.$$

Therefore we obtain a price of one-half for each commodity regardless of the state of the world, and these prices do not reveal any information to the second trader. Conclude that if the second agent's demand does not reflect knowledge of the true state of the world, then the market clearing prices will transmit information from the informed to the uninformed agent, while if his demand does incorporate such knowledge, the resulting market clearing prices cannot reveal the correct state. The information carried by prices cannot be consistent with market clearing if the uninformed agent uses such information. This observation establishes non-existence of rational expectations equilibrium for this two-state example in which prices must either reveal everything or reveal nothing at all about the state of the world. The dilemma also shows that there does not exist an equilibrium for this example in which prices can convey information from informed agents to a second group of the same size of initially uninformed traders, even if both sets consist of an atomless continuum of agents.

The reason that one cannot apply a standard fixed point argument to prove that equilibria exist is a discontinuity in the dependence on price functions of initially uninformed traders' demands. Information transmission – or discontinuities in the operation of forming conditional expectations – causes the problem. When prices in the two states are precisely equal in the rational expectations model, the second trader must revert to maximization of his unconditional expected utility, while whenever prices differ even slightly across states of the world, an uninformed agent is able to infer exactly whether the correct state is heads or tails. Notice also that, in a rational expectations framework, equilibrium prices must be determined for both states of the world simultaneously because the relationship between these prices determines the information that they convey.

My non-existence example appears to be an otherwise extremely well-behaved economy. In this sense, it forms an extremely compelling counter-example. However, one can easily verify [see Allen (1984)] that perturbations of almost any of the parameters describing the economy restore the existence of equilibria. Moreover, in general, such equilibria do permit the uninformed agent to infer from prices the correct state of the world.

On the other hand, obtaining a robust (i.e., "open" or stable under perturbations) counterexample to the existence of rational expectations equilibrium is considerably more complicated. An example of this type was constructed by Green (1977). A second such example, which I personally find easier to understand, was devised by Jordan and Radner (1979), and summarized in their 1982 introductory essay, for the case of $\Omega = \Delta = [0,1]$. These examples are necessarily more difficult than the two-state non-robust example described above. For robust counterexamples, an infinite set of states of the world is needed [see Radner (1979), Allen (1984), and the discussion in the next section] and one must show that there are no partially revealing rational expectations equilibria. The simple dichotomy leading to the contradiction establishing non-existence in the counterexample analyzed above does not suffice to prove non-existence of any rational expectations equilibrium in the general case.

6. *Revealing rational expectations equilibria*

This section concerns existence results for rational expectations equilibria which reveal all information initially possessed by any agent in the (finite – or with finitely many types) economy. Obviously complete revelation is not realistic; prices on the stock market do not transmit all relevant information. Moreover, the conditions needed to ensure that there are such equilibria are too restrictive to be completely satisfactory as economic assumptions.[13] These observations are formalized by Jordan (1983), who proves that if the uncertainty is sufficiently complicated (more precisely, if payoff-relevant states of the world can only be described by a parameter set of higher dimension than the dimension of the set of normalized prices), then (fully) revealing rational expectations equilibria exist only if agents' utilities belong to the hyperbolic absolute risk aversion class which has been studied so extensively in finance.

[13] Note, however, that for many financial assets, a large number of futures and options contracts are marketed, so that as many as a dozen or more prices may pertain to the future prospects of a single corporation, albeit applicable to different dates. On the other hand, the prices of different classes of common and preferred stock probably reflect the value of voting rights and bankruptcy priority claims rather than additional information about the company.

By definition, a rational expectations equilibrium price function $p^*: \Omega \to \Delta$ is said to be *revealing* if, on a subset of Ω which is of full μ-measure, p^* is one-to-one. More generally, say that a price function p^* is revealing if, except possibly on a null subset of Ω, $p^*(\omega) = p^*(\omega')$ implies $u_i(\cdot\,;\omega) = u_i(\cdot\,;\omega')$ for every agent i in the finite economy (or for every type i if the economy has a finite number of types of agents). Conditioning on a revealing equilibrium price function is equivalent to knowing the pooled information of all agents in the economy.

The demonstration that there are revealing rational expectations equilibria is relatively easy because of the trick due to Grossman (1978) and Radner (1979) [and also used in Allen (1981a, 1982b, 1984)] of appealing to an artificial full information economy. If there are market clearing price functions, for excess demands calculated under the supposition that all agents are initially completely informed, which are revealing then such price functions form revealing rational expectations equilibrium price functions. Therefore, the existence question reduces to the problem of whether the artificial full information economy has injective market clearing price functions. Such prices can be obtained for each state of the world separately,[14] thereby avoiding the issue of finding a fixed point in a space of entire price functions.

Radner (1979) uses the artificial full information economy device to show that generically (i.e., for a "typical" such economy) there exists a revealing rational expectations equilibrium provided that the relevant uncertainty can be described by a finite set of parameters. More precisely, he specifies a space of economies by arrays of conditional probabilities on the signals observed by other agents given one's own signal. These signals represent all of the available payoff-relevant information in the economy at the time the market meets; they are correlated with the true payoff-relevant states of the world entering agents' utility functions. For an open and dense subset of such conditional probability arrays which is also of full Lebesgue measure, revealing rational expectations equilibria exist and there are no rational expectations equilibria which are not (fully) revealing. Moreover, the dimension of the set of economies for which rational expectations equilibria fail to exist is larger than the dimension of the set for which there are non-revealing rational expectations equilibria, which is then larger than that of the economies possessing both revealing and non-revealing rational expectations equilibria.[15]

[14] For instance, by conveniently appealing to the existence result stated in Dierker (1974, p. 78).

[15] Here the term non-revealing refers to the absence of full revelation; in particular, it includes partially revealing equilibria which can "generically" be expected to arise whenever there are non-revealing rational expectations equilibria and at least three payoff-relevant signals. The case of complete non-revelation, or constant equilibrium price functions, is a special case of non-revealing equilibria, or price functions which fail to be injective.

A similar generic existence result in a framework in which pertubations are performed with respect to state-dependent utilities rather than conditional probability arrays appears in Allen (1984) for the case of a finite set Ω of states of the world.

Theorem 1. Assume that Ω is finite. Then, given $(u_i(\cdot;\omega_1),\ldots,u_i(\cdot;\omega_{\#\Omega})) \in C^0(\Omega, C^2(\mathbb{R}^l_{++},\mathbb{R}))$ for each agent $i \neq 1$, there is an open and dense subset S of $C^0(\Omega, C^2(\mathbb{R}^l_{++},\mathbb{R}))$ of state-dependent utilities for the first agent (or for the first type of agent having positive measure in a large economy) such that if $(u_1(\cdot;\omega_1),\ldots,u_1(\cdot;\omega_{\#\Omega})) \in S$, then any selection from the equilibrium price correspondence for the artificial full information economy constitutes a revealing rational expectations equilibrium price function. In particular, an open and dense subset of economies possess revealing rational expectations equilibria.

Sketch of proof. First note that competitive equilibria exist for all artificial full information economies; if $E\colon \Omega \to \Delta$ denotes the full information equilibrium price correspondence, then for every economy, for each $\omega \in \Omega$, $E(\omega) \neq \varnothing$. Moreover, for an open and dense subset (i.e., the regular economies) of maps from Ω into $C^2(\mathbb{R}^l_{++},\mathbb{R})$, the full information equilibrium price correspondence assumes only finitely many values – for every $\omega \in \Omega$, $\#E(\omega)$ is finite. This is a variant of Debreu's (1970) result on the finiteness of the equilibrium price set. Sufficiently small C^2 perturbations don't destroy regularity or change the number of equilibria. Use the results in Allen (1981a) to alter the first agent's state-dependent utilities so that $E(\omega) \cap E(\omega') = \varnothing$ whenever $\omega \neq \omega'$. Hence, for an open and dense subset S, any selection from E is one-to-one. This injectivity proves that each selection is a revealing rational expectations equilibrium price function for an open and dense subset S of economies in which prices convey information. See Allen (1984, theorem A) for details. □

The result can be extended to some cases with infinitely many states of the world. Again, one appeals to the artificial full information economy trick. Differential topology provides conditions for the generic injectivity of smooth mappings, and results in Allen (1981a) establish that utility perturbations can be used technically to transfer generic statements about functions to generic properties of economies. The simpler device of perturbing initial endowments is inadequate because such endowment perturbations must generally be state dependent and hence possibly not measurable with respect to the agent's private information. Of course, this would be permissible under the additional

assumption that one agent (or one type of agent) is completely informed initially about the true state of the world.

When Ω is infinite, the relevant object is the support (by definition, the smallest closed subset of full measure) in $(C^2(\mathbb{R}^l_{++},\mathbb{R}))^{\#A}$ of the image measure describing the distribution (under the probability measure μ on events) of state-dependent utilities for each agent. This approach has the virtue of avoiding compactness and dimensionality assumptions on the abstract set Ω of states of the world – which axiomatically does not have such a mathematical structure – although for the special case in which utilities depend smoothly on a set of parameters (describing states of the world), it suffices to check that the parameter set is a compact manifold of an appropriate dimension. To simplify notation, I write $\tilde{\Omega}$ as an abbreviation for either the parameter set or the support (which is a subset of a topological vector space, so that its dimension is well defined, albeit possibly infinite) of the image measure.

Theorem 2. If $\tilde{\Omega}$ is a compact manifold (or is contained in one, in which case take $\tilde{\Omega}$ to be the manifold) and (a) $2\dim\tilde{\Omega} + 1 \leq \dim\Delta$ or, alternatively, (b) the distribution (according to μ) on $\tilde{\Omega}$ is diffuse and $\dim\tilde{\Omega} < \dim\Delta$, then there exists a revealing rational expectations equilibrium for a "generic" subset of economies.

For technical conditions and proofs, see Allen (1981b) for part (a) and Allen (1982b) for part (b). The proofs rely on (a) the Whitney embedding theorem [see Hirsch (1976, p. 55)] and (b) the genericity of transversal intersection in the form of multijet transversality.

Here generic means that, given an economy for which no revealing rational expectations equilibrium exists, a suitable arbitrarily small perturbation restores existence, while given an economy having a revealing rational expectations equilibrium, all sufficiently close economies also have such equilibria. These properties are the analogues of density and openness, except for the detail that I do not define a topology on the space of supports $\tilde{\Omega} \subset (C^2(\mathbb{R}^l_{++},\mathbb{R}))^{\#A}$. They do, however, correspond to an open and dense subset of the space of smooth maps from a fixed compact parameter manifold $\tilde{\Omega}$ into $(C^2(\mathbb{R}^l_{++},\mathbb{R}))^{\#A}$.

Diffuseness means that only those subsets which contain relatively open (in the manifold $\tilde{\Omega}$) subsets can be of positive measure. If $\tilde{\Omega}$ were a Euclidean space, this would be the same as absolute continuity of the image measure with respect to Lebesgue measure. This is a mild dispersion condition on the distribution of state-dependent utility functions (or parameters).

Compactness requires that, as states of the world vary, agents' characteristics do not differ too much in a topological sense. This mild condition is needed for the conditional expected utility maximization step as well as for the application of differential topology methods in the proof of Theorem 2.

The geometric intuition for Theorems 1 and 2 can be explained quite easily. Consider the case where $\tilde{\Omega}$ is one-dimensional, say $\tilde{\Omega} = [0,1]$ without loss of generality. Think of a flexible rope in which the points represent points in $\tilde{\Omega}$, and in which the position of the rope represents the (selection from the) full information equilibrium price correspondence. If I throw the rope onto a table, the rope may cross over itself. These crossings represent parameters where the full information equilibrium price function fails to be injective. However, notice that the crossings affect only a few (finitely many) points generically, so that they receive probability zero under a diffuse measure. On the other hand, if I throw the rope into the air and freeze it in position, then it's very unlikely that the rope will touch itself anywhere; if it does, wiggling the rope slightly gets rid of the phenomenon. Hence, if $\dim \tilde{\Omega} = 1$, then with two normalized prices (or three commodities) and diffuseness, generically there are revealing rational expectations equilibria, while for more general image measures (in the absence of dispersion), three prices after normalization (or four commodities) are needed for the generic existence of revealing rational expectations price functions. If $\dim \tilde{\Omega} = \dim \Delta = 1$, as in the open counterexamples of Green (1977) and Jordan and Radner (1979, 1982), only the strictly monotone mappings are one-to-one, and they're not dense, even in the space of C^∞ functions from $[0,1]$ into $[0,1]$. Finally, if $\tilde{\Omega}$ is a finite set (as in Theorem 1), regardless of the dimension of Δ, it's extremely unlikely that randomly picking finitely many points from the uncountable set Δ will result in selecting exactly the same point more than once. This explains why the finite case yields generic existence results without restrictions on the number of commodities.[16]

The existence of non-revealing rational expectations equilibria is established in Allen (1981c) for a quite restricted class of economies. In these examples, $\dim \tilde{\Omega} = 2$ and $\dim \Delta = 1$, so that smooth price functions cannot be revealing. Hence, in contrast to the Beja (1976) and Grossman and Stiglitz (1976) objections discussed in Section 4, traders can have a strictly positive incentive in equilibrium to gather/purchase and utilize private information. However, these rational expectations equilibria are essentially revealing in the sense that, when combined with any agent's private information, the information conveyed by prices is sufficient to reveal totally the payoff-relevant state of the world.

[16] Of course, in this case, $\dim \tilde{\Omega} = 0$ in the notation above, so that both the inequality in part (a) of Theorem 2 and the weaker one in part (b) are automatically satisfied. This argument shows that Theorem 1 is simply a special case of Theorem 2.

7. Modifications of the equilibrium concept

In view of the counterexamples and the lack of general positive existence results for (especially non-revealing) rational expectations equilibrium, one might wish to modify its definition somewhat. Although there are good arguments (based on considerations mentioned in Section 9) for a more stringent definition, in order to try to solve the existence problem, one wants to weaken the requirements (while, of course, still preserving the spirit of obtaining information from prices). My search for approximations to rational expectations equilibrium has resulted in the exploration of two alternative concepts – approximate rationality and approximate market clearing.

Approximate equilibria with rational expectations always exist whenever, in the framework of the preceding section, $\tilde{\Omega}$ is a compact set. In fact, this result from Allen (1982a) requires only continuous utility functions (defined, for instance, on $\mathbb{R}^l_+$) which are strictly monotone and strictly concave. The construction uses compactness to chop $\tilde{\Omega}$ into some finite number of sets of arbitrarily small diameter. The idea then is to pick a different price for each event in the resulting finite partition of $\tilde{\Omega}$, so that one obtains approximate revelation (prices reveal which of the finitely many small sets contains the true state of the world) and approximate market clearing (from the continuity of aggregate excess demands). However, these price functions fail to be related in a natural way to market demands; a central planner is needed to set prices. Allen (1982a) contains two additional results for rational expectations ε-equilibria which reveal the "correct" amount of information but which exhibit the same decentralization problem. Note that none of these are merely generic results.

The other obvious approximation possibility is to consider notions of approximate rationality. Here large economies with a continuum of negligible agents[17] are required in order to disperse agents' forecasts. Then discontinuities in the dependence of conditional expectations on prices occur at different price vectors for different economic agents and may therefore be removed by aggregation. The result is that ε-rational expectations equilibria exist for all economies with $\tilde{\Omega}$ finite; see Allen (1983). For an attempt (under quite restrictive assumptions) to extend this idea to situations (i.e., $\dim \tilde{\Omega} = \dim \Delta$) not included in other existence results, see Allen (1985a).

[17] Perfectly informed agents may be large relative to the economy.

8. More recent existence results

To this point, my survey has discussed those existence results which were circulating among economic theorists prior to 1981. In January and February of that year, three new working papers appeared. Each of them provided a rational expectations existence result for situations in which revelation cannot hold. Both the paper by Allen [revised and published as two articles, Allen (1985b, 1985c)] and the one by Anderson and Sonnenschein (1982) invoked the use of noise, but these works used noise in entirely different ways to obtain existence results for different modifications of the equilibrium concept. The paper by Jordan (1982) fits more in the tradition of the results discussed in Section 6; it concerns generic existence results for rational expectations equilibria which are, in fact, virtually revealing. In the remainder of this section, each of these independent and simultaneous contributions is discussed in turn.

In Allen (1985b, 1985c), the key idea in the model is the notion of noisy price observations. They permit the removal of discontinuities in the dependence of demand on prices due to the rational expectations features of the economy. In Allen (1985b), aggregation over uncountably many negligible agents implies the desired continuity properties. Note, however, that the mere presence of a continuum of traders, even with dispersed preferences and endowments, does not suffice to solve the discontinuity problem. Diffuseness is needed in the discontinuities arising from conditioning on price information. The appropriate spreading of discontinuities is obtained from the assumption that different agents observe suitably dispersed (i.e., whose distributions have smooth density functions) noisy prices. In the companion paper, Allen (1985c), traders (of which there may be only finitely many) condition also on the fact that they observe such noisy prices – hence the term "fully rational". In both papers, the noisy price observations imply that, in general, excess demands violate Walras' Law in the aggregate; this phenomenon is the source, in the definition of my equilibrium concept, of an ε-market-clearing qualification for one commodity only, where ε is an arbitrarily small positive number. On a more conceptual level, the major drawback of the noisy price observations approach is that it requires a slight retreat from the usual idealization in microeconomic theory that all agents trade at the same prices. (Exact market clearing may be had at the expense of approximate rationality by the story that agents calculate conditional expected utilities based on noisy prices and then trade in the market at the "correct" prices without bothering to recompute conditional expectations.) However, while the hypothesis of suitably dispersed noisy price observations solves the discontinuity problems (that for a single price function, conditional probabilities may exhibit discontinuities at some price vectors and that even though price functions converge in any obvious reasonable sense, their associated conditional distributions over states of the world may fail to converge), other obstacles to an existence proof remain. One would like to perform a fixed point argument on entire price functions (or correspondences). Unfortunately, the Walrasian correspondence is generally not a function, it generally fails to be convex valued, and it need not admit a continuous selection whenever some state of the world gives rise to a critical economy (which is generically expected to occur). Even if the market-clearing price correspondence were single valued, there seems to be no natural compact and convex set of price functions to which one can apply a standard fixed point theorem.[18] To solve these remaining problems, my second innovation is to remodel the fixed point argument so that it is performed on a compact set of uniformly bounded, uniformly equicontinuous, smooth state-dependent aggregate excess demand functions. The main result is that, under some assumptions[19] [see Allen (1985b, 1985c) for technical details], a residual subset of economies have rational expectations equilibria in which all but one markets clear exactly almost surely and aggregate excess demand is of arbitrarily small magnitude in the remaining market. In addition, equilibrium prices exhibit the usual smooth manifold structure of the Walrasian correspondence and generically reveal the correct amount (but not necessarily the most desirable type) of information. Although the story is not completely satisfactory, the noisy price observations approach has the virtue of explaining why and how one can remove the discontinuities which are absolutely inherent to the rational expectations problem.

Anderson and Sonnenschein (1982) provide a rational expectations existence result for a weaker equilibrium concept which Jordan and Radner (1982) call smooth rational expectations equilibria. In the first step, Anderson and Sonnenschein (1982) smooth traders' models by a convolution operation, so that any individual's conditional expectations and demands become continuous, almost by assumption. Equilibrium is defined to be a joint probability distribution on states of the world and prices, in contrast to the usual practice of taking a price function or correspondence from states of the world to prices as the object of interest. This makes it easy to apply a fixed point theorem to the set of joint distributions (which are compact and convex for the topology of convergence in distribution or weak convergence of probability measures) inducing the correct marginal distributions on states of the world. The disad-

[18] This sentence is equally valid if one substitutes weaker hypotheses (i.e., contractible in place of convex) for a fixed point theorem. Nor does a contraction mapping theorem appear to work, except possibly for extremely special examples.

[19] My transversality and equicontinuity assumptions are frankly rather strong.

vantage of this weaker equilibrium notion is that it eliminates the possibility of discussing the amount of information conveyed by prices. Note also that Anderson and Sonnenschein (1982) deviate from complete rationality in their model in that exogenous noise is needed to remove the discontinuities caused by lower hemicontinuity failures in the Walrasian correspondence, but yet agents neglect to condition on this noise in their maximization of expected utility given the available information. The more serious objection to their rather elegant model is that they do not derive continuity in their model or explain why expectations are smooth rather than discontinuous.

Jordan (1982) addresses the case in which $\dim \tilde{\Omega} > \dim \Delta$ and derives a generic existence result (for a residual subset of economies with C^∞ utility functions) for the standard definition of rational expectations equilibrium. However, his equilibrium price functions are (exactly) two-to-one mappings almost everywhere, so that they must be discontinuous on a dense set. The Jordan (1982) equilibria are moreover essentially revealing, in that the inverse image of every price vector either is empty or consists of exactly two parameter values (or states of the world) which can be made arbitrarily close to each other. I don't think that this is a desirable feature; one wants genuinely partially revealing equilibria for the cases covered by this theorem. Moreover, it's extremely difficult to imagine how economic agents could possibly know or use such artificial and pathological price functions in their conditional expected utility maximization problems.

9. *Open problems*

I wish to stress that, in my opinion, the existence problem for rational expectations equilibrium is definitely still not fully solved in a satisfactory manner. I disagree strongly with the conclusion of Jordan and Radner (1982, p. 204) that we have "an essentially complete answer". To be fair, Jordan and Radner (1982, p. 204, emphasis in original) do qualify their remark by adding the statement in a footnote that "the question of existence of *suitably defined* REEs may not yet be properly posed", so that one might view my disagreement as a quibble over semantics or a question of where emphasis should be placed. Yet I'm firmly convinced that our lack of a completely satisfactory solution deserves more emphasis. Better results for the existence of genuinely partially revealing rational expectations equilibria are definitely needed. Indeed, many of the research topics listed below are of most interest in a truly partially revealing rational expectations world.

Other significant open problems include the following: (1) the decentralization or implementation of rational expectations equilibrium, (2) endogenous (costly) information acquisition decisions, (3) the strategic utilization of information, (4) learning behavior or convergence to rational expectations equilibrium, (5) the welfare properties of rational expectations equilibrium allocations,[20] (6) the dynamics of rational expectations in an intertemporal framework, including the case in which the equilibrium concept is modified to permit traders to learn from past prices only, and (7) imperfectly competitive markets with rational expectations, including the study both of Nash (and related) equilibrium oligopoly models and of solution concepts such as the core from cooperative game theory. Each of these areas has been the subject of intense recent research effort, but it seems too early to appraise this (mostly still unpublished) work. For this reason, and also to avoid inevitable errors of omission in citing this burgeoning literature, I shall not attempt to discuss the numerous interesting (but partial and preliminary) answers which have recently become available concerning these topics.

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[20] Observe that the revealing rational expectations equilibrium allocations are ex post Pareto optimal relative to existing markets, although agents might wish, for instance, to purchase insurance contracts which aren't available.

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Chapter 2

NOTIONS OF CORE CONVERGENCE

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1. Introduction

In this paper, we shall survey some of the results on the cores of large finite exchange economies. The subject began with the pioneering work of Edgeworth (1881). Edgeworth gave a geometrical proof, in the case of two commodities and two traders, that as one replicated the economy, the core collapsed to the set of Walrasian equilibria. Edgeworth claimed in passing that his proof generalized to arbitrary numbers of commodities and arbitrary numbers of agents in the base economy being replicated.

The subject lay dormant for nearly a century until Shubik (1959) recognized the importance of Edgeworth's contribution. In 1963, Debreu and Scarf gave the first proof of the theorem that Edgeworth had claimed: that, in replica sequences of economies with strongly convex preferences, the intersection of the cores of the replications coincides exactly with the set of Walrasian equilibria. Their proof is quite different from Edgeworth's.

Aumann (1964) formulated a model of a large economy with a measure space of agents. In this model, he showed that the core coincided with the set of Walrasian equilibria. Moreover, Aumann required only minimal assumptions; for example, neither convexity of the preferences nor boundedness of the endowments are required. Aumann's proof makes use of some of the key ideas in the Debreu and Scarf proof.

In the contributions of Edgeworth, Debreu and Scarf, and Aumann, the conclusion is clean and neat: the core (in Aumann's case) or the intersection of the cores of all replicas (in the other cases) *coincides* with the set of Walrasian equilibria. Moreover, the Debreu and Scarf paper is completely elementary, with a proof that is a model of simplicity and elegance. The Aumann paper, of course, uses more sophisticated mathematics. However, since Aumann found

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