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Elements of Experimental Stress Analysis

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LOW-PRICED TEXTBOOK

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by

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PREFACE

THE object of experimental stress analysis is to deduce the stress conditions in a structural element subjected to some specified loading either by observation of the physical changes brought about in it by the applied loads or by measurements made on a model or analogue.

Ever since the principles of mechanics were first applied to engineering problems in the seventeenth century, recourse has been made to experiment as a means of elucidating the behaviour of structures. Early attempts at experimental analysis were inevitably crude by modern standards but, gradually, reliable methods and precise instruments were evolved which made possible detailed and accurate stress determination.

Sensitive strain measuring devices were available in the latter part of the nineteenth century, but they were generally too cumbersome for use on small elements and too delicate for use under field conditions. These limitations were eventually overcome but substantial progress in this branch of stress analysis was made only when electrical resistance strain gauges were introduced in the 1930's.

Photo-elasticity was known as a possible means of stress analysis from its discovery by Brewster in 1816, but again substantial progress did not take place until well into the twentieth century, in this case because suitable materials were not available until that time. Very few other methods for detailed stress measurement were developed before the 1930's but since then there has been very rapid progress and there are now techniques for the investigation of practically every kind of stress problem. Associated with these techniques is a range of equipment for applying and measuring loads and for measuring deflections, accelerations and other quantities which may be required for the assessment of stresses in particular cases. As previously suggested, the development of experimental stress analysis depends very much on developments in other fields of technology; for example, advances in strain gauge technique are closely tied to instrument technology in general and extension of the scope of photo-elasticity is almost entirely dependent on the emergence of new transparent plastics which happen to have suitable optical properties for this purpose. Those concerned with stress analysis must therefore look beyond the limits of their specialism in order to distinguish new possibilities and improved methods.

The purpose of this book is to describe the principles of the more important and widely used techniques and pieces of equipment and, by reference to a number of selected examples, to suggest appropriate applications of these in laboratory and field investigations. Although the examples used to illustrate the various methods of analysis are taken from the field of Civil Engineering, the book will be of use to all undergraduate and postgraduate students who require a basic knowledge of experimental stress analysis. It is also hoped that the book may be of use to practising engineers who may be concerned with experimental investigations in one way or another.

A book of this size covering such a wide field cannot possibly deal in great detail with any particular topic; a brief bibliography has therefore been given at the end of each chapter which should be sufficient to permit the reader to follow up the outline of principles given here in as great depth as may be required. Attention is directed in particular to the publications of the American Society for Experimental Stress Analysis; these include the very comprehensive *Handbook of Experimental Stress Analysis*, edited by Hetényi (Wiley, 1953) and the Proceedings of the Society which contain valuable papers on all aspects of the subject. Many papers relating to Experimental Stress Analysis are of course published in the journals of various scientific and professional institutions, in the technical press and by many research institutes. These may be traced by reference to publications such as Building Science Abstracts (H.M.S.O., London) and Applied Mechanics Reviews (American Society of Mechanical Engineers).

In conclusion, it may be added that although constant reference to books and periodicals is essential, real knowledge of experimental stress analysis can only be obtained by first hand experience in the laboratory.

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I

MODELS, SCALE FACTORS AND MATERIALS

MODELS AND SCALE FACTORS

WHEN experimental stress analysis techniques are applied in the design of structures or machines, it is usually necessary to employ reduced scale models in the investigations. Furthermore, it may be inconvenient or impracticable to make the model of the same material as the prototype or again, it may be desirable to use a model material of low elastic modulus in order to obtain easily measurable strains. It is thus essential that we should know the relationship between phenomena observed in a scale model and the corresponding effects in the full sized construction which it represents. It is, of course, necessary that this knowledge should be available at the outset in order that the model may be designed so as to represent correctly the behaviour of the prototype.

The problem of scales may be approached in two ways: firstly, by making use of formulae which represent the solution of a simpler problem of the same general type as that being investigated or, alternatively, by determining the required relationship by dimensional analysis.

As an example of the first approach, let us suppose that it is required to investigate the behaviour of a particular type of beam: suppose that the beam carries some specified loading, as shown in Fig. 1.1. Denoting the load on the prototype and on the model by appropriate subscripts and assuming geometrical similarity and homologous load systems, we can write:

	Prototype	*Model*
Load	W_p	W_m
Shear force	F_p	F_m
Shear stress (mean)	F_p/A_p	F_m/A_m
Bending moment	$k \,.\, W_p \,.\, L_p$	kW_mL_m
Bending stress	$M_p \,.\, y_p/I_p$	M_my_m/I_m
Deflection	$k_1 \,.\, \dfrac{W_pL_p^3}{E_pI_p}$	$k_1 \dfrac{W_mL_m^3}{E_mI_m}$

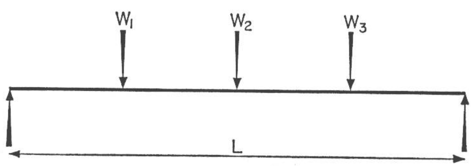

Fig. 1.1. Arbitrary loading on a simple beam.

Comparing the various quantities in the prototype and model we may derive the following ratios:

Load ratio = shear force ratio : $W_r = W_p/W_m$

Shear stress ratio: $$\frac{F_p}{F_m} \cdot \frac{A_m}{A_p} = \frac{W_r}{(L_r)^2}$$

Bending moment ratio: $$\frac{W_p}{W_m} \cdot \frac{L_p}{L_m} = W_r \,.\, L_r$$

Bending stress: $$\frac{W_p}{W_m} \cdot \frac{L_p}{L_m} \cdot \frac{y_p}{y_m} \cdot \frac{I_m}{I_p} = \frac{W_r}{(L_r)^2}$$

Deflection: $$\frac{W_p}{W_m} \cdot \left(\frac{L_p}{L_m}\right)^3 \cdot \frac{E_m}{E_p} \cdot \frac{I_m}{I_p} = \frac{W_r}{E_r \,.\, L_r}$$

In this way it is possible to determine the relationship between corresponding quantities in the model and prototype and thus to interpret model observations in terms of full size. Examination of the relationships for bending stress and deflection show that it is not necessary for the cross section of the model member to be geometrically similar to that of the prototype, provided that the ratio of the moments of inertia is introduced. The final result quoted above of course assumes geometrical similarity throughout. This method can be applied to any problem for which a type solution is known.

The alternative approach by dimensional analysis possesses greater generality as knowledge is required only of what variables control the behaviour of the system. It is beyond the scope of this book to deal with the subject of dimensional analysis and only an indication of the method of approach will be attempted; a full treatment of the method will be found in reference 1.

If we consider the same problem as discussed above, it is easily seen that the stress in the beam depends only on the loading on it and on its dimensions, i.e.

$$\sigma \propto W^a L^b \tag{1.1}$$

Now the dimensions of the various terms in (1.1) expressed in fundamental units of force F and length L are:

$$\sigma \quad [FL^{-2}]; \qquad W \quad [F]; \qquad L \quad [L]$$

Thus:

$$[FL^{-2}] = [F]^a[L]^b \tag{1.2}$$

Since the equation must be dimensionally homogeneous we may equate indices of like dimensions on the two sides, so that:

$$a = 1 \qquad b = -2$$

This gives:

$$\sigma \propto \frac{W}{L^2} \tag{1.3}$$

Comparing the stress in the model with the stress at a corresponding point in the prototype:

$$\frac{\sigma_m}{\sigma_p} = \frac{W_m}{W_p} \Big/ \left(\frac{L_m}{L_p}\right)^2 = \frac{W_r}{(L_r)^2} \tag{1.4}$$

which is the same result as obtained by the alternative method.

The deflection scale ratio can be found in a similar manner as follows:

$$y \propto W^c L^d E^e \tag{1.5}$$

Deflections are proportional to strains and therefore to stresses; these are in turn proportional to the applied load. Thus in (1.5) $c = 1$. Expressing this equation in terms of dimensions we obtain:

$$L = (F)(L)^d(FL^{-2})^e$$

$$= F^{1+e}L^{d-2e}$$

and thus:

$$1 + e = 0 \qquad d - 2e = 1$$

whence:

$$e = -1 \qquad d = -1$$

and

$$y \propto \frac{W}{EL}$$

The deflection scale ratio is therefore:

$$\frac{y_m}{y_p} = \frac{W_m}{W_p} \cdot \frac{E_p}{E_m} \cdot \frac{L_p}{L_m} = \frac{W_r}{E_r L_r} \tag{1.6}$$

MATERIALS FOR EXPERIMENTAL STRESS ANALYSIS

The material adopted for the construction of a model depends on the nature of the structure or element to be represented. Obviously, a first requirement is that the stress–strain relationship of the prototype material should be correctly represented: if this is elastic then the model material must also be elastic although, as we have seen in the preceding paragraph, the elastic moduli need not be identical. The model material must of course be sufficiently strong to withstand the imposed stress. A second requirement is that the material should be capable of being cast or machined to the shape of the prototype with reasonable ease.

Table 1.1 shows Young's modulus and Poisson's ratio for a variety of materials which have been found valuable for experimental stress analysis.

Table 1.1

Properties of Materials

Material	E lb/in^2	*Poisson's ratio*	*Remarks*
Metals			
Steel	30×10^6	0·30	
Brass (70–30)	16×10^6	0·33	
Duralumin	10×10^6	0·33	
Plaster and Cement			
Plaster of Paris	330×10^3	0·15	
Portland cement mortar	2×10^6	0·15	Water–cement ratio approx. 1·25:1
Plastics and Rubber			
Methyl methacrylate	500×10^3	0·38	Perspex
Phenol formaldehyde	300×10^3	0·42	Catalin
Polyethylene	53×10^3	0·50	Alkathene
Polyester resin (15°C)	450×10^3	0·40	Araldite MY753
,, ,, (80°C)	$1{\cdot}9 \times 10^3$	0·49	,,
Rubber	100–200	0·50	

Metals have well defined elastic properties, but their comparatively great rigidity and the difficulty of forming them to complex shapes limits their use for model analysis. The high elastic moduli of metals means that large loads may be necessary to produce measurable strains requiring substantial and expensive loading equipment. There are, of course, situations in which the high accuracy to which metals can be machined and their definite elastic properties may outweigh the difficulties. In general, however, it will be found convenient to use a plastic or other non-metallic material. The properties of these materials are very variable depending on the precise composition and in many cases on their age; the values quoted in Table 1.1 are thus to be regarded as typical. Furthermore, all these materials are subject to mechanical creep, i.e. for a given stress, the resulting strain increases with time. This factor is most important in experimental stress analysis and must be allowed for in the experimental technique. Fortunately, most of the materials employed for models are almost perfectly visco-elastic, that is the stress–strain relation at any instant after loading is linear although the modulus of elasticity varies with time. This behaviour is illustrated by the stress–strain–time curves shown in Figs. 1.2 and 1.3 which are for the material Alkathene.

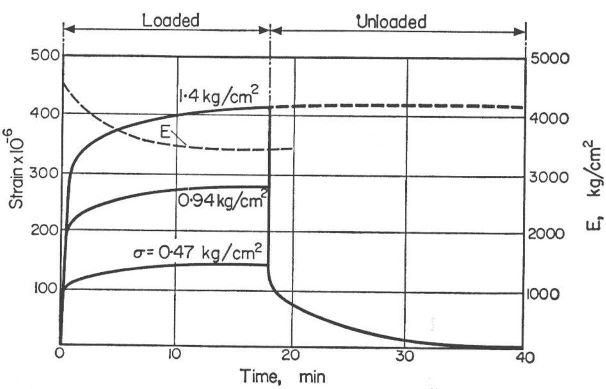

Fig. 1.2. Creep tests on Alkathene prisms (after Rocha[2]).

Difficulties arising from creep may be overcome either by deferring measurement until the increase of strain with time is very small or by taking readings at a particular time interval after loading; calibration tests to determine elastic moduli must, of course, be carried out at the same length of time after loading as in the main experiment. It is also important to see that there is no appreciable creep during the time necessary to take readings on the model. A method for obtaining a considerable number of readings from a model of visco-elastic

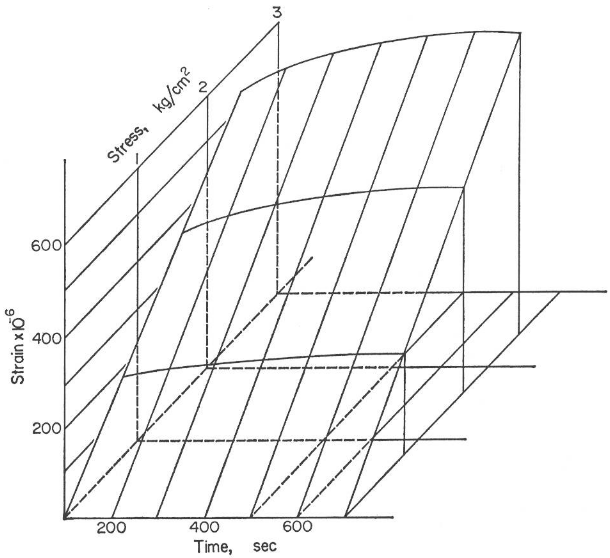

Fig. 1.3. Creep surface for Alkathene (after Rocha[2]*).*

material has been described by Rocha[2]; this may be understood by reference to Fig. 1.4 which shows the strain produced by repeated loading and unloading of such a material. It will be seen that after two or three equal cycles of loading and unloading, the difference in strain observed between loading and unloading becomes constant. In other words the material behaves as if it were elastic; the effective elastic modulus is based on the strain difference between the beginning of loading and the beginning of the following unloading.

As may be seen from Table 1.1, all the plastics have high values of Poisson's ratio. In models of structures in which deformations are controlled predominantly by one of the

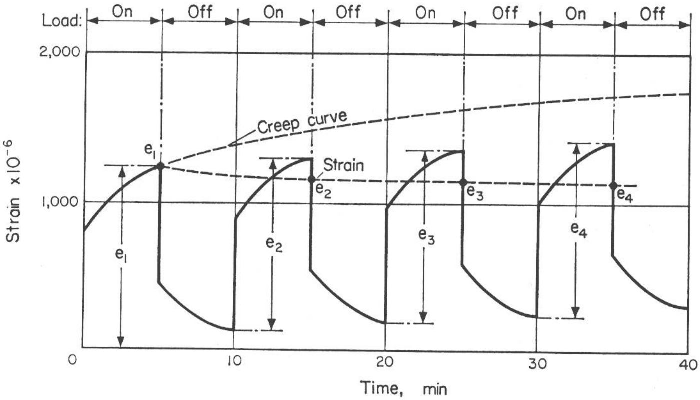

Fig. 1.4. Repeated loading of a visco-elastic material showing constant strain increment after first loading cycle (after Rocha[2]*).*

elastic moduli, as in a beam problem where the elementary Bernoulli–Euler bending theory is applicable, the question of relations between the elastic moduli does not arise. In cases involving complex stress systems the stress distribution may be appreciably influenced by the value of Poisson's ratio [3,4]. It has been shown theoretically, and confirmed by experiment, that the stresses in elastic plates are independent of the ratio of the elastic contents provided that there are no holes. If there are holes this statement is only true if the forces applied to the boundary of any one hole form a system in equilibrium or reduce to a couple. If there is a resultant load on the boundary of a hole, a correction is necessary if the results are to be transferred from a plate of one material to a plate of a different material. Fortunately, the correction is usually small and the