

**ELECTRONIC  
ENGINEERING**

# Folded Unipole Antennas

Theory and  
Applications

**JEREMY K. RAINES**

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# Folded Unipole Antennas: Theory and Applications

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*In memory of*  
John H. Mullaney, P.E.  
(1920–1994)  
*Friend & Mentor*

## ABOUT THE AUTHOR

JEREMY KEITH RAINES was born in Washington, D.C. He received the B.S. in Electrical Science & Engineering from M.I.T., the M.S. in Applied Physics from Harvard University, and the Ph.D. in Electromagnetics from M.I.T. He is a registered Professional Engineer in the State of Maryland. He is a senior member of IEEE and has served as President of the Association of Federal Communications Consulting Engineers.

Since 1972, he has been a consulting engineer in electromagnetics. Projects for government and private industry have concerned antennas, radio wave propagation, radar, architectural electromagnetics, and bioelectromagnetics. Antennas designed by him span the spectrum from ELF through SHF, and they may be found on satellites deep in space, on ships, on submarines, on aircraft, and at a variety of terrestrial sites. Modeling antennas on complicated platforms and in complex environments are chief interests.

Dr. Raines enjoys sharing his knowledge. He was an instructor in the faculty of electrical engineering at M.I.T. He has also taught courses about antennas at the Naval Research Laboratory and at George Washington University. He was a guest lecturer on the subject of bioelectromagnetics at Rockefeller University.

## **Purpose of This Book**

The chief motivation for writing this book was to preserve a body of knowledge that the author believes is unique and valuable, with respect to both antenna design and mathematical techniques. Some parts of it already exist, widely dispersed in the technical literature; however, many others, if not most, parts are original research. In any case, to the best of the author's knowledge, all of the parts have never before been brought together in a single volume such as this one.

The subject, folded unipole antennas, is still timely. Folded antennas and arrays of folded antennas can be extremely compact. They can also be broadband compared with other antennas. Conducting obstacles that cause electromagnetic interference can be reconfigured as folded unipoles and thus detuned, or rendered electronically invisible. Many additional applications remain to be explored and developed.

The mathematical techniques, which are mostly analytic and not numerical, are also still timely. Today's computers are both powerful and affordable; however, they are not always the first choice for design purposes. Sometimes exploring an elegant formula from electromagnetic theory provides more insight and is otherwise more productive. In the author's experience, the folded unipole antenna is an outstanding example in this respect.

The book is mostly theoretical. That the folded unipole antenna actually does work has been well established since the 1940s. Thousands of them have been constructed and are presently operating in the AM broadcast and military bands. Rigorous theoretical analyses have been performed for a very few specific configurations. On the other hand, a comprehensive theory explaining why the general class of folded unipole antennas works, and how their designs can be systematically modified, improved, and applied elsewhere, seemed elusive.

### Intended Audience

This book has been written with at least three different readers in mind. They are: (1) the systems engineer who is seeking new options for antennas and arrays, especially those that offer extreme compactness and broadband performance; (2) the antenna engineer who is not satisfied by sifting through the mountains of data generated by general purpose, wire antenna, numerical analysis programs; and (3) the electromagnetic generalist who is intrigued by the synergy that results when antennas and transmission lines are integrated into a single radiating device. At one time or another, the author has identified with each of these prospective readers.

For the systems engineer, this book should fill a void. When seeking candidate antennas, his first impulse is to consult a handbook or compendium about antennas. At best, he will find a few pages about folded antennas, and they will only touch on the simplest designs. They may not use the term “folded unipole” at all. The discussion will likely be a section or subsection within a chapter about VLF antennas or AM broadcast antennas, giving the wrong impression that they are limited to use in those bands. In fact, the folded unipole embodies design principles that apply throughout the electromagnetic spectrum from ELF through EHF.

For the antenna engineer, this book should provide some refreshing insights concerning the physics of antenna operation, harkening back to the Golden Age of Antenna Analysis before general purpose, numerical programs were widely available. In the opinion of the author, these programs have largely displaced antenna *design* by numerical antenna *analysis*. In the case of design, mathematical formulas, often elegant, can be manipulated to obtain either an optimal configuration, or at least a well-reasoned starting point. In contrast, numerical analysis involves subjecting countless combinations of wires to number crunching until an acceptable configuration is eventually obtained. Why the configuration happens to work, or if it even approaches an optimum, remains a mystery buried under a mountain of numerical data. To be fair, sometimes this is the only way to go, but it should not be the only choice, or even the first choice, in the tool kit of the antenna designer. The elegance of electromagnetic theory is still an excellent alternative.

For the electromagnetic generalist, this book explores in depth what happens when linear antennas are merged with transmission lines into a single device. New formulas and mathematical approaches are developed as needed. Some of them will have application beyond the folded unipole antenna. For example, in Chap. 9 we will derive the exact formulas for the electromagnetic fields of traveling waves of current, valid both in the near and radiation zones. We will also take a critical look at the definition and calculation of characteristic impedance in Chap. 7. In the literature, the characteristic impedance of antennas has been a topic of controversy for decades. We hope to resolve that controversy here.

### Notes Concerning the Examples

The reader will observe that the contour plots in Chaps. 2 through 5, and in Chap. 9, are in shades of gray; however, the captions and text refer to colors. The contour plots were in color for the original manuscript, but they were converted to gray scale to control publication costs. There should be little or no confusion, however, because the “priority color” (i.e., the color of the contour corresponding to optimal design values) is always the darkest gray, nearly black.

The original color contour plots are available at two different websites. They are the author’s website, [www.rainesengineering.com](http://www.rainesengineering.com), and the publisher’s website, [www.mhprofessional.com](http://www.mhprofessional.com).

Readers may also observe that the frequency for most examples is 984 kHz. This is entirely for numerical convenience. It does *not* imply that the folded unipole is primarily intended for the MF band. This particular frequency was selected simply because a wavelength is 1,000 feet in free space. Thus, it is especially easy to mentally translate linear dimensions into fractions of a wavelength when reading abscissas and ordinates from the graphs and plots. For antennas less complicated than the folded unipole, dimensions are often normalized so that the results are independent of frequency. Folded unipoles, however, integrate linear dimensions, lumped impedances, characteristic impedances, and wavelength into a single electromagnetic device. Therefore, formulating a dimensionless, universal parameter is impractical, if not impossible.

Readers will likely be curious concerning the software used to create the examples. They were all generated using a program called GENIPOL (GENeral unIPOL). GENIPOL incorporates the very same mathematical equations that appear in the text.

### The Brief History of This Book

More often than not, an engineering text is introduced as, “This book began as notes for a course offered at the University of \_\_\_\_\_.” This book is nothing of the kind. It concerns a subject that was at the cutting edge of military communications and commercial broadcasting for many decades. The folded unipole antenna still has much unrealized potential and a bright future, in the opinion of the author. He was just lucky enough to enter the world of antenna design at a time when interest in the folded unipole was cycling through a peak. He was also lucky enough to be mentored by some of the eminent contributors to antenna and electromagnetic theory, and by some of the most colorful innovators in industry. One goal of this book is to preserve and share that experience.

The author first encountered folded unipole antenna technology in the summer of 1966, when he was an engineering intern at Multronics, Inc.,



Rockville, Maryland, between undergraduate years at M.I.T. The president of Multronics was **John H. Mullaney, P.E.**, who was awarded some of the early patents for the folded unipole. Its surprising capabilities and the challenge to understand its detailed workings recurred throughout the years. Somehow the prolonged and concentrated study that the folded unipole antenna deserved and required was always sidelined by larger and more urgent projects in communications and radar.

About 35 years later, **James F. Pinkham**, a colleague from those first years at Multronics, mentioned in a chance telephone conversation that someone ought to write a book about the folded unipole antenna. For several years thereafter, the author dismissed the suggestion out of hand because he realized it would take twice as long as anyone expected. Also, engineers are fond of saying, "The Devil is in the details." Filling in those theoretical details concerning the folded unipole antenna would require twice as much formula deriving and computing of examples as anyone expected. Despite these reservations, the idea eventually took hold, and the author wrote a sample chapter and shopped it around. To his pleasant surprise, the publisher immediately expressed "intense interest."

So, the author began writing in earnest. He was correct in the assumption that it would take twice as long as anyone imagined. But it was a grand learning experience and a rare opportunity to consolidate, organize, and preserve a worthwhile technology that connects to many diverse topics in electromagnetic theory. He has no regrets.

That the author embarked on this project at all was perhaps unavoidable destiny. His initials are JKR, and  $jkR$  is one of the most omnipresent terms in antenna design and electromagnetic theory in general. This term occurs most frequently in the expressions  $e^{jkR}$  and  $e^{-jkR}$ .

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# Acknowledgments

This book is the logical result of 40 years of education and experience, from that first summer at Multronics in 1966, to the completion of the first draft in the summer of 2006. During that period the author was mentored, assisted, and otherwise influenced by individuals in both industry and academia.

The book itself would never have happened were it not for the encouragement of two individuals. First, **James F. Pinkham**, a colleague from the first years at Multronics, with firsthand experience of designing and installing folded unipoles, planted the idea of the book in a chance telephone conversation circa 2001. Otherwise, the author would never have considered it. Second, once a sample chapter was written, the idea was enthusiastically endorsed by **Ron Nott**, VP Engineering of Nott Ltd., Farmington, New Mexico. Ron has made a career from installing folded unipole antennas and has hands-on knowledge concerning how well they perform. His constant encouragement and comments sustained the author's creative energy long enough to complete the book, and, as mentioned in the previous section, that was twice as long as anyone anticipated.

The book would never have materialized, of course, without a willing and enthusiastic publisher. For that, the author thanks **Wendy Rinaldi**, Editorial Director at the Professional Books division of McGraw-Hill. When Wendy received a sample chapter, she quickly recognized the potential contribution a book could make to the engineering community. For the ensuing 18 months of intense writing, Wendy provided the rare combination of constant encouragement plus freedom to write and organize with minimal constraints.

There would have been nothing to write were it not for numerous conversations and joint projects with colleagues concerning the folded unipole antenna. **James J. Karaganis** was the engineer who initially hired the author as a summer intern at Multronics. Under the leadership of the company president, the late **John H. Mullaney, P.E.**, a talented team of engineers pursued cutting-edge applications of the folded unipole antenna for military communications. That team included the

late **George P. Howard**, who was especially intrigued with the use of folded unipoles for multiplexing, and **Lawrence Behr**, who is now president of Lawrence Behr Associates in Greenville, North Carolina.

Multronics evolved into a broadcast consulting firm, Mullaney Engineering, in Gaithersburg, Maryland. As a result, application of the folded unipole antenna continued with emphasis in the AM radio band. The author pursued a variety of projects with John H. Mullaney and the engineers at Mullaney Engineering. A pivotal one occurred in 1973, with the creation of a computer program to model some of the simplest configurations of the folded unipole. It was remarkably accurate, sometimes to four significant figures, and it is still in use today.

The 1990s was a sad decade in the history of the folded unipole with the passing of two of its pioneers. George P. Howard passed away in 1993, and John H. Mullaney followed in 1994. Mullaney Engineering, Inc., Gaithersburg, Maryland, continues to provide innovative folded unipole designs to the broadcast industry. The author has been privileged to participate in projects and discussions with **Jack Mullaney**, President of MEI, and with **Alan E. Gearing, P.E.**, Senior Engineer, MEI, concerning the folded unipole antenna. The author is especially grateful to Jack Mullaney for researching the fascinating histories of both the configuration and name of the NORD antenna. These are related in Sect. 1.2.

Before his death, John H. Mullaney shared many historical anecdotes and design techniques concerning the folded unipole with **R. Morgan Burrow, Jr., P.E.**, formerly with Mullaney Engineering, and now principal of his own consulting firm. During several conversations, Morgan shared those reminiscences with the author. For example, in 1953, Mullaney was responsible for the first U.S. broadcast antenna to be licensed by the FCC as a folded unipole, Station WLOU.

When she learned of the author's book project, **Nellie Fleet Mullaney**, widow of John H. Mullaney, generously provided original notes and circa 1953, patent applications concerning the folded unipole antenna.

In addition to the knowledge acquired from industrial experience, the author was privileged to stand on the shoulders of some academic giants. They provided a solid foundation in electromagnetic theory for this book.

The author pursued both his B.S. and Ph.D. degrees in the M.I.T. Dept. of Electrical Engineering. During that time, he was especially influenced by two of the faculty. The late **Prof. Robert L. Kyhl** was his dissertation supervisor. He was also a brilliant microwave engineer and veteran of the M.I.T. Radiation Laboratory. While the author was consumed with the derivation and numerical solution the most formidable vector, integro-differential, eigenvalue equation he has ever encountered before or since, Prof. Kyhl became a good-humored thorn in his side. He often remarked

to the author that a little physical intuition and insight was worth a thousand rigorous equations. It took decades for that wisdom to sink in; however, that philosophy should be apparent throughout this book. We have eschewed the general purpose, numerical, wire antenna analysis programs, which can generate matrices with millions of elements, for a much simpler boundary value approach.

The other great influence at M.I.T. was **Prof. Steven K. Burns**. For many years, the author taught Prof. Burns' circuit design Course 6.702, *Bioelectronics Project Laboratory*, and this experience influenced him in at least two ways. First, analyzing, building, and testing circuits for medical instrumentation provided both hands-on experience with and a critical understanding of the distinction between common mode and differential mode currents, voltages, and impedances. That distinction is the cornerstone of the theory of folded unipole antennas. Second, it generated a lasting interest concerning electromagnetic fields and their interaction with living systems. That interest is apparent especially in Chap. 10 of this book.

The author pursued his M.S. degree at Harvard University's Division of Applied Physics. There he was greatly influenced by the late **Prof. Ronold W. P. King**, an eminent authority well known and highly regarded in the antenna community. Circa 1958, Prof. King and his student **Charles W. Harrison, Jr.**, were among the first electromagnetic theorists to subject a version of the folded unipole antenna to a rigorous analysis using integral equations. Though an alternative mathematical formulation is pursued in this book, Prof. King articulated the essential physics for our approach. That physical foundation is that there are two modes of current on the folded unipole. They are the common mode (also known as the antenna mode or unbalanced mode) and the differential mode (also known as the transmission line mode or balanced mode).

With respect to the production details of this book, the author certainly would have thrown in the towel had it not been for the technical know-how of some very skilled individuals who generously shared their time and experience. Engineering colleague **Michael Millard**, Managing Partner, TowerSwitch, LLC, Deerfield Beach, Florida, guided the author along the many meandering learning curves for desktop publication, and especially for Microsoft Equations 3.0, which is a formidable tool, indeed, once it is mastered. **Leslie A. Gelman**, Falls Church, Virginia, similarly guided the author along a learning curve for converting hard copy illustrations and graphics into a digital format suitable for publication. **Jean Bubley**, Manager of the Esther Bubley Photography estate, Brooklyn, New York, and **Johanna Kovitz**, Allston, Massachusetts, are experts concerning Adobe Photoshop, and they labored tirelessly to convert color contour plots into publishable gray scale presentations.

The author also wishes to thank, in advance, readers who discover and report errors in this book. Beyond the simplest configuration of the folded unipole, most of the material here is original research. Numerous sanity checks have been performed along the way, and they have been satisfied and documented here. Still, as engineers are fond of saying, "The Devil is in the details." As a result, errors may yet linger, and the author would appreciate being informed of them.

*jkR*  
Rockville, Maryland  
2006

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