

APPLIED MECHANICS UPDATE 1986

edited by

C. R. STEELE

G. S. SPRINGER



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STANFORD UNIVERSITY**

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PREFACE

For nearly 30 years **APPLIED MECHANICS REVIEWS** has been publishing review articles by leading experts in their fields. These articles, covering an interesting range of topics, have gained wide acceptance by the scientific and engineering communities. It was decided, therefore, to update many of the previously published reviews and present them in a single volume. It is hoped that experienced researchers as well as newcomers to a given area will find this collection useful.

Charles R. Steele
George S. Springer

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Effects of Sound and Vibrations on Heat Transfer

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Abstract

The possibilities of using sound or vibrations to increase convective heat transfer rates have received much attention. Until recently, no analyses had been developed for the effects which have been found experimentally. This review begins with a discussion of some relevant acoustics and fluid mechanics, followed by an outline of the analysis of heat transfer by acoustic streaming, and its promising comparison with data for a circular cylinder, and, finally, a short account of other published results.

Introduction

There is now ample experimental evidence that sound and vibrations affect heat transfer. The effect is most usually to increase the average heat transfer. In a few cases, a decrease in heat transfer has been observed, and where local measurements have been made it has often been found that simultaneous increases and decreases occur at different positions on the surface. From the practical point of view, the effects are usually stronger on natural convection than on forced convection for a given intensity. The intensities of the sound fields for which effects have been found are significantly higher than those normally encountered, even in a noisy engineering environment, and exceed the normal level for discomfort to humans.

One of the practical problems which originally inspired interest in the effects of intense sound and vibration on heat transfer was encountered in rocket propulsion motors; when a combustion instability of high amplitude occurred in such motors, the local heat transfer to the motor walls was drastically increased and the wall temperature would often rise to the point where the motor was destroyed. Subsequently, some expedients were found which helped to suppress the combustion instabilities, but combustion is a sufficiently complicated phenomenon that the possibility of encountering combustion oscillations in new motor designs persists [Ref. Group I]. In some special situations, the use of vibrations or sound to increase the rate of heat or mass transfer is especially attractive since it can reduce or eliminate the possibility of chemical or bacteriological

contamination, e.g., in artificial kidney machines [H17]. To stimulate further studies, there is also the desire to be able to increase heat transfer coefficients by any reasonable means possible, especially in gases where heat transfer coefficients tend to be small.

Until very recently, it was not possible to compare experimental measurements with any analysis based upon an acceptable fluid-mechanical representation. Even now, the analysis has been developed only for a portion of the total field of interest, and an understanding of the interaction of strong sound fields with truly turbulent flows may require protracted investigations.

This brief review begins with a discussion of some relevant acoustics and fluid mechanics (including a rough classification of flows and configurations, with the current status indicated), followed by an outline of the analysis of heat transfer by acoustic streaming and its promising comparison with data for a circular cylinder with transverse oscillations, and, finally, a short account of other published results.

Some acoustics

Many experimenters have given their measurements directly in acoustical terms rather than in a more usual fluid mechanical form. In particular, values for the sound pressure levels (SPL) are often cited. For the oscillating flow fields involved it is, in practice, more accurate and convenient to make *pressure* fluctuation measurements with a microphone than to make *velocity* fluctuation measurements with a hot wire. In resonant tubes, the fluctuating pressure can be measured as a function of position along the direction of propagation; the boundaries on the flow are clearly defined and the behavior is close to that predicted by simple, one-dimensional analysis. If a single source propagates a sound field within an enclosure with absorbing walls, the pressure and velocity amplitudes decrease as one moves away from the source. If two such sources (driven from a common electrical source but with a phase gap between them) are held opposite one another, it is possible to establish a standing sound field between them. The maximum pressure amplitude varies in each

pressure-antinode surface, similarly for the maximum velocity amplitude.

A parameter which deserves attention is the standing-wave ratio, which is the ratio of the pressure oscillation amplitudes at neighboring pressure antinodes and nodes. The significance of the standing wave ratio can be explained quite simply. In many experiments, the pressure amplitude was measured at a pressure antinode, and this amplitude was then used to predict the velocity amplitude at a pressure node (which is also a velocity fluctuation antinode). This prediction usually has been based on one-dimensional acoustics with the assumption of infinite standing-wave ratio; thus, the estimated velocity amplitude is incorrect if the standing-wave ratio is not large. In a resonant standing field, the errors are likely to be small.

Acoustical standards are commonly based on air at standard atmospheric pressure and 20°C temperature. This gives a standard characteristic impedance (density \times velocity of sound) of 415 kg/m²sec (the unit is also known as a rayl). The common reference standard of intensity I_0 for airborne sounds is 10⁻¹² watts/m², which is the approximate audibility threshold of a 1000 cps pure tone for human ears. The effective (i.e., root-mean-square) pressure which corresponds to this is

$$P = \sqrt{\rho c I_0} = 0.0000204 \text{ newton/m}^2.$$

The audible range of intensities encompasses about 13 decades. In order to compress the range of numbers required to describe this, and to represent the subjective response of the human ear, a logarithmic scale is used. Values on such a logarithmic scale are called sound levels, the units being called decibels. Thus, the *intensity level* is

$$IL = 10 \log (I/I_0) \text{ dB}.$$

Similarly, the *sound pressure level* is

$$SPL = 20 \log (P/P_0) \text{ dB},$$

where the commonly-used reference is $P_0 = 0.00002$ newton/m² (equivalent to 0.0002 dyne/cm² or 0.0002 microbar). The *IL* and the *SPL* correspond very closely in plane progressive waves, but differences can be expected in more complex sound fields. The particle velocity U in a sound field is related to the pressure fluctuation P by

$$U = P/\rho c.$$

This makes the particle velocity-amplitude at reference conditions smaller than 10⁻⁷ m/sec, which is hardly a useful reference level for measurements of effects on heat transfer (especially since the induced convective motion is proportional to the square of the particle velocity amplitude, for any chosen frequency). If a sound pressure level of 140 dB is considered, the corresponding pressure amplitude is 10⁷ larger than the reference value (i.e., it is about 6 lb_f/ft² or 3 \times 10⁻³ bar) and the corresponding particle velocity amplitude is about 0.7 m/sec (the exact value depends upon the impedance, ρc , of course).

The correspondence between *SPL* values and velocity fluctuation values is clearly a direct one, and measure-

ments of either *SPL* or of velocity fluctuations can be used in an initial presentation of experimental data. However, the *SPL* scale does not represent in itself a characteristic dimensionless parameter, since the reference value used is extrinsic to the fluid-mechanical situation. *SPL* values do not have a significance like that of Reynolds numbers for example. Since the *SPL* scale of values is proportional to the logarithm of the particle velocity, even those fluid-mechanical changes which are smoothly progressive can appear sharp and sudden when plotted on the *SPL* scale. A change of 2 dB at 140 dB might be thought small, but it corresponds to a change of about 30 percent in the particle velocity amplitude [Acoustics—Ref. Group II].

Despite its significance in engineering applications, relatively little attention has been given to sound propagation combined with steady flow in ducts; some recent investigations are cited here [M13, P4].

Fluid Mechanics

It must be possible to explain and represent the effect of sound fields and vibrations on heat transfer in convection by a proper application of fluid mechanics. For all practical purposes, the fluid can be regarded as a continuum. It is very well-known that a disturbance of small amplitude and particular frequency in a fluid gives rise immediately to a characteristic length scale, the acoustical wavelength λ . This can be compared with the length scale of the heat-transferring body in the direction of acoustic propagation, ℓ . Another natural length scale is the so-called a.c. boundary layer thickness, $(\nu/\omega)^{1/2}$, where ν is the kinematic viscosity and ω is the frequency. This boundary layer thickness is independent of the amplitude of the oscillations and can be very small; thus, in air with oscillations at 50 cps, the thickness is only 10⁻³ feet (0.3 mm). A third length scale intrinsic to the acoustic field is the amplitude of particle oscillation, a . This can be made dimensionless with respect to the body size, or can be used to form the oscillation Reynolds number, $Re_{osc} = a\omega\ell/\nu\sqrt{2}$, or the streaming Reynolds number, $Re_s = (a\omega)^2/\nu\omega$. The flow around a body or inside a duct can be described in terms of the dimensionless parameters listed if there are no other sources of motion to be considered.

In most cases, the unsteady motion associated with the propagation of the sound field or with vibration of body surfaces gives rise to a second-order steady motion. To be more specific, when there are spatial gradients of fluctuating velocity products, e.g. of $\overline{u'v'}$, associated with a sound field, a steady mean motion (acoustic streaming) ensues. The gradients may exist because a body surface is long compared with the acoustic wavelength (this happens in a Kundt tube, where steady motion of dust particles can be observed in the presence of axially-propagated sound) or because of the effects of the body shape on the flow outside the a.c. boundary layer even when the ratio ℓ/λ is small (this can happen with a cylinder held transverse to the direction of acoustic propagation or vibration) [Simple oscillating flows and consequent streaming—Ref. Group III].

For any chosen configuration, it is possible, in principle, to compute the acoustic streaming and the con-

sequent heat transfer associated with it. This recently has been done for transverse relative oscillations between a circular cylinder and a fluid; it has been found that there is agreement with experimental results (when the right limits are approached). The streaming motion becomes more complicated to analyze when oscillations are applied in more than one direction. For example, if a circular cylinder is subjected to relative axial or rotational oscillations alone, neither will produce a steady secondary flow. However, if transverse oscillations occur simultaneously, the resulting streaming motion will be three-dimensional and strongly dependent on the frequency and phase relationships between the two oscillations [K3].

Additional Forces

When the fluid is subjected to other forces beyond those associated with the sound field or vibrations, it is necessary to introduce further parameters to describe the motion. In most situations, the boundary layer thickness is dependent on the Reynolds number or Grashof number, and is also a function of position on the surface. The typical new parameters which arise in the description of the motion are: the ratio of the a.c. boundary layer thickness to the thickness which would be found in the flow if oscillation-free; and the local ratio of the Reynolds numbers, or of the oscillation Reynolds number to the square root of the Grashof number.

It is difficult to find configurations in which one can generate simple similarity solutions of the type which have proved so useful in the understanding and representation of steady boundary layer flows. For some bodies, it is possible to find similarity, however, in restricted regions; for example, at the bottom of a horizontal heated cylinder in a gravitational field and subjected to transverse vertical or horizontal oscillations. The calculation of instabilities and separation for flows carrying intense oscillation fields has yet to be studied analytically in any thorough manner. For many oscillations of practical interest the amplitudes of the oscillations are large enough (compared with the mean velocities which occur in the absence of the oscillations) to lie beyond the probable validity of studies which assume linearization and small perturbations as a starting point [Oscillations superimposed on steady flows—Ref. Group IV].

Turbulent Motions

The analysis of turbulent shearing motions is in a primitive state compared with the analysis of laminar motions. The semi-empirical theories which have enjoyed some success offer no general prospect of incorporation of the effects of sound fields. The turbulence in a shearing motion involves a fluctuation spectrum which covers a large range in frequencies, and it is possible that a sound field of a single frequency could selectively couple into the energy spectrum of a flow; as the turbulent flow evolves in time, any energy which is added at a particular frequency will be distributed over parts of the spectrum due to the natural processes which occur in turbulence. This offers a fascinating problem for research. Despite the difficulties of pro-

viding an analytical understanding, study of the influence of intense sound fields and vibrations on heat transfer under turbulent flows appears to be worthwhile for the development of high-performance heat exchangers and exchange columns for the chemical processing industries [Ref. Group V]. The mean motion of a turbulent flow is often described as being driven by the Reynolds stresses associated with the correlations of the fluctuating velocity components; the laminar acoustic streaming which is responsible for heat transfer is similarly generated by a Reynolds stress distribution, the fluctuating components in this case being associated with the unsteady boundary layer motion. The ratio of the a.c. boundary layer thickness to the laminar sub-layer thickness of a turbulent boundary layer should be a pertinent parameter in these circumstances. For some published measurements, this ratio is about unity and there is some prospect for an analysis to be completed.

Summary

One can attempt a classification of flows and configurations which provides a framework for analyses. Table 1 summarizes the parameters for flow analyses. The flow can be grouped as involving: (A) oscillations alone; (B) oscillations plus buoyancy effects; (C) oscillations plus steady forced flow. The configurations can be grouped as: (1) body \ll wavelength; (2) body \gg wavelength; (3) body \sim wavelength. The current status on analysis and experiment is roughly as follows:

Body \ll wavelength

- A: analysis performed for cylinder; good agreement with experiment.
- B: analysis performed for bottom of horizontal cylinder, including (currently) precise numerical solution; good agreement with experiment. Analysis and experiments also available for vertical heated plate oscillated in the plane of its surface and normal to it.
- C: some interesting experiments point to significant effects in separated regions. Other data suggest that effects of oscillations can be swamped as steady flow grows sufficiently strong.

Body \gg wavelength

- A: found when sound is propagated axially exterior to a long cylinder or parallel to a plate. No analysis yet completed. This should be approached by experimental results for B (following) in the limit $Gr/Re_{osc}^2 \rightarrow 0$.
- B: some experimental results available, together with a rough analysis of the initial influence of a relatively weak sound field; for axial propagation around a circular heated cylinder. (Perhaps the liquid phase in nucleate pool boiling should be considered in this category; no analysis for it has yet been made in the form described here.) Analysis for vertical heated plate also available.
- C: a few experimental results available in external flows; with plenty of results for internal flows (pipes). Analysis based on superposition of flows has been attempted.

Body ~ wavelength

- A: no analysis completed and no experimental data.
 B: some experimental results available; their trend is similar to results for body \ll wavelength but relatively weaker effects on heat transfer.
 C: no extensive results available.

TABLE I
TABLE OF PARAMETERS FOR FLOW ANALYSES

General parameters

λ/ℓ	acoustic wavelength/body length scale
a/ℓ	particle displacement amplitude/body length scale
$(\nu/\omega)^{1/2}/\ell$	a.c. boundary layer thickness/body length scale
$a\omega\ell/\nu\sqrt{2}$	oscillation Reynolds number (velocity is that of primary, oscillatory flow), Re_{osc}
$a^2\omega/\nu$	streaming Reynolds number (velocity is that of secondary, steady flow), Re_s

With natural convection

$(\ell/Gr^{1/4})/(\nu/\omega)^{1/2}$	natural convection boundary layer thickness/a.c. boundary layer thickness
$Re_s/Gr^{1/4}$	streaming Reynolds number/square root of Grashof number; a measure of Reynolds stress/buoyancy stress

With forced flow

$(\ell/Re_\ell^{1/2})/(\nu/\omega)^{1/2}$	forced convection boundary layer thickness/a.c. boundary layer thickness
Re_s/Re_ℓ	streaming Reynolds number/steady forced flow Reynolds number
$5(\mu\omega/\tau)^{1/2}$	viscous sublayer layer thickness/a.c. boundary layer thickness (τ -wall shear stress under turbulent flow)

Oscillatory Temperature Distribution

The temperature distribution in the fluid near a body subjected to sound or vibrations can be regarded as the sum of a steady temperature distribution and an oscillatory temperature distribution. For most practical circumstances, the time-average heat transfer is the feature of interest, so that most attention is devoted to the steady temperature distribution. In subsequent sections, the time-average heat transfer will be discussed.

The oscillatory temperature distribution tends to have a larger relative amplitude when oscillations are superimposed upon a steady flow than when oscillations alone are present. Lighthill [L14] found that there are different phase relationships between the oscillatory surface temperature gradient, the oscillatory surface velocity gradient, and the free-stream oscillations over a flat plate. The oscillatory skin friction had a phase lead over the velocity oscillation of the stream, and surface temperature gradient a phase lag; the phase shifts are functions of the frequency. Subsequently, various studies were made of a similar nature for closely related problems and similar conclusions concerning phase shifts were reached.

Some experimental investigations of oscillatory temperature distributions also have been attempted. Tsui [T8] utilized a Mach-Zehnder interferometer to look at effects in natural convection; Feiler and Yeager [F13] made studies of a flat plate in an oscillatory flow, using a high-speed schlieren system and a hot-wire anemo-

meter, and interpreted their results as including evidence of transient flow reversal within the boundary layer; Mori and Tokuda [M21] made studies of a heated cylinder transverse to a steady stream and vibrated in the direction of flow. Clearly, optical methods are attractive because of the lack of interference with the phenomena under study, but difficulties are encountered in providing enough light to obtain a photographic record as the frequency is increased. None of the experiments produced results at variance with expectation.

Heat Transfer from a Cylinder by Laminar Acoustic Streaming

The steady streaming motion around a cylinder in transverse oscillations is rather complicated; a symmetrical sector is illustrated in Figure 1. In each quadrant there are two circulatory motions, separated by a dividing streamline. The size of the inner streaming

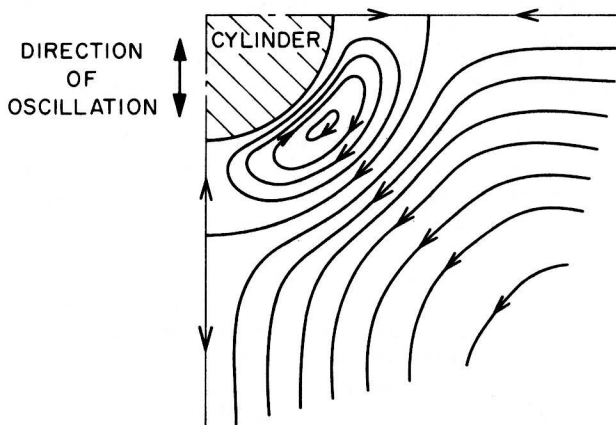


Figure 1. Isothermal streaming pattern near a cylinder subjected to transverse oscillations.

region has been greatly exaggerated; it is the same order of size as the a.c. boundary layer itself. Schlichting [S2] found that the azimuthal velocity became constant at distances z from the surface where $z \gg (\nu/\omega)^{1/2}$. The flow calculated by Schlichting adequately describes the motion unless the ratio of $(\nu/\omega)^{1/2}/d$ exceeds about 10^{-1} , beyond which the inner streaming region becomes progressively larger [H14, R3], or the streaming Reynolds number exceeds about 10, beyond which the outer streaming develops its own boundary layer (Stuart [S20] describes its asymptotic form, $Re_s \rightarrow \infty$). Figure 2 (a) is a chart which illustrates the regimes of flow, and Figure 2 (b) indicates the status of flow analysis in these regimes. Heat transfer experiments have been performed in each of the zones [Ref. Group VI].

An interesting departure from ordinary boundary layer behavior is found in the calculation of heat transfer. In ordinary steady boundary layer flows, an increase in flow velocity is accompanied by simultaneous decreases in the thicknesses of both the hydrodynamic and the thermal boundary layers. Many people have come to regard the ratio of the boundary layer thicknesses found on isothermal surfaces as simply related to the Prandtl number, (ν/α) . In acoustic streaming flow, a basic dif-

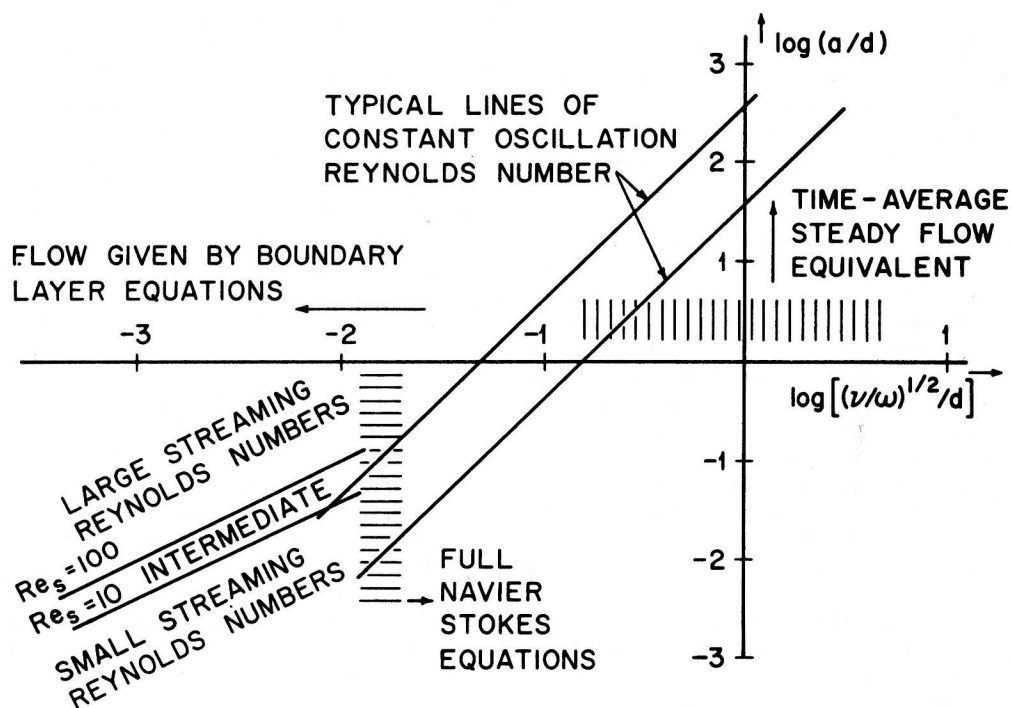


Figure 2a. Flow charts for a circular cylinder: Flow regimes.

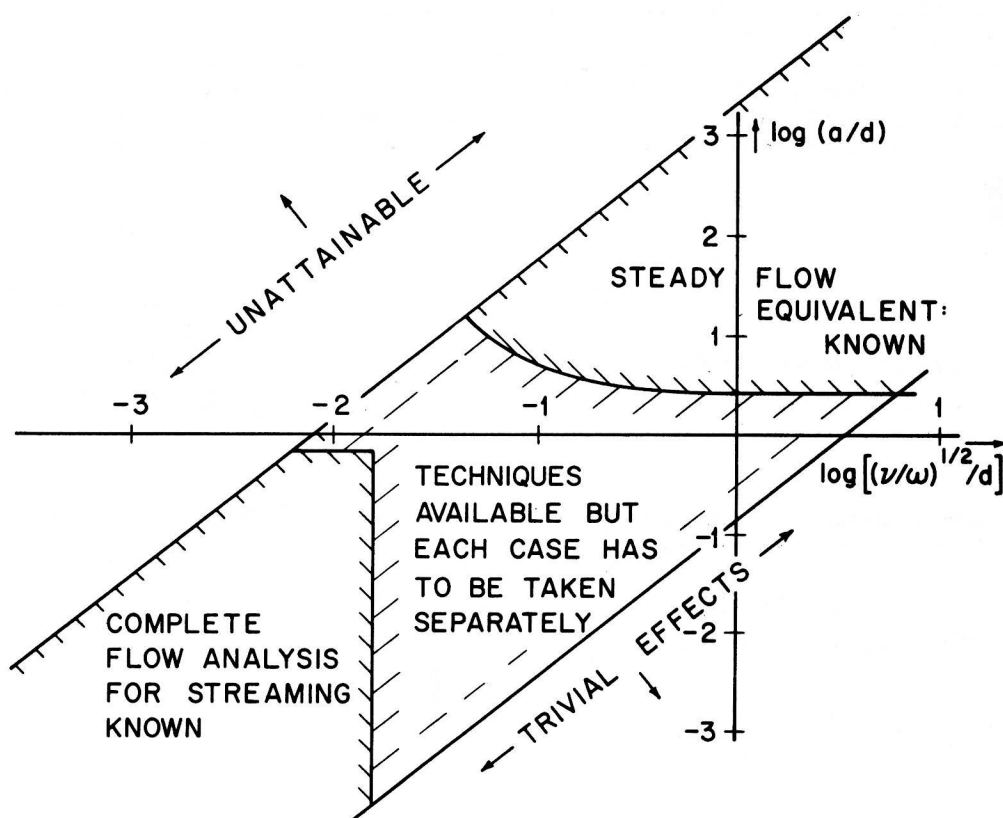


Figure 2b. Flow charts for a circular cylinder: Status of flow analysis.

ference occurs; the physical size of the streaming pattern does *not* change when the amplitude of oscillation is increased (unless $Re_s \gg 1$), but the streaming velocities increase. The latter should increase heat transfer, so that the corresponding thermal boundary layer becomes thinner. Here, therefore, the boundary layer thickness ratio is not related simply to the Prandtl number.

This departure from ordinary boundary layer behavior leads directly to a framework for analysis. Whatever the Prandtl number may be, it is always possible to find an amplitude small enough that the thermal boundary layer is much larger than $(\nu/\omega)^{1/2}$. Then the techniques developed for ordinary flows with $Pr \rightarrow 0$ become applicable; the slug-flow nature of the outer streaming (found in Schlichting's solution) makes the asymptotic analysis straightforward. At large streaming Reynolds numbers (found at large amplitudes and low frequencies), the slug flow gives way to an outer boundary layer flow; the smaller streaming velocities are not so effective then in transporting heat away. Richardson [R12] has presented details of the analysis.

The other extreme in ordinary convection analysis is the asymptotic solution for $Pr \rightarrow \infty$. The thermal boundary layer is then so thin that the velocity distribution in it is essentially linear. A corresponding case is found in convection by acoustic streaming, provided that the amplitude of oscillation is large enough and that the Prandtl number is large. Figure 3 (a) is a chart which indicates the two regimes where the heat transfer is due to the inner or the outer streaming motions. It may be noted that the ordinate is different from that used in Figure 2. The dip in the boundary between the regimes when $(\nu/\omega)^{1/2}/d > 0.01$ is due to the relative growth of the inner streaming region size which coincides with the increasing effect of boundary layer curvature. Figure 3(b) illustrates the current status of the analysis.

A difficulty arises in analysis for convection by inner streaming. When the thermal boundary layer is much smaller than $(\nu/\omega)^{1/2}$, the flow carrying heat away from the surface does not move off to infinity after separating from the surface; it goes to the separating streamline and subsequently returns to the surface. Another exchange process must occur across the separating streamlines between the inner and outer streaming. This provides another resistance to transport of heat. The experimental data available for comparison involve $(\nu/\omega)^{1/2}/d \gg 0.1$, where the inner and outer streaming undergo rather drastic changes in scale. Richardson [R12] calculated the resistance to transport at the cylinder surface itself, expecting this to be an upper bound to the experimental results. This proved to be so, with data coming as close as 85 percent of this bound.

The analyses outlined take no account of the effects of buoyancy. This is not important for the cases where the Prandtl numbers are large, but effects are found when the amplitudes of oscillation are small and also the Prandtl number is not large. Richardson found that the data agreed very well with the predictions of analysis when plotted on a scale of Gr/Re^2 , with the measurements extrapolated to $Gr/Re^2 \rightarrow 0$. However, the task of showing that the departures of the data at finite Gr/Re^2 from this analysis can be predicted by corresponding extension of the analysis has yet to be completed. (Some results appear now to be close at hand.) Some idea of the complicated flow patterns which can arise may be gained from looking at Figure 4. This displays the computed first-order solution for natural convection in the vicinity of the bottom of a heated horizontal cylinder in a vertical standing sound field. As the sound intensity is increased, a doubly-reversed flow develops.

So far the analysis of heat transfer has not been carried out for the acoustic streaming around bodies of

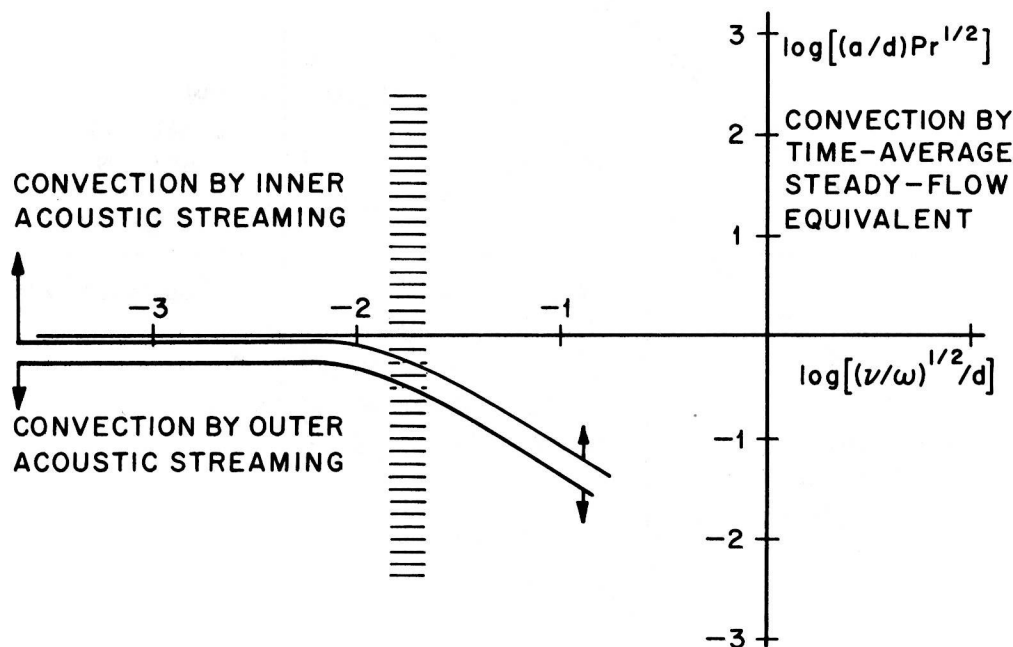


Figure 3a. Heat transfer charts for a circular cylinder: Convection regimes.

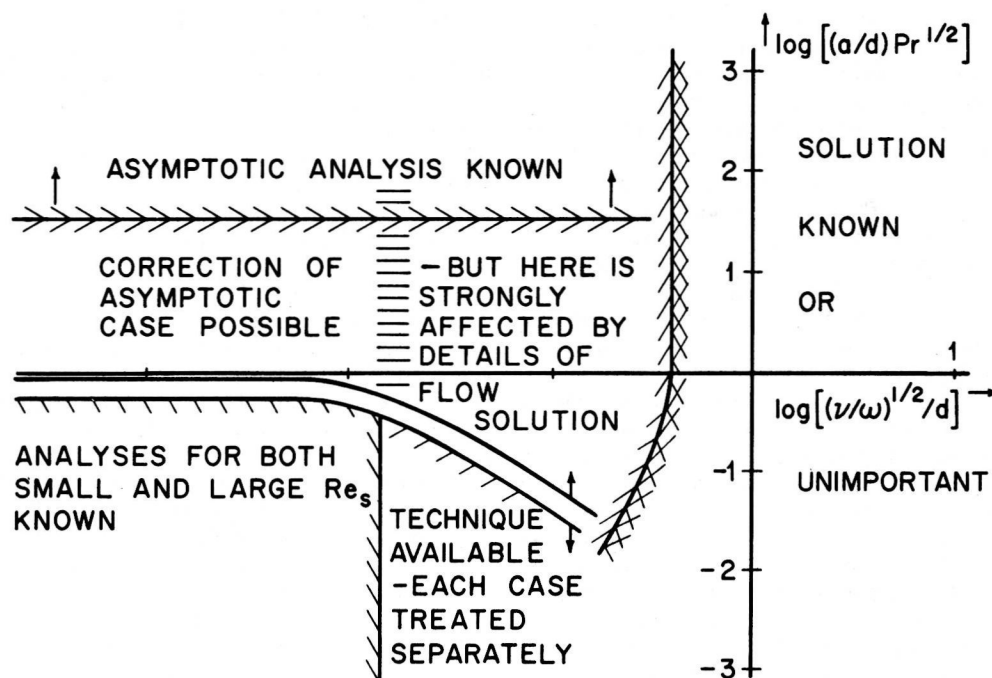


Figure 3b. Heat transfer charts for a circular cylinder: Status of convection analysis.

any shape other than the circular cylinder in transverse oscillations, but this should now be an easy task. For example, the streaming motion around a sphere has been calculated [W4]; however, few experimental data are available for comparison [cf. F14, H16].

The effect of amplitude and frequency on heat transfer by acoustic streaming can be summarized as follows: If the frequency is held constant, the heat transfer increases as the relative amplitudes (displacements) of the surface and the outer fluid (i.e., outside the viscous region) increase. If the displacement amplitude a is held constant, and the frequency ω is increased, the heat transfer increases. If the velocity amplitude $a\omega$ is held constant (e.g., the sound pressure level is held constant) and the frequency is increased, the heat transfer decreases. This latter effect obviously leads

to the suggestion that, by and large, a low frequency should be used; however, when the frequency becomes low with $a\omega$ constant, the amplitude a becomes very large—larger than the body size, and the effects of the oscillations approach a quasi-steady behavior.

Excitation at a Critical Frequency

The effect of transverse oscillations on convective heat transfer from a cylinder has been explained (to the extent that analyses have been completed) as due to the creation or modification of the steady flow by the Reynolds stresses. When other examples of oscillatory flow are considered, one may expect that this explanation will also hold for many cases. However, some cases cannot be explained simply as the consequence

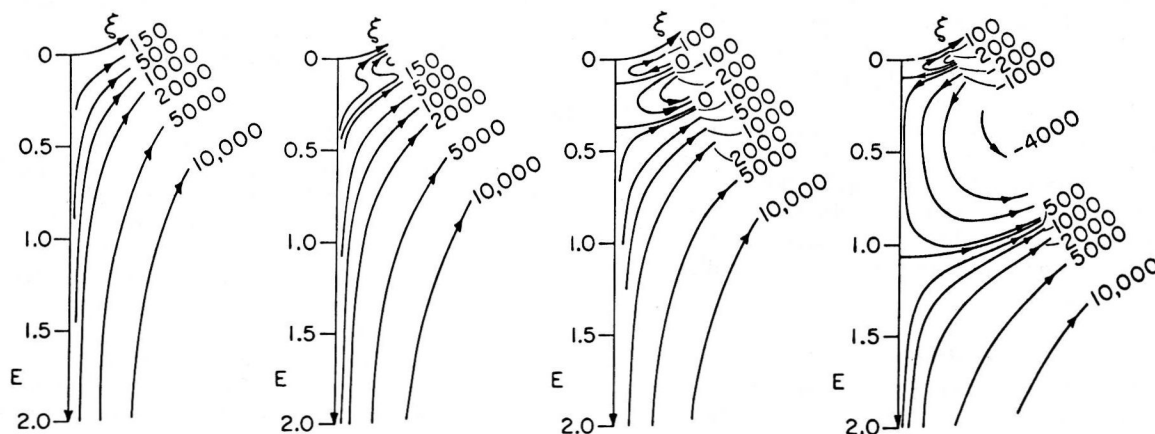


Figure 4. Streaming patterns of the bottom of a heated horizontal cylinder in a vertical sound field. The sound intensity is zero for the left-hand pattern, and increases progressively for the successive patterns.

of the Reynolds stresses introduced by the oscillatory flow, so that other mechanisms must be considered, such as excitation at critical frequencies.

Excitation at a critical frequency implies that in a particular laminar flow situation there are certain frequencies in the neighborhood of which an applied disturbance experiences a high-gain amplification. The nonlinear character of the Navier-Stokes equations assures the introduction of distortions and often the attainment of turbulent flow. Excitation at a critical frequency will generally act as a trigger, leading to considerable effects. (However, it is not an invariable principle of fluid mechanics that applied oscillations promote instability, e.g. [D3].)

For some flows, there are clearly-defined frequencies critical for instability of a significant portion of the flow. In a few experiments, there has been a deliberate attempt to perturb the flow at such frequencies, or optimum effects have been found to correspond to such frequencies. In some recent measurements with a transversely-oscillated circular cylinder in a steady cross-flow, Kezios and Prasanna [K4] found increases in over-all heat transfer of about 20 percent when the applied oscillation frequency equalled the vortex-shedding (Strouhal) frequency. Their resonant heat transfer curves are similar to the fluctuating drag curves of Bishop and Hassan [B7]. Another critical frequency for instability in the same geometry has been identified by Bloor [B13] and Gerrard [G2]; this is the order of 10 to 50 times the Strouhal frequency. The heat transfer measurements of Fand and Cheng [F3] for a cylinder in crossflow with a transverse sound field exhibited a maximum increase when applied frequencies were about twice the Bloor-Gerrard frequency; the applied frequencies were not small enough to match the Bloor-Gerrard frequency directly. The increases were found to occur predominantly on the back of the cylinder, and were larger than could be accounted for by simple acoustic streaming associated with the sound field. A possible mechanism for this effect has been discussed [R15]. In a combined visual and hot-wire anemometer study of instability in a boundary layer formed under a free stream of low turbulence intensity, Knapp, Roache and Mueller [K10] recently found that it is possible to "lock" the instability wave frequency to that of an applied sound field when the applied frequency was sufficiently close to that which occurs naturally. In this case (attached boundary layer flows), it is less probable that there are heat transfer benefits to be gained by excitation restricted to a critical frequency, because similar effects can be achieved more cheaply by increasing the free-stream turbulence. The latter method introduces a blunderbuss approach to critical frequencies, since the turbulence energy is distributed over a wide spectrum.

Ultrasonics

Experiments have demonstrated that ultrasonic agitation of a liquid increases the convective heat transfer [Ref. Group VII]. Several authors have suggested that effects are due to acoustic streaming, but without completing the analyses necessary for quantitative comparison. In a typical investigation, McCormick and Walsh

[M11] performed experiments on a small wire-wound resistor with power inputs up to 100 watts. Increases in heat transfer coefficient were about 30 to 50 percent at 10 kc, and rose to about 220 to 280 percent at 50 kc. The influence of power supplied to the barium titanate transducers was investigated only roughly; optimum performance was found with a power supply of about 50 watts to the ultrasonic generator, of which the efficiency was unknown. No measurements were made of pressure or velocity fluctuation in the open-topped water tank, nor was any flow visualization reported. A simple forced-convection experiment in a tube showed that improvements of the order of 20 percent could be achieved, independently of the flow rate (using Reynolds numbers up to about 30,000). One investigation has been reported recently [H17] where ultrasonic vibrations were applied to improve transport in an artificial kidney machine. Medical machinery offers considerable scope for use of sound and vibrations since oscillations can be applied without risk of chemical or bacteriological contamination. The principal limitation appears to be the damage that can be caused to cells by over-intense ultrasonic agitation.

It is worthwhile to mention that the term "ultrasonic" merely implies that the frequencies involved are above the range of normal human hearing.

Heat Transfer in Further Specific Cases

Flat plates

The flat plate is a hardy perennial in convection analysis. It is not surprising that some investigators have turned to it in attempts to come to an understanding of the influence of oscillations on laminar flows. It can be realized now that the Reynolds stresses will be small, and the influence of the oscillations weak, if the plate itself is oscillated or if the wavelength of a sound field (applied by a separate source in the fluid) is large compared with the plate size. This has largely been borne out by analytical and experimental studies [Effects on heat transfer from flat plates—Ref. Group VIII].

Heat transfer from nichrome ribbon heater strips on a flat plate placed in a tunnel and subjected to a travelling sound field in the main stream flow have been reported [F13]; the sound field was produced by a siren placed upstream of the plate, and contained several harmonics as well as the fundamental frequencies (34 to 680 cps). The flow approaching the plate also had a relatively large turbulence intensity (circa 10%) attributed to the grid-effect of the siren plates. The values of $(\rho U)_{rms}/\rho U$ were of the order of 30 to 60 percent, so that the siren provided a truly brutal disturbance to the average flow; the heat transfer was increased by as much as 65 percent. Measurements with the hot-wire anemometer and the high-speed schlieren indicated the existence of transient flow reversal within the boundary layer. The effects of oscillations of a heated flat plate normal to a column of water have been investigated [S1]. Bayley, Edwards and Singh [B1] performed experiments with a stubby, 8-inch long flat plate supported diametrically in a 4-1/4-inch pipe, subjected to oscillations due to rotation of a downstream

valve with pulsation frequencies from 10 to 100 cps (thus, $\lambda \gg \ell$ throughout). They were not able to determine the flow conditions on the plate surface—it is not clear whether the rounded nose of the plate led to an early separation or transition—and local heat transfer measurements were not made. Some heat transfer measurements were made on a heated vertical plate subjected to the progressive sound field produced by a siren having its center line normal to the plate [J11]. The results were all qualitatively consistent with the analysis described; the nonuniformity of the intensity over the plate surface makes a quantitative comparison difficult.

J. A. Clark, together with his co-workers, made an extensive series of analytical investigations on the effects of oscillations on the flow and temperature distribution over vertical heated surfaces. They predicted the effects in laminar flow to be small (almost too small to measure) but they did observe advancement of transition.

Kubanskii performed some experiments on a flat plate with cavities recessed in its surface [K19].

Ducts and tubes

Reynolds stresses can be produced by fluid oscillations in ducts or tubes when the wavelength of sound is not large compared with the duct length. This circumstance prevailed in many of the experiments which have been reported for this configuration, and the results appear to be qualitatively consistent with the modifications in local heat transfer which would be introduced by the Reynolds stresses. An adequate analysis based on this effect has not yet been completed; analyses have concentrated instead on simple superposition or even have excluded the possibility of Reynolds stresses by assuming that the speed of sound was essentially infinite, e.g., [S9]. It would seem that a good way to increase the Reynolds stresses is to make the wavelength small, and therefore to make the frequency high. Some workers have found that high frequencies are favorable to appreciable change in local heat transfer, e.g., [H5, L16, R18], but have attributed the effectiveness to momentary flow reversal. [Effects in ducts and tubes—Ref. Group IX.]

For flow inside ducts and tubes, the earliest measurements appear to be those of Martinelli et al. [M5] in a vertical tube. This investigation involved oscillations produced by a reciprocating pump but these oscillations did not consist of a simple harmonic motion. Some early work also was performed by Hooper but reported only recently [H15]. Hooper investigated the effects on heat transfer of longitudinal vibration of a heated tube connected to fixed tubing by two flexible "serpents;" water was passed through the system. Increases in over-all heat transfer were found. Raben [R1, N6] described some experiments on heat transfer from a circular cylinder oscillated transversely to its axis inside a coaxial outer tube; through the annulus a steady flow of water was maintained. The largest increases (up to 450 percent) were found with the smallest steady flow rates. The configuration is rather close to that analyzed by Richardson (where the outer cylinder was absent, as also was the axial, annular flow) and Raben's data follow the pattern of Richardson's analysis in the appropriate range.

Thus, it appears that at large amplitudes and small axial flow, the heat transfer is caused by laminar acoustic streaming (outer streaming at large streaming Reynolds numbers) but that this can be progressively dominated by the superimposed steady axial flow.

The practical significance of oscillations in rocket motors led to studies in tubes for which the flow oscillations were generated acoustically. A major experimental program was carried out by Jackson and his co-workers, using a circular tube with a bell-mouth entry and a powerful horn at the exit. The heated section was isothermal and permitted local zone measurements of heat transfer. In the first series of experiments, Jackson et al. placed their heated section closely downstream of the entrance bell-mouth, so that the flow in the heated section was under the initial boundary layer growth and before the tube flow was "fully-developed." A standing sound field was established in the tube at a frequency of the order of 200 cps. In what seems to be the region of the tube wall covered with a laminar boundary layer, the local heat transfer coefficient exhibited a spatially-periodic ripple with maxima at velocity antinodes and minima at velocity nodes; the average was scarcely different from that found in the absence of sound. For conditions where the wall was apparently covered by a turbulent boundary layer, the positions of maxima and minima were reversed and the average heat transfer was smaller than found in the absence of sound. An analysis of the steady, laminar, fully-developed pipe flow as modified by the sound field was made [P8], subject to certain assumptions which limited applicability to $\bar{M}/M'^2 \leq 1$; since the measurements corresponded to a much weaker oscillation intensity (the threshold for significant local effects was about $\bar{M}/M'^2 = 63$), it was not possible to make quantitative comparison.

Jackson et al. carried out further measurements after adapting the system so that the heated section was preceded by a much longer tube which served as a hydrodynamic starting length. The flow through the heated section was to be turbulent and fully developed as a pipe flow. In this later series of experiments with the longer tube, it was necessary to reduce the frequency to about 90 cps in order to obtain a sufficiently high intensity. In this turbulent flow, it was found that the effect of the resonant sound field was to reduce the local heat transfer coefficient by an amount proportional to the local particle velocity amplitude of the sound field (with an average spatial lag of about 20°). In both series of investigations, very high intensities were used (155–165 dB SPL). In the first series, the effect of a given intensity level decreased as the mean-flow velocity was increased, but with a very interesting exception. For conditions where the tube wall was apparently covered by a turbulent boundary layer, the heat transfer was relatively much more sensitive to a given intensity than the laminar boundary layer. This increase in sensitivity coincides with a shift in the region of the boundary layer flow which offers most of the resistance to heat flow to a relatively thin layer on the wall (the laminar sublayer, cf. [K6]), and this thin layer has the same order of thickness as the a.c. boundary layer for these experiments. It would seem that there is

a prospect for analysis of the results of Jackson and his co-workers, even for turbulent flow conditions; the relative sensitivity of the heat transfer makes an analysis significant for practical use.

The naphthalene mass transfer experiments of Low and Hodgins [L18] are noteworthy for measurement of the ablated naphthalene in the effluent air by ultraviolet spectrophotometry. Enhancement of mass transfer was found to increase with the amplitude of the standing sound field, and to decrease with increase in frequency. The mass transfer rate was observed to increase at an antinode and to decrease at a node.

Cylinders in crossflow and parallel flow

Following the discussion of heat transfer from cylinders by acoustic streaming, one may expect that the effects of providing a crossflow or a parallel flow will depend upon the relative strengths of the two flows. If the forced flow is mild, while the oscillations are strong, the effects of the oscillation predominate; as the forced flow increases in strength, there is a redistribution of flow and heat transfer until the forced flow predominates. This expectation is borne out by the experimental data, but with exceptions [Effects in forced convection on cylinders—Ref. Group X]. These exceptions arise when the vibrations or sound fields have a frequency close to the known, naturally-arising shedding (Strouhal) frequency or to the Bloor-Gerrard frequency for waves in the separated shear layer (see preceding section on excitation at a critical frequency). It may be noted that the a.c. boundary layer thickness associated with wake oscillations at a Strouhal number of 0.2 (which prevails over a large range of Reynolds numbers) is the same order as the forward-stagnation-region boundary layer thickness.

Miscellaneous

Most important heat transfer applications involve transfer from solid surfaces, but transport within a fluid may also be significant sometimes. An example is the loudspeaker-amplifier built at Stanford Research Institute [A7], in which a stream of combustible gas is modulated as it passes into the region between the diaphragm of a conventional electromagnetic acoustic driver and its horn; the modulated gas stream expands through a throat and passes through a wire-mesh flameholder. The acoustical output is considerably increased when combustion takes place (it is not surprising that this fire-breathing loudspeaker has been dubbed "The Dragon"). Related work of interest includes studies of thermoacoustic transduction [M9] and the intensification of turbulence by combustion [E6].

Heat transfer between gases and combustion chamber walls is important in reciprocating internal combustion engines, and some investigations have been directed to this problem [Ref. Group XI]. The growing public concern with air pollution by road vehicles should lead to further work on this.

Results of experiments with heat exchangers and transfer columns are encouraging for the use of vibrations in practice, but the data are far too scanty to be compared with the analysis described earlier [Ref. Group V]. It must be admitted that rather little is known about local heat transfer in any except the sim-

plest heat exchanger configurations, so that lack of comparison with analysis is not a significant detractor to the theory developed so far for the influence of vibrations.

A few investigations have been made for effects on boiling or condensation [G5, G7, K13, M7]. Gibbons made his studies for heat transfer from a wire in liquid in a pipe, with oscillations produced by a piston at the end of the pipe. He reported that heat transfer with nucleate boiling was increased by as much as 300 percent, but with film boiling only by as much as 30 percent.

If vibrations are not used deliberately to increase convection under conditions of weightlessness (where natural convection is absent), there may be some convection due to random motions of a vehicle anyway [G1].

Practical Utility

Is it worthwhile in any sense to use vibrations or sound fields in a deliberate attempt to increase heat transfer rates? Is it better, so to speak, to shout at your soup rather than to blow on it?

If the question is posed for the case of the cylinder, with oscillations proposed as a rival to steady forced flow, then it appears that under some circumstances heat transfer by oscillations will exceed that by forced convection when compared on a mean velocity basis. This occurs with amplitudes of oscillation so large that they may not be obtainable in practice, and, on a cost and weight basis, forced convection is an outright winner.

When the possibility of chemical or biological contamination must be avoided as far as possible, the use of oscillations to increase transfer rates has advantages over use of pumps.

When the question is asked for circumstances where there is already some forced convection involved, and the comparison lies between choosing to increase the flow rate and to impose a sound field or vibrations, the answer is far less clear. At the present time, there appears to be a distinct possibility that use of a well-chosen sound field may be advantageous.

Future Prospects

Future work on the effects of vibrations and oscillations on heat transfer will be divided between distinctly scientific and distinctly practical studies. Now that the fluid-mechanical basis for many of the effects observed in experiments has essentially been established, the subject has been made safe for further analytical studies of heat transfer in the pattern of those already made for the circular cylinder, but for other boundary geometries and for more complicated oscillations. There are also some interesting problems of flow separation and stability. Other fields of heat transfer studies—the effects of free-stream turbulence on convection, nucleate boiling, and some types of two-phase flow—should benefit from this work.

Practical studies should be directed towards the specialized application (e. g., medical machinery) and the development of more efficient exchange equipment. If