

# symposia on theoretical physics



E7963067

Lectures presented at the 1965 Third Anniversary Symposium of the Institute of Mathematical Sciences Madras, India



Edited by **ALLADI RAMAKRISHNAN**Director of the Institute



PLENUM PRESS • NEW YORK • 1967

Library of Congress Catalog Card Number 65-21184

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M. H. Stone

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V. Weisskopf

### Introduction

The Third Anniversary Symposium, held in January 1965, was devoted mainly to various topics in elementary particle physics, with a few lectures on many-body problems and a short supplementary program in mathematics.\*

In the Introductory Address Professor V. Weisskopf, Director-General of CERN, Geneva, presented a broad survey of the then current scene in elementary particle physics, the most dominant trend in which is the concept of symmetry. He traced the use of the concept of rotational invariance and symmetry under permutation of identical objects in the realm of atomic spectra and how, with the inclusion of isotopic spin, such use was extended to the study of properties of nuclei. Professor Weisskopf also described how, in addition, elementary particles are characterized by a new quantum number, the hypercharge, which, with isotopic spin, is part of a wider symmetry SU(3). He mentioned three classes of experiments at CERN, one in search of quarks, one to investigate the existence of vector bosons suggested by theories as possible mediators of weak interaction, and one to test the existence of cosmic forces to explain CP or T violation. The quotations from Newton's Opticks at the beginning and the end of the lecture were strikingly relevant.

Two lectures dealt with the application of SU(3) symmetry to weak and strong interactions, respectively. Ph. Meyer of the University of Paris, Orsay summarized his work on the conserved vector current hypothesis in relation to broken symmetries. By assuming that the violation of SU(3) invariance can be described by a local

<sup>\*</sup>While this introduction discusses all the lectures delivered at the Symposium at Matscience in January 1965, it has not been possible to include all of them in this volume.

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Lagrangian which transforms like a member of an SU(3) multiplet with isospin and strangeness equal to zero, he proved that the first order correction to the vector decay amplitude in the limit of zero momentum transfer can be accounted for by using unrenormalized coupling constants but with wave functions corresponding to the physical masses. The multiplet assignments of various observed particles in the SU(6) scheme (in which one fuses the internal symmetry group for the hadrons with the ordinary spin) and the different sum rules between the masses of the hadrons which are in good agreement with the observed masses, were described by V. Singh of the Tata Institute of Fundamental Research, Bombay.

The concept of an equivalent potential in quantum field theory and S-matrix theory and the use of a nonlocal potential in calculations in elementary particle physics formed the subject matter for three talks. R. Blankenbecler of Princeton University showed how one could obtain upper and lower bounds for the phase shifts and the K-matrix elements in nonrelativistic problems. He then extended these ideas to the relativistic case where, starting from the Bethe-Salpeter equation and including multiparticle states, a potential can be constructed in a nonperturbative fashion. Application of the method to the ρ-meson bootstrap problem does not lead to any self-consistent solution. L.A.P. Balazs of the Tata Institute discussed a generalization of the work of Charap and Fubini in which the potential is constructed by requiring that it reproduce the relativistic amplitude at any energy, and is obtained by calculating the absorptive parts in the crossed channel reactions for increasingly larger values of the momentum transfer by interactions using the strip approximation to the Mandelstam representation. Starting from a nonlocal potential corresponding to a repulsive interaction, A. N. Mitra of the University of Delhi explained how by using the potential in a Schrödinger-type equation a detailed examination of the phase shifts can lead to an understanding of some pion resonances. A model for peripheral interactions below 10 GeV in which the K-matrix element for quasi two-particle reactions were replaced by the corresponding Born terms while the remaining K-matrix elements for higher particle final states were assumed to have a statistical distribution with zero mean value was presented by K. Dietz of CERN, Geneva.

J. Lukierski of the University of Wroclaw, Poland, presented

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some considerations regarding the renormalizability of theories of particles of spin higher than, or equal to, ones which are traditionally considered to be unrenormalizable. He showed how the choice of suitable projection operators may lead to renormalizability conditions less stringent than the usual ones.

A possibility of renormalizing theories (considered usually unrenormalizable) using Caianiello's approach involving Pfaffians and Hafnians was suggested by N. R. Ranganathan and R. Vasudevan of Matscience, Madras. M. Gourdin of Orsay, France, gave a compact formula for calculating the matrix element of electron scattering from a target of arbitrary spin in a covariant way, and K. Venkatesan of Matscience, Madras, explained how the notion of a group representation breaks down for certain values of the group parameter in the case of complex angular momentum.

The three lectures on many-body problems started with that of C. de Dominicis of Saclay (France) who dealt with the quasiparticle formulation of quantum statistical mechanics which is based on partial summations in diagrammatic expansions and discussed the relationship with the Landau theory of Fermi liquids. A method of obtaining sum rules for any system, given the Hamiltonian of the system and the main variables which one wants to occur in the sum rules, described by P. T. Landsberg and D. J. Morgan of the University College of Cardiff, Wales. A. Sjölander of the Institut för Mekanik in Göteborg, Sweden, gave an account of the concept of lattice waves or phonons and described in detail the inelastic neutron scattering technique and its theory as a means by which the experimental determination of phonon dispersion curves and polarization directions are exclusively done at present.

In the supplementary mathematics session, M. H. Stone of the University of Chicago, emphasized the need for enquiry into the techniques of model construction in the various mathematical sciences and the study of the role of the mathematician in this respect. We are particularly grateful to Professor Harish-Chandra, at the Institute for Advanced Study in Princeton, for his active participation in the symposium by giving a series of two lectures on the problem of constructing the characters for noncompact, semi-simple Lie groups. S. K. Srinivasan of the Indian Institute of Technology, Madras, gave a brief outline of some recent developments in stochastic point processes.

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It is our earnest hope that the supplementary sessions in mathematics will soon grow into a full-fledged symposium to be conducted concurrently with that on theoretical Physics.

Alladi Ramakrishnan

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# **Introductory Address**

### VICTOR WEISSKOPF

DIRECTOR-GENERAL, CERN Geneva, Switzerland

The science of elementary particles has in recent years naturally become the most fundamental of all pursuits in physics. In this lecture. I shall attempt to present a broad and distant view of the current scene in elementary particle physics and indicate to you the general trend of ideas in this field. The most dominant in this trend is the concept of symmetry, which is playing an increasingly fundamental role in the study of interactions of elementary particles. It will be rewarding for us to discuss this in an historical perspective. We shall start with a quotation from Newton's Opticks:

"All these things being consider'd, it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conducted to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary Power being able to divide what God himself made one in the first Creation. While the Particles continue entire, they may compose Bodies of one and the same Nature and Texture in all Ages: But should they wear away, or break in pieces, the Nature of Things, depending on them, would be changed. Water and Earth, composed of old worn Particles and Fragments of Particles, would not be of the same Nature and Texture now, with Water and Earth composed of entire Particles in the Beginning. And therefore, that Nature may be lasting, the Changes of Corporeal Things are to be placed only in the various Separations and new Associations and Motions of these permanent Particles; compound Bodies being apt to break, not in the midst of solid Particles, but where those Particles are laid together, and only touch in a few Points."

We clearly perceive the remarkable vision of Newton in the above quotation. He is obviously worried about elementary particles. He is troubled that elementary particles may lose an edge here or a corner there in course of time. Since nature is of infinite duration, he wonders how the properties of these particles will remain the same. So he wants them to be incomparably hard. Today the elementary particle physicist is able to answer Newton's concern.

Rutherford discovered in the 1920's that atoms are not incomparably hard, and indeed they are composed of electrons and nuclei. It is then hard to understand how the atoms can keep their properties. We are now aware of the phenomenal success of the concepts of quantum mechanics, which provide an explanation of a wide variety of properties of various atoms. If we probe deeper into the cause of this tremendous success, we perceive that the properties of the quantum states are defined by an underlying symmetry. The shape of the wave functions of an electron in a Coulomb field is a good illustration of the situation.

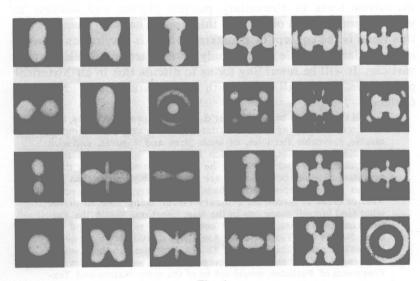


Fig. 1

We now observe that the atomic states possess two distinct symmetries: (a) rotational symmetries, which also dictate spin symmetry and (b) symmetry under permutation because of identity of electrons. The latter is known as the *Pauli principle*. It is this principle which gives the different varieties of atoms distinct properties, since the electrons are forced to occupy various orbital states with different energies.

These symmetries are all related to certain conservation principles. We know that rotational symmetry gives rise to conservation of total angular momentum, while the identity of particles is due to symmetry under permutation. We also believe that the conservation of charge is due to invariance under a gauge transformation. In fact, this invariance is the basis of the light-quantum hypothesis and of our study of quantum electrodynamics. In short, the principles of symmetry described above ensure that the properties of particles are preserved. This remark does in a sense answer Newton's problem.

In our picture of atoms given above, the nuclei seem to be incomparably hard. Thus, Newton's worry merely gets shifted from the atoms to nuclei. It was again Rutherford who soon established that even nuclei are not incomparably hard. They consist of protons and neutrons, and evidently they cannot be elementary particles. How is it that nuclei have characteristic unchanging properties? It is remarkable again that the answer should be based on symmetry.

The analogy between atomic physics and nuclear physics is surprisingly closer than we should perhaps expect. We have in nuclear physics shell model nuclear wave functions, a periodic table for describing the properties of nuclei, and so on. If we look more closely into the underlying symmetries, we discover that the nuclear interactions also possess invariance under rotations and permutations. Besides these, there is a new symmetry known as *isospin symmetry* which treats the proton and neutron as two states of a particle called the *nucleon*. Though this symmetry is exact as far as nuclear forces are considered, it is violated by electromagnetic interactions.

Let us look at some relevant numbers in these two systems. A rough estimate of the size of the atoms is provided by the Bohr radius

$$a = \frac{\hbar^2}{mc^2} = 5 \times 10^{-9} \, \mathrm{cm}$$

The interaction energy is measured in Rydberg units. A Rydberg in atomic physics is given by

$$Ry_{atom} = \frac{me^4}{\hbar^2} = 27 \text{ eV}$$