Berry/Rice/Ross

Solutions Manual

Physical Chemistry

Solutions Manual for

PHYSICAL CHEMISTRY

by Berry, Rice, and Ross

Prepared by

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Table of Contents

CHAPTER 1

1. We are given two salts, $MgSO_x$ (magnesium sulfite) and $MgSO_y$ (magnesium sulfate). If A represents oxygen and B represents sulfur plus magnesium, then modern data would yield

$$\frac{w_A \text{ (magnesium sulfite)}}{w_B \text{ (magnesium sulfite)}} = \frac{48}{104.3 - 48} = 0.853$$

and
$$\frac{w_A \text{ (magnesium sulfate)}}{w_B \text{ (magnesium sulfate)}} = 1.137$$

and $\frac{x}{y} = R_{12} = \frac{0.853}{1.137} = \frac{3}{4}$ from Eq. (1-2). If there were a 20% error in the apparent weight of sulfite, one would have

$$\frac{w_A \text{ (magnesium sulfite)}}{w_B \text{ (magnesium sulfite)}} = \frac{0.8 \times 48}{24.3 + 0.8 \times 32} = 0.770$$

and then $R_{12} = \frac{0.770}{1.137} = 0.677 \approx \frac{2}{3}$.

The formulas of the sulfite and sulfate would be taken to be $MgSO_2$ and $MgSO_3$ respectively. The percentage composition (by weight) of oxygen in magnesium sulfite would be taken as

$$\frac{w_A}{w_A + w_B} = \left[1 + \frac{w_B}{w_A}\right]^{-1} = \left[1 + \frac{1}{.770}\right]^{-1} = 0.435 .$$

From the formula $MgSO_2$ for magnesium sulfite, the atomic weight of Mg would be found as follows:

$$\frac{2 \times 16}{AW(Mg) + 32 + 2\times16} = 0.435$$

or $$AW\,(Mg) = 9.6$$

2. a. Represent a pinhead by a disc 1 mm in diameter and 0.25 mm in thickness. Then

$$6.022 \times 10^{23} \times \pi \times (0.5 \times 10^{-3})^2 \times 0.25 \times 10^{-3} \text{ m}^3$$

$$\approx 10^{14} \text{ m}^3$$

b. One mole of electrons = 96486.7 C

$$\frac{96486.7 \text{ C}}{0.1 \text{ C/s}} = 964867 \text{ s} \simeq 11 \text{ days}$$

c. $10^4 \times 0.1 \text{ mm} = 10^3 \text{ mm} = 1 \text{ m}$

d. $$\frac{0.1 \text{ C/s} \times 1 \text{ s}}{1.76 \times 10^{11} \text{ C/kg}} = 5.7 \times 10^{-13} \text{ kg}$$

3. Use Eq. (1-14). We know $e \simeq 1.6 \times 10^{-19}$ C and $\frac{1}{2} mv^2 \simeq 100$ eV = 1.6×10^{-17} J. If $\ell = 1$ mm and $E = 10^4$ V/m, then $\tan\theta = 0.05$, or $\theta = 2.86°$. If we take θ' in Eq. (1-19) to be equal to θ, then the anticipated range for B is

$$2.38 \times 10^{-2} \text{ T} < B < 3.77 \times 10^{-2} \text{ T}$$

for the muon mass in the range 200-500 times the electron mass.

4. The success of the Milliken method depends on the degree to which the effect of the electron's (or the quark's) charge can be maximized, which implies that one should try to maximize the difference between the magnitudes of v_1 and v_2 (Eqs. (1-23) and (1-24)). Since

$$v_1 = \frac{2r^2}{9\eta} g (\rho_s - \rho_{air})$$

and

$$v_2 = \frac{neE}{6\pi\eta r} - \frac{2r^2}{9\eta} g (\rho_s - \rho_{air}),$$

the determination of e can be made most accurately if r is small. Therefore, a sphere radius of 0.01 mm is preferable. The only property of the electron or quark which enters the analysis of the Milliken experiment is its charge; the particle's mass has no measurable effect.

5. a. $F = q E = 1.60 \times 10^{-19} \text{ C} \times 10^5 \text{ V/m}$

$= 1.60 \times 10^{-14} \text{ J/m} = 1.60 \times 10^{-14} \text{ N}$

Negative z direction.

$a = \frac{F}{m} = 1.76 \times 10^{16} \text{ m/s}^2$

b. $B = \mu_0 H = 4\pi \times 10^{-7} \text{ N/A}^2 \times 10^4 \text{ A/m}$

$= 4\pi \times 10^{-3} \text{ N/Am} = 4\pi \times 10^{-3} \text{ T}$

$F = 1.60 \times 10^{-19} \text{ C} \times 10^5 \text{ m/s} \times 4\pi \times 10^{-3} \text{ T} \times \sin 45° = 1.42 \times 10^{-16} \text{ N}$

If the velocity is in the positive x-y quadrant, the force points in the negative z direction.

$$a = \frac{F}{m} = 1.56 \times 10^{14} \text{ m/s}^2 .$$

c. $F_{elec} = 1.60 \times 10^{-19}\ C \times 10^4\ V/m = 1.60 \times 10^{-15}\ N$

Negative y direction

$F_{mag} = 1.60 \times 10^{-19}\ C \times 10^5\ m/s \times 0.1\ T = 1.60 \times 10^{-15}\ N$

Positive y direction

$F = F_{elec} + F_{mag} = 0\ ; \quad a = 0$

6. From Eq. (1-34),

$$\frac{Ed}{B^2} = \frac{r^2 q}{2m}$$

With $r = 0.15$ m, $q = 1.60 \times 10^{-19}$ C, and 1 amu < m < 200 amu, the restriction on the fields is

$$5.42 \times 10^1\ V/T^2 < \frac{Ed}{B^2} < 1.08 \times 10^4\ V/T^2$$

For m = 1 amu, $\frac{Ed}{B^2} = 1.08 \times 10^4\ V/T^2$

With $d = 10^{-4}$ m, $E = 10^6$ V/m, a magnetic field of $B = 9.62 \times 10^{-2}$ T will be suitable.

Similarly, for m = 50 amu, $d = 10^{-4}$ m, one can use $E = 2 \times 10^5$ V/m and $B = 3.04 \times 10^{-1}$ T, and for m = 200 amu, $d = 10^{-4}$ m, one can use $E = 2 \times 10^5$ V/m and $B = 6.07 \times 10^{-1}$ T.

7. Let m = mass of ion before fragmentation

m' = mass of ion after fragmentation

From Eq. (1-33) $v_0 = \left(\frac{2qEd}{m}\right)^{1/2}$

and from Eq. (1-31) $r = \frac{m' v_0}{qB}$

so that Eq. (1-34) will now read

$$r = \frac{(2Ed)^{1/2}}{B}\left(\frac{m'^2}{mq}\right)^{1/2}$$

The apparent mass is therefore m'^2/m. If m = 150 amu and m' = 135 amu, the apparent mass is 122 amu.

8. A charge e which is accelerated through a potential difference V acquires a kinetic energy given by

$$\frac{1}{2} mv^2 = eV.$$

The radius of curvature in a magnetic field H is given by

$$r = \frac{mv}{eB} = \frac{(2V)^{1/2}}{B} \frac{m}{e}^{1/2}$$

where $B = \mu_0 H$. In practice, the radii of EL_1 and EL_2 would be measured separately and then averaged to yield a reliable value for r. The charge to mass ratio is then given by

$$\frac{e}{m} = \frac{2V}{r^2 B^2} .$$

9. Let ℓ = distance between slit and plate. The time during which the ions are accelerated by the field is $\tau = \ell/V_x$, and the magnitude of the acceleration is eE/m , where E is the electric field and m is the ionic mass. The deflection of the ions in the y direction is therefore

$$d = \frac{1}{2}\frac{eE}{m}\tau^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{\ell}{V_x}\right)^2$$

$$d(^6Li) - d(^7Li) = 10^{-2}\ m = \frac{eE\ell^2}{2V_x}\left(\frac{1}{m(^6Li)} - \frac{1}{m(^7Li)}\right).$$

Since $m(^6Li) = \dfrac{6}{6.022 \times 10^{26}}$ kg and

$m(^7Li) = \dfrac{7}{6.022 \times 10^{26}}$ kg ,

one finds $E\ell^2 = 2.18 \times 10^3$ Vm .

If ℓ = 1 m, for example, the field has the value $E = 2.18 \times 10^3$ V/m.

10. If the angle between the slots is ϕ (in degrees), then the condition for passage of the particles is that they travel 0.1 m in the time it takes the disks to rotate through the angle ϕ. If the rotation speed is Ω revolutions/sec, this time is $\phi/360\ \Omega$. The velocity of the particles which pass is therefore

$$v = \frac{0.1}{\phi/360\ \Omega}\ m/s = \frac{36\ \Omega}{\phi}\ m/s .$$

For example, if $v = 10^3$ m/s and $\phi = 45°$, then Ω = 1250 revolutions/sec.

A slot 2 mm wide at the rim of a disc of radius 50 mm subtends an angle of

$$\frac{2}{50}\ \text{radians} = 2.29° .$$

If ϕ is set at 45°, then the actual range of angles is $42.71° < \phi < 47.29°$. If Ω = 1250 revolutions/sec, then

$$952\ m/s < v < 1054\ m/sec .$$

CHAPTER 2

1. The interpretation of E_{thresh} is the amount of energy required to excite an atom from the ground state to some particular excited state. The maximum energy that can be released upon relaxation from the excited state is therefore E_{thresh}, which corresponds to a maximum frequency of E_{thresh}/h. In some cases, the relaxation may be entirely to intermediate excited levels rather than to the ground state, so that all of the emission frequencies are less than E_{thresh}/h.

2. The wavelength of radiation corresponding to 2.103 eV = 3.37×10^{-19}J is

$$\frac{hc}{3.37\text{x}10^{-19}\text{J}} = 5.90\text{x}10^{-7}\text{m} = 589.6 \text{ nm.}$$

The energy corresponding to 670.8 nm is

$$\frac{hc}{670.8\text{x}10^{-9}\text{m}} = 2.96\text{x}10^{-19}\text{J} = 1.848 \text{ eV}$$

3. The exciting energy is 18.72 eV = $2.999\text{x}10^{-18}$J. The emission at 585.24 nm corresponds to an energy of $3.394\text{x}10^{-19}$J. Evidently, the electrons at 18.72 eV excite the neon atoms to a state above the lowest excited state. Partial relaxation to an intermediate state yields radiation at 585.24 nm. The remaining energy, $2.660\text{x}10^{-18}$J, can be radiated and will correspond to a wavelength of 74.68 nm. The total exciting energy can also be radiated at once, corresponding to a wavelength of 66.23 nm.

4. For velocity v, one has $eV = 1/2\, m_e v^2$ and $eV' = 1/2\, m_p v^2$. Therefore

$$\frac{V'}{V} = \frac{m_p}{m_e} \approx 1800. \text{ If } V = 300 \text{ volts,}$$

then V' = 540,000 volts.

5. a. $E = h\nu = h\text{x}10^{15}\text{s}^{-1} = 6.62\text{x}10^{-19}\text{J}$

   b. $E = 6.02\text{x}10^{23}\text{x } 6.62\text{x}10^{-19}\text{J} = 3.99\text{x}10^{5}\text{J}$

   c. 6 km/hr = 1.67 m/s

   $E = 1/2\, mv^2 = 83.3$ J

   d. $E = 3600 \text{ x } 60 \text{ J} = 2.16\text{x}10^{5}$ J

6. For simplicity, consider a simple cubic lattice of atoms whose radius is r. The cross sectional area of the lattice per atom is then $\sigma = (2r)^2 = 4r^2$. The number density is $n = 8/(4r)^3 = 1/8\, r^3$, so the attenuation coefficient is $\sigma n = 1/2r$. If $\Delta I/I = -0.10$, then $\Delta x \approx 0.1 \text{ x } 2r = 0.2r$. From Eq. (2-7), $I/I_0 = 0.90$, and

$x = -2r\ln 0.90 = 0.21\ r.$

7. From Eqs. (2-15) and (2-16) we see that the total cross section for scattering would be proportional to

$$2\pi \int_0^{\pi} e^{-\theta^2/\theta^2_m} \sin\theta d\theta$$

if the plum-pudding model were correct. The fraction of particles expected to be found at angles greater than $\pi/2$ would then be

$$\frac{\int_{\pi/2}^{\pi} e^{-\theta^2/\theta^2_m} \sin\theta\ d\theta}{\int_0^{\pi} e^{-\theta^2/\theta^2_m} \sin\theta\ d\theta}$$

In the range $\pi/2 < \theta < \pi$, $e^{-\theta^2/\theta^2_m}$ is essentially zero. (For $\theta = \pi/2$, $e^{-\theta^2/\theta^2_m} = e^{-8100}$.) The numerator of the above ratio of integrals is therefore ≈ 0, and one would not expect to see any particles scattered at angles greater than $\pi/2$.

8. From Eq. (2-9) we have

$$\frac{1}{\lambda_H} = R_H\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)$$

for a transition in atomic hydrogen and

$$\frac{1}{\lambda_D} = R_D\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)$$

for a transition in atomic deuterium. The constants R_H and R_D are related to $R_\infty = 109737.31\ cm^{-1}$ by

$$R_H = \frac{\mu_H}{m_e} R_\infty\ , \quad R_D = \frac{\mu_D}{m_e} R_\infty$$

where m_e is the mass of an electron ($m_e = 5.48593\times10^{-4}$ au) and μ_H and μ_D are the reduced masses of H and D. Since the total atomic masses are $m_H = 1.007825$ au and $m_D = 2.01410$ au, the reduced masses are

$$\mu_H = \frac{(m_H - m_e)m_e}{m_H} = 5.48295\times10^{-4}\ \text{au}$$

and $$\mu_D = \frac{(m_D - m_e)m_e}{m_D} = 5.48444\times10^{-4}\ \text{au}$$

and $R_H = 0.99456\ R_\infty = 109678\ cm^{-1}$, and $R_D = 0.999728\ R_\infty = 109708\ cm^{-1}$.
For a $3\to2$ transition,

$$\lambda_H = 6564.69 \text{ Å}, \quad \lambda_D = 6562.91 \text{ Å}$$

For a 4→2 transition,

$$\lambda_H = 4862.74 \text{ Å}, \quad \lambda_D = 4861.41 \text{ Å}$$

For a particular transition,

$$\frac{\lambda_D}{\lambda_H} = \frac{R_H}{R_D} = \frac{\mu_H}{\mu_D} = \frac{m_D(m_H - m_e)}{m_H(m_D - m_e)}$$

Therefore, if λ_D/λ_H, m_D, and m_H are known, the above is an equation for m_e.

9. Kinetic energy $= hc \left(\frac{1}{2.537\text{x}10^{-7} \text{ m}} - \frac{1}{4.047\text{x}10^{-7} \text{ m}}\right)$

$$= 2.924\text{x}10^{-19} \text{ J}$$

10. The energy levels for C^{+5} are (Eq. (2-68))

$$E_n = -hcR_\infty \frac{Z^2}{n^2}$$

with $Z = 6$. For a 2→1 transition,

$$\frac{1}{\lambda} = 36 R_\infty (1 - 1/4) \text{ , and}$$

$$\lambda = 33.75 \text{ Å (x-ray region)}$$

The ionization energy is $36 \text{ hc } R_\infty = 489.81$ eV.

For a 3→2 transition,

$$\lambda = 182.25 \text{ Å (x-ray region)}$$

The ionization energy of an n=2 electron is

$$\frac{36}{4} hcR_\infty = 122.45 \text{ eV.}$$

11. We use Eq. (2-66), with an "effective" value of 2 for Z. (That is, we assume that the remaining nuclear charge is effectively screened by the inner electrons.) Then, 7.644 ev corresponds to r = 1.88 Å and 15.031 ev corresponds to r = 0.96 Å. An estimate for the average value of r is therefore 1.42 Å.

12. By trial and error, one can fit the data by

$$\frac{1}{\lambda} = R \left(\frac{1}{(2-\delta)^2} - \frac{1}{m^2}\right), \quad m = 3, 4, \ldots$$

The values of δ found are:

m	3	4	5	6	7
λ/nm	589.18	330.26	285.28	268.04	259.39
δ	0.061	0.281	0.332	0.351	0.360

The value of δ remains reasonably constant for the higher energy transitions.

13. $\sigma(\theta) = A \cos \theta/2$; $\sigma(5°) = 2.584\text{x}10^{-17}$ cm^2
$\Rightarrow A = 2.586\text{x}10^{-17}$cm^2

$$\sigma = 2\pi \int_0^\pi d\theta\ \sigma(\theta)\ \sin\theta = 2\pi A \int_0^\pi d\theta\ \sin\theta\cos\frac{\theta}{2}$$

$$= \frac{8}{3}\pi A$$

14. a. Use Eq. (2-28) as an approximation.

$m \approx 0.1$ kg; $r \approx 0.15$ m

$v \approx 33 \times 2\pi \times 0.15$ m/min ≈ 0.5 m/s

$A \approx 0.05$ J sec

b. Use Eq. (2-42)

$\omega = 2 \quad \times 1000\ \text{sec}^{-1}$
$x_1 \approx 10$ cm
$v_0 = \omega x$
$m \approx 2$ kg
$A \approx 400$ J sec

c. Let time be measured from t=0, when the ball is at its maximum height $h_0 = 1$ m. As the ball falls, its height is given by $h = h_0 - 1/2\ gt^2$, where g is the acceleration of gravity. During this time, the ball's velocity is given by $v = gt$. At time $t_1 = \sqrt{2h_0/g}$ the ball hits the ground (h=0). The total action during the fall is therefore

$$A_{fall} = \int_0^{t_1} mv^2\ dt = mg^2 \int_0^{t_1} t^2\ dt$$

$$= \frac{1}{3} mg^2 \left(\frac{2h_0}{g}\right)^{3/2}$$

It is easily verified that the action during the rise of the ball from the ground up to h_0 is equal to A_{fall}, so the total action for the cycle is

$$A = \frac{2}{3} mg^2 \left(\frac{2h_0}{g}\right)^{3/2} = 5.9\text{x}10^{-3}\ \text{J sec}$$

For the free hydrogen atom, $v = 10^4$ m sec^{-1} and $m = 1.66\text{x}10^{-27}$ kg.

$$A = \int_0^{1\ \text{sec}} mv^2\,dt = mv^2 \times 1\ \text{sec} = 1.66\text{x}10^{-19}\ \text{J sec.}$$

15. The energy of one quantum of radiation is

$$h\nu = \frac{hc}{\lambda} = 1.86\text{x}10^{-20}\ \text{J.}$$

In one second the laser emits 1 J, or $5.38\text{x}10^{19}$ quanta. To raise the temperature of 200 g of water from 25°C to 100°C requires 200 x 4.184 x 75 J = $6.28\text{x}10^4$ J. Since the laser emits 1 J/sec, 17.4 hours of operation is required.

16. Use Eq. (2-63) with n = Z = 1

$$r = 2.82\text{x}10^{-5}\ \text{m}$$

17. Use Eq. (2-76), with the reduced mass of positronium μ_p replacing μ_H. Since

$$\mu_p = \frac{m_e^2}{2m_e} = \frac{m_e}{2},$$

the Rydberg constant for positronium is given by

$$R_p = \frac{1}{2} R_H = 54839\ \text{cm}^{-1}.$$

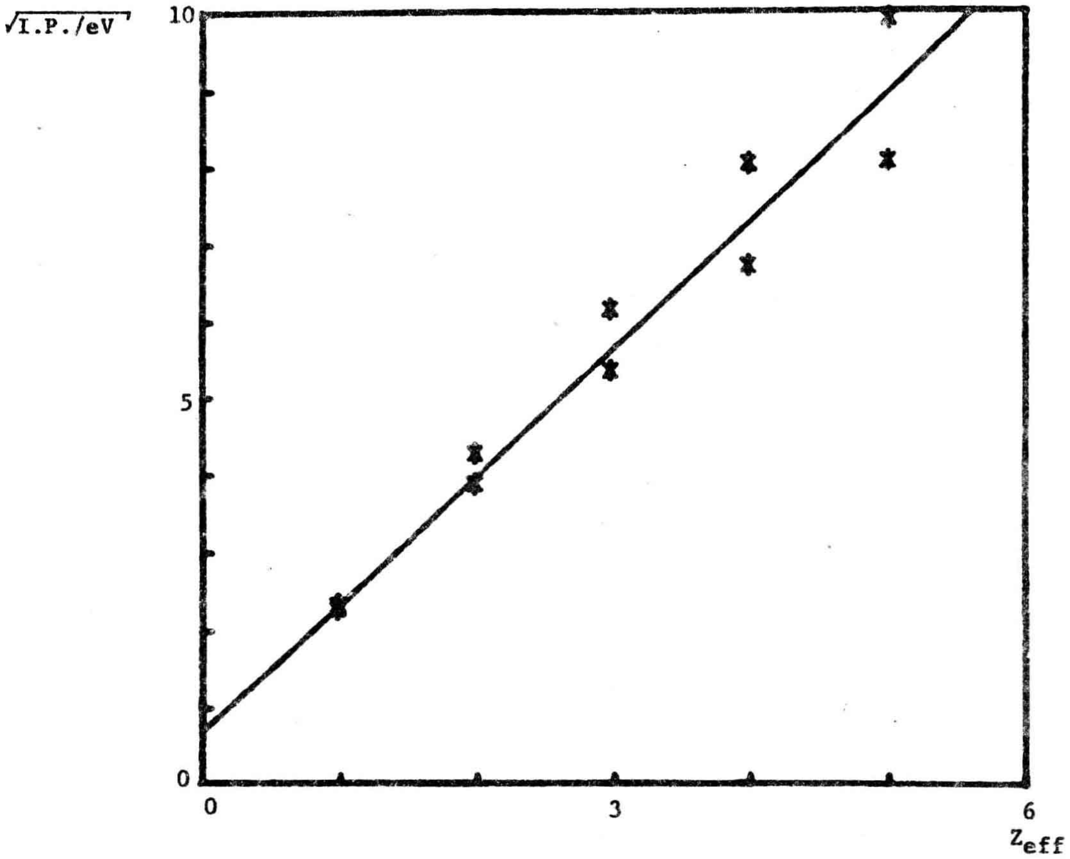
The first line in the Lyman series has frequency

$$\nu = c\,R_p\,(1 - \frac{1}{4}) = 1.23\text{x}10^{15}\ \text{sec}^{-1}$$

and the ionization energy is

$$hc\,R_p = 1.09\text{x}10^{-18}\ \text{J}$$

18. Note that each species contains one electron in the valence shell. From Eq. (2-68), the ionization energy of a one electron species is proportional to the square of the nuclear charge. For the given data, we can plot the square root of the ionization potential vs. an "effective" nuclear charge which is the actual nuclear charge minus the number of core electrons. Such a plot is shown below. The fit of the data to a straight line is reasonably good.



19. $k = m\omega^2 = 166$ N/m

$$\omega = \sqrt{\frac{k}{m}} = 316 \text{ sec}^{-1}$$

20. Since the force in any direction (x, y, and z) is independent of motion in the other directions, the oscillator moves in each direction independently of the others. Thus the action contains three separate contributions:

$$A_{cycle} = \pi m \,(v_{0x}x_1 + v_{0y}y_1 + v_{0z}z_1) \,,$$

an obvious extension of Eq.(2-42). There will be a separate quantization condition for each contribution to the action, so that following Eq.(2-52) for the one dimensional oscillator yields

$$E_{osc} = \nu A_{cycle} = h\nu\,(n_x + n_y + n_z).$$

21. If the particle moves at (constant) speed v, the action per cycle is

$$A = \int_0^{2L} m\,v\,ds = 2\,m\,v\,L \,,$$ since in one cycle the particle moves

through a distance of 2L. The quantization rule requires $A = nh$, so that

$$v = \frac{nh}{2mL} .$$

There is no potential energy, so the total energy of the particle is

$$\frac{1}{2} mv^2 = \frac{n^2h^2}{8mL^2}$$

Therefore, the set of allowed energies is discrete and the energies are proportional to n^2 .

22. Use Eqs.(2-68) and (2-73) with $Z = 10$, $n_1 = 1$ and $n_2 = 2, 3, 4, 5$ and 6. The wavelengths are 1.2150 nm, 1.0252 nm, 0.9720 nm, 0.9492 nm, and 0.9373 nm.

23. Use Table (2-2) for v_n with n=1. For Z=2, v is already greater than 1% of the speed of light. The first excitation energy (1→2) is 2.180×10^{-18} J x Z^2 x (1 - 1/4). The rest mass energy of the electron is 8.19×10^{-14} J. When Z = 23, the excitation energy exceeds 1% of the electron rest mass energy.

24. Use Eq. (2-63), with $Ze^2/4\pi\varepsilon_0$ (for electrostatic attraction) replaced by GMm (for gravitational attraction). Here, G is the gravitational constant (6.67×10^{-11} Nm^2/kg^2), M is the mass of the sun (2×10^{30}kg) and m is the mass of the earth (6×10^{24} kg). Equation (2-63) then becomes

$$r = \frac{n^2\hbar^2}{GMm^2} = 2\times10^{-138} \text{ m} \times n^2 \quad (!!)$$

For $r = 1.5\times10^8$ km , $n \approx 2.5\times10^{74}$ (!!)

CHAPTER 3

1. a. $p = mv = 1.66 \times 10^{-24}$ kg m/sec; $\lambda = \frac{h}{p} = 3.96 \times 10^{-10}$ m

b. $E = 8 \times 10^{-21}$J; $p = \sqrt{2mE} = 1.21 \times 10^{-25}$ kg m/sec;

$\lambda = 5.49 \times 10^{-9}$ m

c. $\lambda = 5.49 \times 10^{-13}$ m

d. $\lambda = 1.12 \times 10^{-11}$ m

e. $\lambda = 6.43 \times 10^{-17}$ m

2. The disturbance represented by $\vec{f}$ (x,y,z) occurs only in the x direction, so the direction of displacement is the x direction. The phase of the wave is $kz - \omega t$, so the direction of propagation is the z direction. The nodes, determined by the condition $\cos(kz - \omega t) = 0$, are not stationary in space, so this is a traveling wave. A parallel longitudinal wave might be given by

$$\vec{f}(x,y,z) = [0, 0, A\cos(kz - \omega t)].$$

3. a. A (very) crude estimate of λ would be $\lambda \approx 4$Å. (The lowest order standing wave in a rigid box has a wavelength twice the box length.)

b. $p = h/\lambda$ and $K.E. = \frac{p^2}{2m} = 1.51 \times 10^{-18}$ J ≈ 10 eV

c. 15–20 eV

4. $E = \hbar\omega = \frac{p^2}{2m} = \frac{1}{2m} \cdot \frac{hcm}{\lambda} = \frac{hc}{2\lambda} = \frac{\hbar ck}{2}$

or $\omega = ck/2$. This dispersion relation is of the same form as is found for light waves ($\omega = ck$).

5. By induction: $\frac{\partial\psi}{\partial x} = \frac{d\psi}{d\phi}\frac{\partial\phi}{\partial x} = k\frac{d\psi}{d\phi}$

Given that $\frac{\partial^{n-1}\psi}{\partial x^{n-1}} = k^{n-1}\frac{d^{n-1}\psi}{d\phi^{n-1}}$, then

$$\frac{\partial^n\psi}{\partial x^n} = k^{n-1}\frac{\partial}{\partial x}\frac{d^{n-1}\psi}{d\phi^{n-1}} = k^{n-1}\frac{d^n\psi}{d\phi^n}\frac{\partial\phi}{\partial x}$$