THE ECONOMETRIC ANALYSIS OF SEASONAL TIME SERIES

ERIC GHYSELS

DENISE R. OSBORN

CAMBRIDGE
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sels and Denise R. Osborn

Foreword by Thomas J. Sargent, *Stanford University*

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The Econometric Analysis of Seasonal Time Series

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themes in modern econometrics

The Econometric Analysis of Seasonal Time Series

Eric Ghysels
Denise R. Osborn

"Seasonal time series data are everywhere, and it is by now well understood that modeling such seasonality is important in various respects. This book gives an up-to-date account of what has been going on in the last 15 years in this area and hence what effectively is the current state of the art. The authors are to be commended for undertaking this [illegible] simply is a good book."

– Philip Hans Franses, *Erasmus University, Rotterdam*

"To many economists, seasonality is some sort of noise that is purged from the data before it is analyzed. Econometric research of the recent decades has revealed, however, that seasonal adjustment is a much more difficult exercise than had previously been assumed, as seasonal and non-seasonal data characteristics are linked. A full understanding of economic variables requires careful modeling of their seasonal features. Eric Ghysels and Denise Osborn have managed to survey a wide variety of statistical models of seasonality and of tests that are useful in discriminating among them. Their work will constitute valuable reference material for researchers and practitioners alike."

– Robert M. Kunst, *University of Vienna*

THE ECONOMETRIC ANALYSIS OF SEASONAL TIME SERIES

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The Econometric Analysis of Seasonal Time Series

In this book, Eric Ghysels and Denise R. Osborn provide a thorough and timely review of the recent developments in the econometric analysis of seasonal economic time series, summarizing a decade of theoretical advances in the area. The authors discuss the asymptotic distribution theory for linear stationary and nonstationary seasonal stochastic processes. They also cover the latest contributions to the theory and practice of seasonal adjustment, together with its implications for estimation and hypothesis testing. Moreover, a comprehensive analysis of periodic models is provided, including stationary and nonstationary cases. The book concludes with a discussion of some nonlinear seasonal and periodic models. The treatment is designed for an audience of researchers and advanced graduate students.

Eric Ghysels is the Edward M. Bernstein Professor of Economics and a Professor of Finance at the University of North Carolina at Chapel Hill. He has been a visiting professor or scholar at several major U.S., European, and Asian universities. He gave invited lectures at the 1990 World Congress of the Econometric Society, the 1995 American Statistical Association Meetings, the 1995 Brazilian Econometric Society Meetings, and the 1999 $(EC)^2$ Conference on financial econometrics. He has served on the editorial boards of the *Journal of the American Statistical Association*, the *Journal of Business and Economic Statistics*, the *Review of Economics and Statistics*, the *Journal of Empirical Finance*, and several Annals issues of the *Journal of Econometrics*. In 1999, he was Chair of the Business and Economics Statistics section of the American Statistical Association. He has published in the leading economics, finance, and statistics journals, and his main research interests are time series econometrics and finance.

Denise R. Osborn is the Robert Ottley Professor of Econometrics in the School of Economic Studies, University of Manchester, where she has taught since 1977. Professor Osborn was the inaugural chair of the Women's Committee of the Royal Economic Society (RES) and a past member of both the RES Executive and Council. She is currently Vice Chair of the Economics and Econometrics Panel for the 2001 Research Assessment Exercise in the U.K. and a member of the Training Board of the Economic and Social Research Council (ESRC). Her research interests have been primarily concerned with improving understanding of the dynamics of macroeconomic time series, recently focusing on seasonality and business cycles. Professor Osborn has published more than 30 papers in refereed journals, including [illegible] *Journal of Business and Economic Statistics*, *Journal of the American Statistical Association*, *Journal of the Royal Statistical Society (Series B)*, and *Journal of Econometrics*.

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To my mother
E.G.

To Nicholas, Sarah, and Philip
D.R.O.

Contents

Foreword by Thomas J. Sargent

Seasonality means special annual dependence. Many weekly, monthly, or quarterly economic time series exhibit seasonality. Eric Ghysels and Denise Osborn's book is about specifying, econometrically testing, and distinguishing alternative forms of seasonality. This subject is of high interest to economists who use dynamic economic models to understand economic time series.

Dynamic economic theory seeks to describe and interpret economic time series in terms of the purposes and constraints of economic decision makers. Economic decision makers forecast the future to inform their decisions. Dynamic theory focuses on the relationship between forecasts and decisions, a relationship characterized by the Bellman equation of dynamic programming.

From the point of view of economic theory, the seasonal dependence of economic time series is especially interesting because substantial components of seasonal fluctuations are predictable. But in the most plausible models – those of indeterministic seasonality – seasonal fluctuations are not perfectly predictable. Optimizing behavior of decision makers combined with the predictability of seasonal time series leads to sharp, cross-equation restrictions between decision makers' rules and the seasonal time series whose forecasts impinge on their decisions. Particular statistical models of seasonality of the varieties described by Ghysels and Osborn differ in the detailed statistical structure of a time series of interest to a decision maker, and so they deliver different restrictions on decision makers' behavior and so on market prices and quantities.

That the predictability of seasonal fluctuations leads to sharp restrictions on decision rules has led to opposing responses from applied economists. One response has been to try to purge the time series of their seasonality – via seasonal adjustment procedures – and to focus analysis on the dynamics of the less predictable components of the time series. The opposite response has been to focus especially hard on the seasonal components, because they contain much detectable information about the preferences and constraints governing decisions. Ghysels and Osborn's book helps us to understand either approach, and to better implement both responses. As emphasized by Christopher Sims' work on estimating approximating models, a decision to use seasonally unadjusted data can be justified by a prior suspicion that one's model is least reliable for thinking about seasonal fluctuations.

Whether or not you want to use seasonally adjusted data, Ghysels and Osborn's book is of interest. Decisions to seasonally adjust data or to use raw data both require accurate specifications of the seasonal components of fluctuations. The strong seasonal fluctuations of economic time series give us a good laboratory for testing dynamic economic theories of asset pricing and accumulation of capital. Ghysels and Osborn's book will improve our use of that laboratory.

Palo Alto
November 1999

Preface

There has been a resurgence of interest in seasonality within economics in the past decade. Since the book by Hylleberg (1986) titled *Seasonality in Regression*, many developments have taken place. Some of these developments are covered in recent books by Franses (1996b) and Miron (1996). The former specializes on the subject of so-called periodic models, whereas the latter emphasizes economic aspects of seasonality. Also, surveys have been written by Franses (1996a), Ghysels (1994a), and Miron (1994), special issues of journals on the subject have been published by De Gooijer and Franses (1997) and Ghysels (1993), and a book of readings has been published by Hylleberg (1992).

The focus of this book is in some ways close to that of Hylleberg (1986), who gave a comprehensive description of the then-current state of the art. Our coverage will be more selective, however, as we mostly concentrate on the econometric developments of the past decade. Consequently, we will not discuss in detail subjects such as "The History of Seasonality," covered by Hylleberg (1986, Chapter 1) and Nerlove, Grether, and Carvalho (1995), or "The Definition of Seasonality," as in Hylleberg (1986, Chapter 2). Nor will we deal, at least directly, with the longstanding debate about the merits and pitfalls of seasonally adjusting series. It should also be made clear that this book focuses on theoretical developments and hence does not have the pretention to be empirical. It is not our purpose to analyze real data, beyond the presentation of some series that illustrate different types of seasonality. Similarly, we give no advice to the practitioner who wishes to seasonally adjust on how this adjustment should be performed. Rather, we aim to present a coherent account of the current state of the econometric theory for analyzing seasonal time series processes.

Important developments have taken place over the past decade. Parallel to the advances in the econometrics of unit root nonstationarity, we have witnessed corresponding progress in the analysis of seasonal nonstationarity. In Chapter 3 we provide a comprehensive coverage of unit root nonstationarity in a seasonal context. Chapter 2 precedes this analysis and covers the more traditional topic of deterministic seasonal processes, although here too there has been substantial progress in the 1990s. Although we are not concerned with the pros and cons of seasonal adjustment, it is a matter of fact that seasonality is frequently modeled implicitly through the use of seasonal adjustment filters. The U.S. Bureau of the Census and Agustin Marvall at the Bank of Spain have both recently developed new procedures for seasonal adjustment, namely the X-12 and SEATS/TRAMO programs, respectively. Chapter 4 is almost entirely devoted to these procedures. Since the seminal papers by Sims (1974) and Wallis (1974), much more has been learned about estimation with filtered data, which is why an entire chapter (Chapter 5) is devoted to it. Periodic models of seasonality were introduced into the econometrics literature during the late 1980s; Chapter 6 summarizes this literature and its subsequent progress.

Traditionally, the topic of seasonality is put in a context of macroeconomic time series sampled at a monthly or quarterly frequency. In one area, however, namely financial econometrics, new high-frequency data sets have become available with series sampled on a transaction basis. The availability of such series has created considerable interest in models for so-called intraday seasonality. This is one of the topics of Chapter 7, together with other topics related to the nonlinear analysis of seasonal time series.

Chapter 1 is a guided tour of the substantive material that follows. We start with a set of examples of seasonal economic and financial time series to motivate the discussion. The examples considered include some empirical series and theoretical models. The analysis in this first chapter is not meant to be rigorous, but rather illustrative, since the main purpose is to provide the reader with a general overview of the different types of seasonal processes considered in later chapters.

We have written this book for an audience of researchers and graduate students familiar with time series analysis at the level of, for instance, Hamilton (1994a). Thus we do not review certain basic results of the econometrics of stationary and nonstationary time series. For example, we refer to I(0) and I(1) processes without reviewing their basic properties. Also, we use spectral analysis, assuming familiarity with spectral domain techniques. There are many other cases in which we assume that the reader has sufficient background knowledge to understand the concepts and methods. We make no apology for this since there are good recent textbooks, including Hamilton (1994a), which cover this material.

This book contains many results that have emerged from joint work undertaken by the authors with others. We have had the privilege and pleasure to work with many co-authors whose intellectual input we would like to acknowledge. Material in this book either directly or indirectly covers joint work of the authors with Chris Birchenhall, Tim Bollerslev, Alice Chui, Clive Granger, Alastair Hall, Saeed Heravi, Hahn Lee, Offer Lieberman, Jason Noh, Pierre

Perron, Paulo Rodrigues, Pierre Siklos, and Jeremy Smith. We would also like to thank Robert Taylor for reading and commenting on some of the material and Paulo Rodrigues for assistance with the figures. Peter Phillips, Patrick McCartan, and Scott Parris provided invaluable guidance and help while we were writing this monograph, and Nicholas Berman patiently proofread the entire manuscript. We also benefited from insightful comments of three referees who read preliminary drafts of the book. Finally, we owe an unmeasurable debt to Sandi Lucas, who keyboarded various drafts of the book with devotion beyond the call of duty.

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Manchester, U.K.

List of Symbols and Notation

D_s	periodic first difference operator for season s;
$f_y(w)$	spectral density of y at frequency ω;
$I(d)$	process integrated of order d;
L	lag operator, $Ly_t = y_{t-1}$;
l_k	$k \times 1$ vector of ones (k may be omitted);
m_s	seasonal mean shifts, $s = 1, \ldots, S$;
$PI(d)$	process periodically integrated of order d;
S	number of seasons per year;
s_t	season in which t falls, $s_t = 1 + [(t-1) \bmod S]$;
$SI(d)$	process seasonally integrated of order d;
T	number of observations in sample;
T_τ	number of years of sample observations;
U_t	vector i.i.d. disturbance, $U_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})'$;
$W_i(r)$	standard Wiener process;
Y_t	generic vector time series, $Y_t = (y_{1t}, \ldots, y_{Nt})'$;
Y_τ	vector time series of the S seasons of year τ, $Y_\tau = (y_{1\tau}, \ldots, y_{S\tau})'$, $\tau = 1, \ldots, T_\tau$ (similarly, U_τ);
$y_{s\tau}$	observation on y_t relating to season s of year τ ($s = 1, \ldots, S$, $\tau = 1, \ldots, T_\tau$) (similarly, $\varepsilon_{s\tau}$);
y_t	generic univariate time series observed over $t = 1, \ldots, T$;
$y_t^{(1)}/y_t^{(2)}/y_t^{(3)}$	linear filtered series of y_t used in HEGY procedure;
y_t^h	holiday component of y_t;
y_t^i	irregular component of y_t;
y_t^{ns}	nonseasonal component of y_t;

y_t^s	seasonal component of y_t;
y_t^{tc}	trend cycle component of y_t;
y_t^{td}	trading-day component of y_t;
y_t^{X11}/y_t^{LSA}	X11/linear seasonally adjusted filtered series of y_t;
z_t	weakly dependent univariate process.

Greek

α_S	coefficient tested in DHF t test;
γ	drift;
Δ_S	seasonal (annual) difference operator, $\Delta_S = 1 - L^S$;
δ_{st}	seasonal dummy variable for observation t and season s, $\delta_{st} = 1$; if $s = 1 + [(t-1) \bmod S]$ and 0 otherwise;
ε_t	univariate i.i.d. disturbance;
$\Theta(L)$	vector moving average operator, $\Theta(L) = \Theta_0 - \Theta_1 L - \cdots - \Theta_q L^q$;
$\theta(L)$	univariate moving average operator, $\theta(L) = 1 - \theta_1 L \cdots - \theta_q L^q$;
$\theta_S(L^S)$	seasonal polynomial MA lag operator;
π_i	coefficient tested in HEGY test;
$\rho(k)$	autocorrelation function at lag k;
$\widetilde{\sigma}$	degrees of freedom corrected estimator of σ;
τ_t	year of observation t, $\tau_t = 1 + \text{int}[(t-1)/S]$;
$\Phi(L)$	vector autoregressive operator, $\Phi(L) = \Phi_0 - \Phi_1 L - \cdots - \Phi_p L^p$;
$\phi(L)$	univariate autoregressive operator, $\phi(L) = 1 - \phi_1 L \cdots - \phi_p L^p$;
ϕ_{ij}^P	periodic autoregressive coefficient for variable i at lag j (j may be omitted for periodic AR of order 1; P may be omitted when periodic context is clear);
$\phi_S(L^S)$	seasonal polynomial AR lag operator.

Processes

ARMA process for seasonal component of y_t	$\phi_s(L)y_t^s = \theta_s(L)\varepsilon_t^s$ (similarly, other components);

k_i	coefficient of cointegrating relationship;
Regression model	$y_t = x_t\beta + u_t$;
Univariate ARMA process	$\phi(L)y_t = \theta(L)\varepsilon_t$;
Vector ARMA process	$\Phi(L)Y_t = \Theta(L)U_t$.

1 Introduction to Seasonal Processes

1.1 Some Illustrative Seasonal Economic Time Series

Economic time series are usually recorded at some fixed time interval. Most macroeconomic aggregate series are released on a monthly or quarterly frequency. Other economic data, particularly financial series, are available more frequently. In fact, some financial market data are available on a transaction basis and hence are no longer sampled at fixed intervals. Throughout this book we will focus almost exclusively on data sampled at a monthly or quarterly frequency. There is an emerging literature dealing with data sampled at higher frequencies, such as weekly, daily, or intradaily, possibly unequally spaced. This literature is still in its infancy, particularly compared with what we know about quarterly or monthly sampled data.

The purpose of this section is to provide some illustrative examples of seasonal time series of interest within economics. It is certainly not our intention to be exhaustive. Instead, we look at several series that reveal particularly interesting features and stylized facts pertaining to seasonality. We also discuss various ways to display seasonal series.

Figure 1.1 has four panels displaying U.K. and U.S. monthly growth rates in industrial production over roughly 35 years. The left panels represent unadjusted raw series, whereas the right panels show the corresponding seasonally adjusted series. Both the U.K. and U.S. raw series show very regular seasonal patterns that are clearly dominant and seem to obscure and dwarf the nonseasonal fluctuations. The nonseasonal fluctuations are filtered from the raw data and appear in the right panels (note the difference in scale to appraise the dominance of the seasonal fluctuations). The seasonally adjusted series are the most often used; that is; they are typically released to the general public, appear in the financial press, and so on. There has been a long debate about the merits and dangers of seasonally adjusting economic time series, a debate that will not be covered in this book. Lucid discussions appear in Ghysels (1994a), Granger (1978), Hylleberg, Jørgensen, and Sørensen (1993),

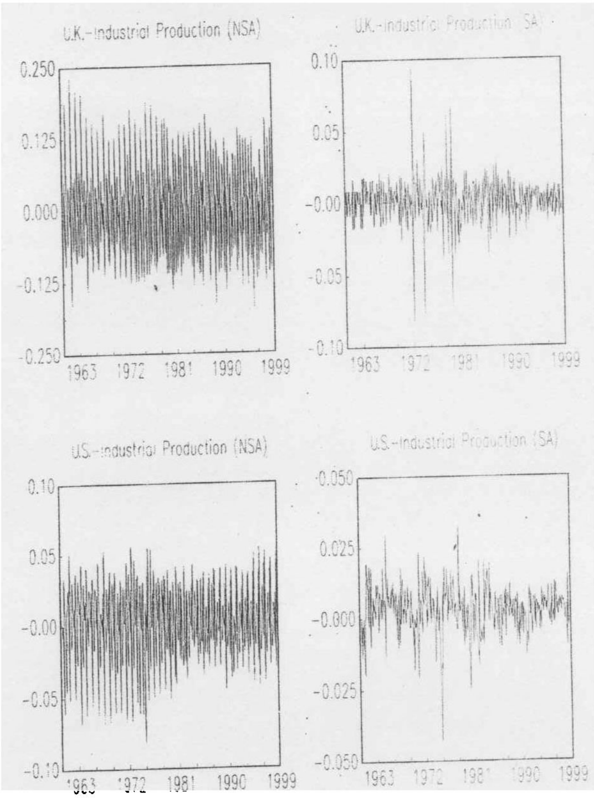

Figure 1.1. U.K. and U.S. industrial production – monthly NSA (not seasonally adjusted) and SA (seasonally adjusted).

and Miron (1994) among many others. Obviously, to obtain a seasonally adjusted series we need to subtract from (or divide) the raw series (by) an estimate of the seasonal component. Whether this component is assumed orthogonal to the nonseasonal one, and therefore void of interest to economists, and whether the seasonal is best viewed as deterministic or not are some of the basic questions being debated. While we do not elaborate explicitly on the pros and cons of seasonal adjustment, we do cover quite extensively some of the basic questions underlying this debate. For example, the distinct features of deterministic and stochastic models of seasonality are discussed in great detail in Chapters 2 and 3.

We can infer from Figure 1.1 that the seasonal patterns in the U.K. and U.S. are not quite the same. U.K. industrial production shows far greater variability and, taking into account the differences in scale, certainly larger peaks. It is clear from the right panels, however, that the greater variability of U.K. series is partly due to the nonseasonal fluctuations.

Two series appear in Figure 1.2; these are U.K. retail sales of clothing (shown as the first difference of the logarithm, in the top two panels) and U.K. retail credit: net lending (lower two panels). Retail sales of clothing show an extremely regular seasonal pattern, except perhaps that the third-quarter dip appears more pronounced toward the end of the sample. The top left plot is a standard time series plot. The top right graph represents the same series plotted as four curves, each corresponding to a specific quarter of each calendar year. Since clothing is a basic consumption good, it is not surprising to find hardly any business cycle movements in the series. The time series plot of annual quarters reveals that a simple model with a distinct mean for each quarter probably goes a long way toward describing the behavior of the series. Plotting each quarter as a separate curve is a particularly useful tool for examining the seasonal behavior of a series. Such plots can be found in Hylleberg (1986), and the construction of the underlying series, that is, a multivariate vector process of quarterly observations sampled annually, forms the basic building block of several test statistics [e.g., Franses (1994) and Canova and Hansen (1995), among others] that have been proposed recently and will be discussed later.

While the retail sales of clothes is highly seasonal and regular, it is clear from the graphs appearing in the lower two panels of Figure 1.2 that net lending of retail credit is also highly seasonal but clearly does not feature stable seasonal patterns. The time series plot shows fluctuations of increasing amplitude. The plot of quarterly curves drift apart and cross each other. The two examples in Figure 1.2 are almost perfect polar cases. Sales of clothing is a relatively easy series to model in comparison to net lending. One obvious implication is that seasonal adjustment of net lending is a rather daunting task. Of the two series, it is ironically also the one in which the nonseasonal component is more interesting.

When we restrict our attention to macroeconomic time series, then the class of seasonal models is confined to processes with dynamic properties at periods of a quarter or a month. However, when financial time series are studied, then our interest shifts to seasonal patterns at the daily level together with seasonal properties in higher moments. The last example, which appears in Figure 1.3, illustrates this. It shows the daily returns and daily returns squared of the Standard and Poor (S & P) 500 stock index as well as its monthly and daily averages. It is clear from Figure 1.3 that distinct patterns appear both in the mean and volatility at the daily and monthly level.[1]

In summary, our selection of empirical series has illustrated that seasonality has many different manifestations. It is no surprise, therefore, that a rich toolkit of econometric techniques has been developed to model seasonality.

[1] One of the early studies on daily returns patterns is Gibbons and Hess (1981). In Chapter 7, we provide further details.

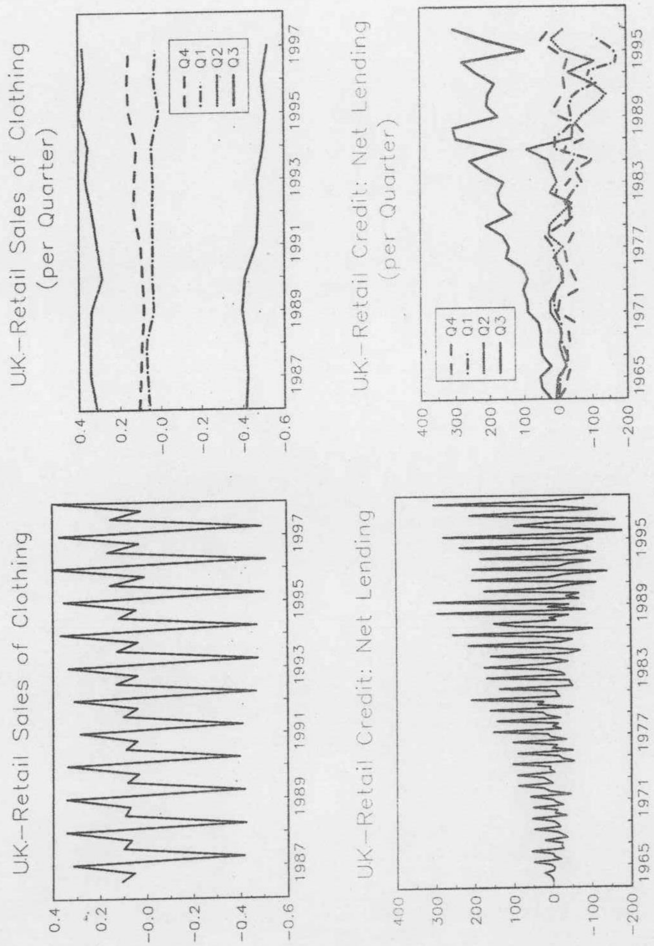

Figure 1.2. U.K. retail sales of clothing and retail credit.

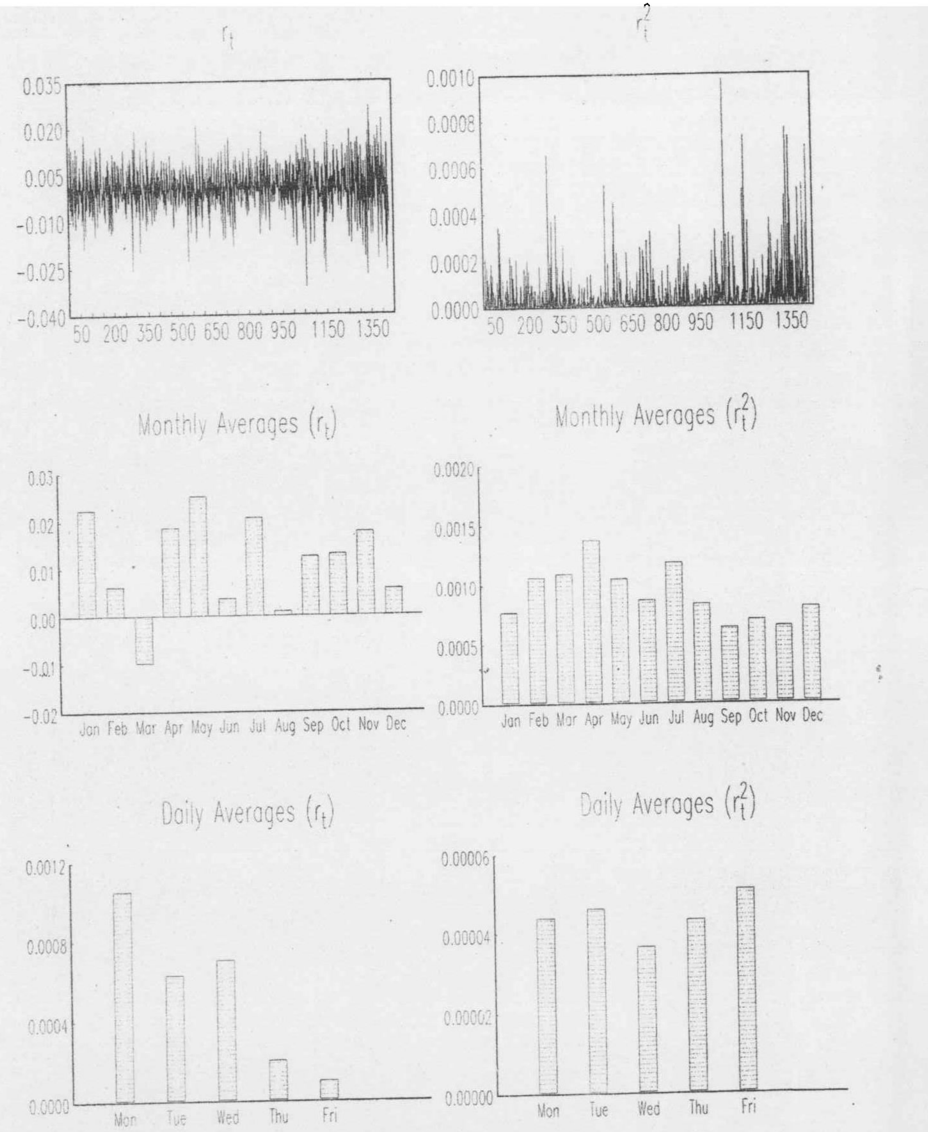

Figure 1.3. Returns and squared returns of S&P 500 index.

1.2 Seasonality in the Mean

In this section, we review a class of time series processes that model seasonal mean behavior. This includes most of the standard processes, such as deterministic seasonal mean shifts, stochastic stationary and nonstationary processes, and unobserved components ARIMA models. A subsection will be devoted to each class of process.

1.2.1 *Deterministic Seasonality*

From knowledge of the season in which the initial observation falls, we can deduce the season for all subsequent values of t. For simplicity of exposition, we assume that $t = 1$ corresponds to the first season of a year (that is, the first quarter for quarterly data or January for monthly observations), and we denote the season in which observation t falls as s_t. With S observations per year, s_t can be obtained mathematically as $s_t = 1 + [(t-1) \bmod S]$. (That is, s_t is one plus the integer remainder obtained when $t-1$ is divided by S.) It will also be useful to have a notation for the year in which a specific observation falls, and we refer to this as τ_t. This can be found as $\tau_t = 1 + \text{int}\,[(t-1)/S]$, where int denotes the integer part.

Let us consider a univariate process y_t, which has the following representation:

$$y_t = \sum_{s=1}^{S} \delta_{st} m_s + z_t, \qquad t = 1, \ldots, T, \tag{1.1}$$

where δ_{st} is a seasonal dummy variable that takes the value one in season s (more formally $\delta_{st} = 1$ if $s_t = s$ for $s = 1, \ldots, S$) and is zero otherwise. The process z_t is assumed to be a weakly stationary zero mean process.[2] The first term on the right-hand side of (1.1) represents deterministic level shifts, while the second term z_t is stochastic and may even exhibit seasonal features as will be discussed in the next subsection. Hence, for any given season s the unconditional mean of y_t is m_s. In Figure 1.4, Panel A, we display a time series plot of sample size $T = 40$ of a deterministic seasonal process with z_t i.i.d. $N(0, 1)$ and $S = 4$ with $m_1 = -1.5, m_2 = -0.5, m_3 = 0.5$, and $m_4 = 1.5$; DGP stands for data-generating process. Figure 1.4 has four panels, similar to those used for the retail sales of clothing and net lending time series in the previous section. Namely, Panel B of Figure 1.4 displays the data as four annual time series plots of quarters. Panels C and D repeat this exercise for a large sample of 50 years ($T = 200$) instead of 10 years. It is clear from this simulated series example that the four curves of quarterly data appearing in panels B and D can cross and show considerable variability, unlike the U.K. retail sales of clothing series. In large samples (Panel D) the quarterly curves do not drift apart, however, since the process features mean reversion.

[2] For a definition of weakly stationary or covariance stationary processes, see, e.g., Hamilton (1994a).

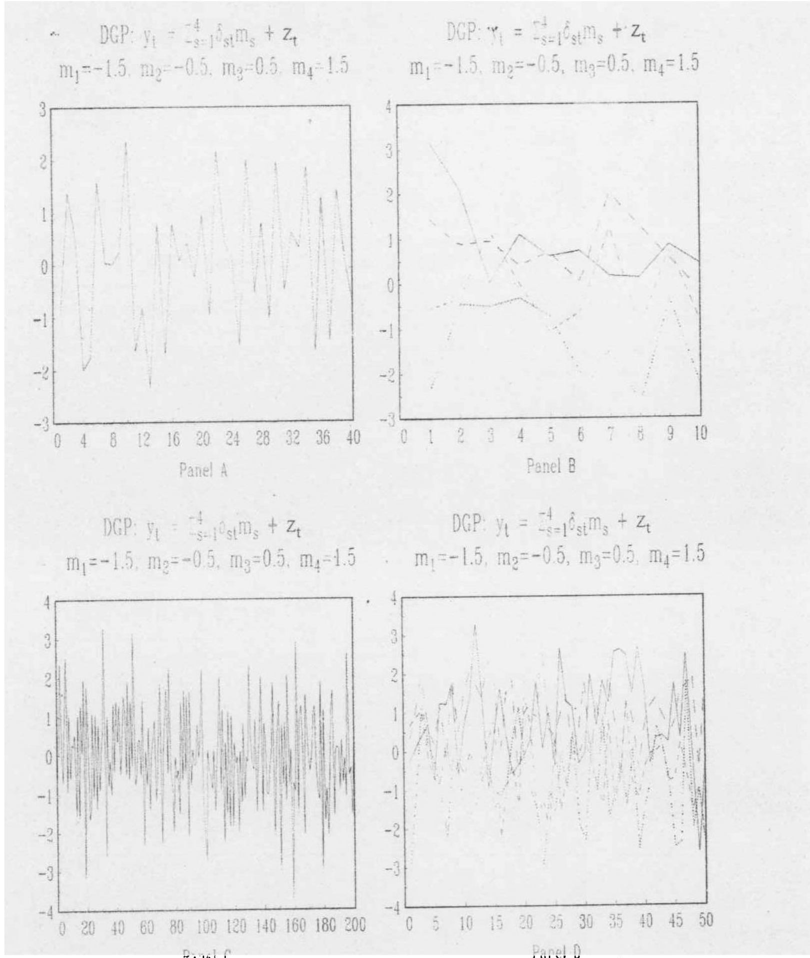

Figure 1.4. Two examples of a deterministic seasonal process.

A deterministic function with period S can also be equivalently written in terms of sines and cosines, namely

$$\sum_{s=1}^{S} \delta_{st} m_s = \sum_{k=1}^{S/2} \left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi tk}{S}\right) \right] \tag{1.2}$$

for $t = 1, \ldots, T$, where

$$\alpha_k = \frac{2}{S} \sum_{s=1}^{S} m_s \cos\left(\frac{2\pi kj}{S}\right), \qquad k = 1, 2, \ldots, \frac{S}{2} - 1, \tag{1.3}$$