CALCULUS
WITH ANALYTIC GEOMETRY

THE APPLETON-CENTURY MATHEMATICS SERIES

CALCULUS

WITH ANALYTIC GEOMETRY

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Raymond W. Brink and John M. H. Olmsted, *Editors*

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PREFACE

This book presents a first course in calculus and analytic geometry, with enough material for three semesters. It is sufficiently rigorous for the most highly screened students in our best universities, yet each new concept is motivated so carefully by a natural, intuitive introduction that students with less preparation can profit from it and enjoy it.

Seven great basic concepts are stressed: *function*, *limit of a function*, *continuity*, *derivative*, *antiderivative*, *definite integral*, and *infinite series*. Every effort is made to impress on the student that a mastery of these ideas is indispensable in acquiring a genuine understanding of calculus as contrasted with the mere use of its formulas and techniques in solving problems.

At the same time, an abundance of material is provided dealing with the degree of accuracy of computed results, and with other aspects of the computational work that is now so important for progress in science and technology.

Since ϵ, δ methods are necessary for a proper definition of the limit of a function, a very thorough treatment of inequalities and absolute values precedes it.

It has been the author's experience that many first-year students of calculus fail to make a clear distinction between the very different concepts of the indefinite integral (or antiderivative) and the definite integral as a limit of a sum. This is partly due to the similarity of their names and symbols. Not only is their difference stressed in the present book when these concepts are first introduced, but an antiderivative of a function f is symbolized by $D^{-1} f$, so strikingly different from the symbol $\int_a^b f(x)\,dx$ for a definite integral, until the chapter on technique of integration is reached. At that point the traditional name "indefinite integral" is adopted along with its customary symbol $\int f(x)\,dx$. By that time the student sees clearly the distinction between these two concepts and is ready to start using the commonly accepted name and symbol without confusion.

Set notation is introduced early. It is usually employed when advantageous, but not slavishly or as a fetish. Instructors who wish to use it more often can ask their students to express other theorems and definitions in set notation.

Vectors in two- and three-dimensional space are presented with a firm mathematical basis and are applied widely. Vectors do not supplant a sound foundation in Cartesian analytic geometry but complement it and make possible a more concise formulation of some of the theorems that were first derived in the classical Cartesian manner.

Throughout this book the principal definitions and theorems are prominently labeled, numbered and displayed, both for easy reference and to keep the main structure of the material before the student's eyes. The number 7.3.4, for example, refers to the fourth numbered definition or theorem in §3 of Chapter 7. The number 14.6 refers to §6 of Chapter 14.

There is a wide variety of exercises, some asking the student to prove theorems, some to fix ideas presented in the text and to help him acquire facility with the techniques of calculus, and some in verbal form to apply the calculus to idealized situations from daily life.

The author has tried to write this book in a simple and straightforward style, without unnecessary verbiage or involvement, so that it will be unusually suitable for the student to study by himself or with minimum help from a teacher.

I am glad to have this opportunity to express my gratitude to Professor Raymond W. Brink, Editor of the Appleton-Century Mathematics Series, for his many suggestions and aid in the preparation of the manuscript. Thanks are also due to my colleague, Charles J. Merchant, for some of the exercises, to my wife, Bernice Lee Purcell, for making the index, and to Ruth Gayle Fones and Jack Newsbaum for checking the answers to the odd-numbered exercises.

E. J. P.

Calculus with Analytic Geometry

Errata

Page 69, middle of page:
replace **Example 2** with **Example 4**

Page 85, definition 4.2.1: The last line of the definition should read
and x is in the domain of f.

Page 94, near the middle of the page:
replace **4.3.7 Theorem** with **4.3.6 Theorem**

Page 225, third line from the bottom (displayed equation):
replace F with f

Page 286, third line above Exercise 19:
replace (see 5.6 Example 2) with (see 2.11, Example 4).

Page 312, Exercise 11: The last equation in this exercise should read
$|F'P| + |FP| = 2a$

Page 312, Exercise 12: The last line of this exercise should read
F are the points whose coordinates are $(-\sqrt{a^2 + b^2}, 0)$ and $(\sqrt{a^2 + b^2}, 0)$, respectively.

Page 356, next-to-the-last line on the page:
change BAB' to $\widehat{BAB'}$

Page 357, beginning of line 4:
change BAB' to $\widehat{BAB'}$

Page 357, line 8 [displayed equation (2)]:
change $|BA|$ to $|\widehat{BA}|$

Page 584, fourth line following the heading "Exercises":
replace tangent with velocity

(over)

Page 597, next-to-the-last line of Definition 19.3.1:
replace *region R* with *domain of f*

Page 682, third line:
replace *sum* with *series*

Page 700, second and third line from the bottom of the page: change these two lines to read

Of course, if k is a positive integer or zero, the Taylor's series has but a finite number of nonzero terms, and $R_n(x) = 0$ for some n; this is the binomial theorem of elementary algebra.

Page 709, exercises 19 and 20: change the next-to-the-last line in each exercise to read
show that, formally,

Page 751, lines 4 and 5 of the proof: change these lines to read
for all values of x in the domains of both f and g and satisfying $0 < |x - a| < \delta$, and thus that $\lim_{x \to a} [f(x)g(x)] = 0$.

Page 753, the ninth and tenth lines from the bottom of the page: change these lines to read

In Case 1, a is either positive or negative; in either event it follows that there is an open interval containing a in which all numbers x

Page 757, the last seven lines of the page: replace this section with the following:
Therefore the set

$$T = \left\{ \frac{1}{U - f(x)} \middle| \, a \leq x \leq b \right\}$$

has 0 for a lower bound and (by A.2.3) some number L for an upper bound. Thus

$$(5) \qquad 0 < \frac{1}{U - f(x)} \leq L \text{ for } a \leq x \leq b,$$

which implies that

$$(6) \qquad U - f(x) \geq \frac{1}{L}, a \leq x \leq b.$$

Page 842, line 2 of the right-hand column:
change cyclindrical to cylindrical

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