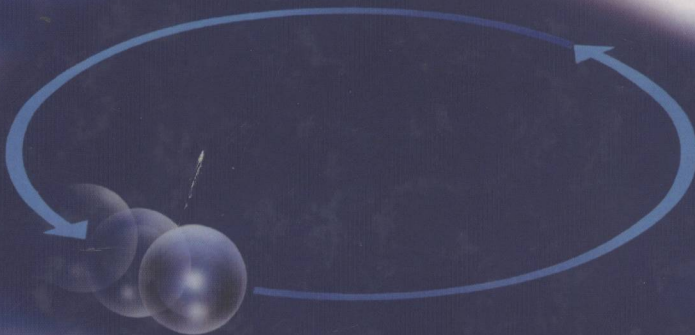


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# Physics of Semiconductors in High Magnetic Fields

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Noboru Miura



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# Physics of Semiconductors in High Magnetic Fields

**Noboru Miura**

*Institute for Solid State Physics, University of Tokyo*



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Great Clarendon Street, Oxford OX2 6DP

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Published in the United States  
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First published 2008

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Printed in Great Britain  
on acid-free paper by  
Biddles Ltd. [www.biddles.co.uk](http://www.biddles.co.uk)

ISBN 978-0-19-851756-6 (Hbk)

1 3 5 7 9 10 8 6 4 2

## PREFACE

The recent progress of magnet technology has enabled us to study solid state physics in very high magnetic fields. Steady fields up to about 40–45 T are available with hybrid magnets in several facilities in the world. Using pulsed fields, 50–90 T can be easily produced with a long duration of 10–100 ms, and we can now obtain a field even in the megagauss range (above 100 T) by special techniques, and these high fields are routinely employed for various measurements in solid state physics.

High magnetic fields quantize the electronic states in substances, and the effects of the quantization give rise to many new phenomena. In semiconductors, the Landau quantization effect is particularly significant because of the small effective masses and high mobilities. Thus, semiconductor physics is one of the areas where high magnetic fields are most useful.

Recently, remarkable progress in semiconductor technology has made it possible to grow many new systems that have never before been available. The development of techniques such as molecular beam epitaxy (MBE), metal organic chemical vapor deposition (MOCVD), or various fine processes have created low dimensional electron systems. In low dimensional systems, the quantization effect by magnetic fields is even more enhanced, combined with the quantization by the potentials built in the samples. The integer and fractional quantum Hall effects are among the most remarkable manifestations of the quantization effects. A variety of quantum phenomena have been observed in artificial new low dimensional systems.

The author has devoted himself to developing techniques for generating very high pulsed magnetic fields up to several megagauss (several hundred tesla) by various kinds of approaches, and applied them to various solid state experiments for many years. Many new results have been obtained using such high fields in a wide range of physics. Many interesting phenomena have also been observed by collaborating scientists who come from outside including overseas to use our facilities. Among them, typical results on semiconductor physics will be described in each chapter of this book.

There have been many excellent textbooks on semiconductor physics. There have also been many valuable proceedings of international conferences on semiconductor physics in high magnetic fields as listed at the end of the book. However, there have not been so many books in the form of a monograph sharply devoted to the physics of semiconductors in high magnetic fields. The aim of this book is to give a review of recent progress in this area with a particular emphasis on the new phenomena observed in the very high field range, mainly in the pulsed field range above 20 or 30 T up to a few megagauss.

This book is partly based on the notes for a series of lectures which the author gave at the graduate course of the University of Tokyo, with the additional descriptions of new phenomena observed in high magnetic fields. In order to make the entire book consistent as a monograph, considerable effort has been directed to enriching the introductory part of each chapter. Therefore, the book will hopefully have the dual character of textbook and monograph giving a review of the latest progress. In the last chapter, the experimental techniques for generating pulsed high magnetic fields for the data acquisition are described.

The author strongly wishes that this book will serve as a guide for young scientists, students, and engineers to increase their interest in high magnetic fields.

*Noboru Miura*  
Tokyo, Japan  
September 2007

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## INTRODUCTION

Magnetic fields quantize the energy states of the conduction bands and the valence bands in semiconductors. In high magnetic fields, the quantization of energy states in semiconductors becomes very prominent and the radius of the cyclotron motion of conduction electrons or the electron wave-function extension is much reduced. The quantization of the electronic states and the modification of the electronic wave function cause many new quantum phenomena in high magnetic fields. Electrons in most semiconductors possess high mobility or long relaxation time between scattering. Today, electron mobility of the best GaAs crystals reaches higher than  $10^7 \text{ cm}^2/\text{Vs}$  at low temperatures, which corresponds to a mean free path of mm. Moreover, in most cases, the effective masses of electrons and holes are smaller than the free electron mass. These features are most favorable for observing the effects of quantization.

The recent advances in semiconductor technology have enabled us to fabricate high quality samples in which electrons and holes are confined in artificial quantum potentials embedded in them. They are heterostructure interfaces, superlattices, quantum wells, quantum wires, and quantum dots. In these samples, the low dimensional electrons confined in quantum potentials show unique properties in magnetic fields. For instance, the two-dimensional electron systems in quantum wells or heterostructures are fully quantized when magnetic fields are applied in a perpendicular direction to the system. Therefore, under the unique situation of full quantization, many new quantum phenomena have been observed. The integer and fractional quantum Hall effects are the highlights of these. When fields are applied parallel to the two-dimensional systems, on the other hand, a variety of phenomena take place originating from the interplay between the magnetic field potential and the artificially formed quantum potentials. In quantum dots or quantum wires, phenomena originating from the potential interplay are also observed, and they are extremely useful for the characterization of new quantum devices. Thus the physics of low dimensional electron systems in high magnetic fields is one of the most important subjects of semiconductor physics.

Figure 1.1 shows the cyclotron energy  $\hbar\omega_c$ , the cyclotron radius  $r_c$ , and the spin Zeeman splitting  $g\mu_B B$  as a function of magnetic field  $B$ . The cyclotron energy and the Zeeman splitting of free electrons increases as a linear function of magnetic field, and reaches 11.6 meV (136 K) at 100 T. As  $\hbar\omega_c$  is inversely proportional to the effective mass  $m^*$  which is smaller than the free electron mass  $m$  in most semiconductors,  $\hbar\omega_c$  is even larger than that of free electrons. In Fig. 1.1,

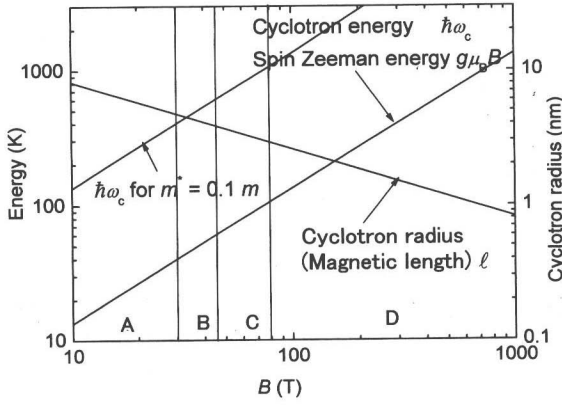


FIG. 1.1. Cyclotron energy  $\hbar\omega_c$  and the spin Zeeman splitting  $g\mu_B B$  for a free electron in a unit of K, and the cyclotron radius of the ground state  $r_c$  which is the same as the magnetic length  $l$ , as a function of magnetic field  $B$ . The range of the magnetic fields available with the present technology is shown: A. Superconducting magnet, B. Water-cooled or hybrid magnet, C. Non-destructive long pulse magnet, D. Destructive short pulse magnet.

the line for an effective mass  $m^* = 0.1m$  is also shown. In high enough magnetic fields,  $\hbar\omega_c$  becomes larger than various excitation energies in semiconductors, such as the longitudinal optical (LO) phonon energy  $\hbar\omega_o$ , the band gap energy  $\mathcal{E}_g$ , the plasmon energy  $\mathcal{E}_p$ , the binding energies of excitons or impurity states, *etc.* Under such unusual circumstances, we can observe new phenomena that have never been observed in the lower field range. The cyclotron radius of the ground state  $l$ , on the other hand, decreases with increasing magnetic field in proportion to  $1/\sqrt{B}$ . It is reduced to 2.6 nm at 100 T and 0.81 nm at 1000 T, and becomes smaller than various characteristic lengths in semiconductors. In other words, very high magnetic fields by themselves create electronic states of nano-scale structure. For example, it may become smaller than the wave-function extension of excitons or impurity states. This situation also causes various interesting phenomena in a new regime. In artificial quantum structures, it can become smaller than the wave-function extension of electrons confined in the quantum potential. In very high magnetic fields,  $l$  can be reduced to a very small value: even smaller than the lattice constant of crystals. The effective mass approximation that is quite adequate to treat the electronic states in a magnetic field may break down in extremely high magnetic fields. It is an interesting question what will happen under such conditions.

Today, as high field magnet technology develops, the maximum field available for experiments has increased more and more, and we can study semiconductor physics in very high magnetic fields. Table 1.1 shows various techniques for

**Table 1.1** Different techniques for generating high magnetic fields and the maximum fields generated by them.

Technique	Maximum field (T)	Duration
Electromagnet	3	DC
Supeconducting magnet	20	DC
Water cooled magnet	33	DC
Hybrid magnet	45	DC
Wire-wound pulse magnet	90	1–100 ms
Helical pulse magnet	70	~100 $\mu$ s
Single turn coil technique	300	~ $\mu$ s
Electromagnetic flux compression	620	~ $\mu$ s
Explosive driven flux compression (Bellow type)	200	~ $\mu$ s
Explosive driven flux compression (Cylindrical type)	> 1000	~ $\mu$ s

producing high magnetic fields. In the 1960s, the Francis Bitter National High Magnetic Field Laboratory (FBNHMFL) was built at the MIT, and steady high magnetic fields were generated by using a large DC generator of 10 MW and a large water cooling system for the magnets heated by the large current (water-cooled magnet). High magnetic fields up to about 20 T were generated and the fields were employed for many different experiments, but the major application was in the field of semiconductor physics. In those days, the FBNHMFL was one of the main centers of semiconductor research in the world. In later years, several high magnetic field facilities have been built in Europe and Japan as well, and the maximum field has increased. The technology of superconducting magnets using Nb-Ti alloy or Nb<sub>3</sub>Sn wires also made great progress in the 1960s. By using commercial superconducting magnets, it has become possible to generate high magnetic fields (around 15–17 T) easily in ordinary laboratories. The maximum field now available with superconducting magnets is almost 21 T. However, because of the limited critical field and the critical current of presently available superconducting materials, in order to obtain higher fields, we have to use the water-cooled magnets in large facilities as mentioned above. At present, the maximum field obtained with such water-cooled magnets is 33 T at the new National High Magnetic Field Laboratory (NHMFL) in Tallahassee succeeding the FBNHMFL in USA. In most large DC field facilities, hybrid magnets are constructed combining a water-cooled magnet with a superconducting magnet set outside. The hybrid magnet can generate higher magnetic fields than simple water-cooled magnets. The highest field now at Tallahassee is 45 T.

For generating a field above this field level, pulse magnets are employed. By supplying a large pulse current from a capacitor bank, a wire-wound solenoid type magnet can generate magnetic fields up to 60–90 T or so, within a duration time of 1–100 ms. For pulse magnets, it is not necessary to install a large power supply which consumes an enormous electric power, nor a large water cooling system. The use of pulse magnets extended the available field range significantly.

Together with the progress of fast data acquisition equipments, researches using pulse magnets have become more and more popular since the late 1970s.

However, there is a limitation to the maximum field that can be generated by pulse magnets. The main reason is a large magnetic stress (Maxwell stress) exerted on the magnet due to the high magnetic fields. The Maxwell stress is proportional to the square of the field, and exceeds the mechanical strength of the magnet. Thus the magnets are inevitably destroyed if the field exceeds some limit. A great deal of effort is being made worldwide to generate the highest possible field non-destructively. So far, the maximum non-destructive field is limited to nearly 80 T. A number of laboratories in the world are now competing in the generation of higher magnetic fields aiming at 100 T.

Above this field level, we have to employ destructive techniques, explosive method, electromagnetic flux compression, the single turn coil technique, *etc.* The first attempt of the destructive technique was made at Los Alamos in USA and Alzamas-16 in the former Soviet Union (now known as Sarov in Russia) by using chemical explosives to compress the magnetic flux. Generation of very high magnetic fields up to 600 T or even higher has been reported. Although the explosive techniques can generate extremely high magnetic fields, the use of the generated fields is not so easy in ordinary experimental rooms because of the destructive nature. Electromagnetic flux compression or the single-turn coil technique is more suitable for the generation of megagauss fields, as they use capacitor banks as an energy source. At the Institute for Solid State Physics (ISSP) of the University of Tokyo, these two techniques have been developed since 1970 so that they may be applied to solid state experiments. By electromagnetic flux compression, magnetic fields up to 622 T have been generated. This is the highest field which has ever been generated by indoor experiments.

By these techniques mentioned above, fields above 100 T can now be obtained. As  $B = 100$  T corresponds to 1 MG (megagauss) in cgs units, magnetic fields higher than 100 T are often called "megagauss fields". Except for the single turn coil technique among different megagauss generating techniques, samples are also destroyed at every shot. The rise time or the duration time of the field is a few microseconds. Although the time is very short, the present measuring techniques enable us to obtain high accuracy data with good signal to noise ratio in these high fields. Thus generated megagauss fields have been conveniently employed in many different kinds of experiments and many new phenomena have been observed in a variety of substances.

More details of these techniques are given in Chapter 7. The state of art of the magnet technology world-wide is seen in a recently published reference [1].

This book is devoted to describing the fundamental physics of semiconductors in high magnetic fields and to introduce interesting new phenomena actually observed in recently available very high magnetic fields. The topics include transport phenomena, cyclotron resonance and related phenomena, magneto-optical spectroscopy, and diluted magnetic semiconductors. Each chapter includes mostly experimental data obtained in these very high magnetic fields, up

to 40–60 T with long pulse magnets, up to  $\sim 260$  T with the single turn coil technique, and up to  $\sim 600$  T with electromagnetic flux compression, in addition to typical basic data of high field physics of semiconductors. In many of the figures in this book, readers will be impressed by the scale of the magnetic fields usually represented on the horizontal axis. In the following chapters, you will be invited to exploit this fantastic world of high magnetic fields.

## ELECTRONIC STATES IN HIGH MAGNETIC FIELDS

In this chapter, we overview the electronic states in high magnetic fields for mobile electrons and bound electrons. This provides an important basis for understanding the contents of the subsequent chapters. Readers who are already familiar with these basics can skip this chapter and jump to Chapter 3.

### 2.1 Free electrons in magnetic fields

First, let us consider the motion of free electrons in magnetic fields and their kinetic energy. The Hamiltonian of free electrons is given by

$$\mathcal{H} = \frac{1}{2m} \mathbf{p}^2, \quad (2.1)$$

where the momentum  $\mathbf{p}$  is represented by

$$\mathbf{p} = m\mathbf{v}. \quad (2.2)$$

The energy eigenvalue of Eq. (2.1) is

$$\mathcal{E} = \frac{p^2}{2m}. \quad (2.3)$$

When a magnetic field  $H$  is applied, free electrons conduct a cyclotron motion. The classical kinetic equation for electrons in an electric field  $E$  and a magnetic flux density  $B = \mu_0 H$  is

$$m\ddot{\mathbf{r}} = -e\mathbf{E} - e(\mathbf{v} \times \mathbf{B}). \quad (2.4)$$

If  $E = 0$  and  $B$  is parallel to the  $z$ -axis, this leads to

$$\begin{aligned} m \frac{d^2 x}{dt^2} &= -eBv_y, \\ m \frac{d^2 y}{dt^2} &= +eBv_x. \end{aligned} \quad (2.5)$$

The solution of (2.5) is represented as follows:

$$\begin{cases} x(t) = X + \xi \\ y(t) = Y + \eta, \end{cases} \quad (2.6)$$

with constants  $X$  and  $Y$ , and



$$\begin{cases} \xi(t) = r_c \cos \omega_c(t - t_0) \\ \eta(t) = r_c \sin \omega_c(t - t_0) \end{cases} \quad (2.7)$$

Here,  $(X, Y)$  is the center of the cyclotron motion, and  $(\xi, \eta)$  is the relative co-ordinate around the center.

$$\omega_c = \frac{eB}{m} \quad (2.8)$$

is the frequency of the cyclotron motion called the cyclotron frequency. The kinetic energy of the electron in the plane perpendicular to  $\mathbf{B}$  is given by

$$\mathcal{E}_\perp = \frac{1}{2}mv^2, \quad (2.9)$$

and the radius of the orbit is

$$r_c = \frac{mv}{eB}. \quad (2.10)$$

In quantum mechanics, the energy of the cyclotron motion is quantized. The magnetic flux density  $B$  and electric fields are represented by a vector potential  $\mathbf{A}$  and scalar potential  $V$  as

$$\mathbf{B} = \nabla \times \mathbf{A} = \text{rot} \mathbf{A}, \quad (2.11)$$

and

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V. \quad (2.12)$$

It should be noted that  $\mathbf{A}$  and  $V$  cannot be uniquely determined to give the same  $\mathbf{B}$  and  $\mathbf{E}$ . We can easily show that when  $\mathbf{A}$  and  $V$  are the potentials that satisfy (2.11) and (2.12),  $\bar{\mathbf{A}}$  and  $\bar{V}$  with a gauge transformation

$$\begin{aligned} \bar{\mathbf{A}} &= \mathbf{A} + \nabla \lambda \\ \bar{V} &= V - \frac{\partial \lambda}{\partial t} \end{aligned} \quad (2.13)$$

also give the same values of  $\mathbf{B}$  and  $\mathbf{E}$ . This provides unique phenomena in connection with the gauge invariance in a magnetic field.

Putting the Lagrangian as

$$\mathcal{L} = \frac{mv^2}{2} + eV - e(\mathbf{v} \cdot \mathbf{A}), \quad (2.14)$$

we obtain a kinetic equation

$$\frac{d}{dt} \frac{\partial \mathcal{L}(q_i, \dot{q}_i)}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}(q_i, \dot{q}_i)}{\partial q_i} = 0, \quad (2.15)$$

from which Eq. (2.4) is derived [GT-2]. The Hamiltonian is

$$\mathcal{H}(p_i, q_i) = \sum_i (\dot{q}_i p_i - \mathcal{L}). \quad (2.16)$$