Optimal Combining and Detection

Statistical Signal Processing for Communications

Jinho Choi

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Optimal Combining and Detection

Statistical Signal Processing for Communications

With signal combining and detection methods now representing a key application of signal processing in communication systems, this book provides a range of key techniques for receiver design when multiple received signals are available. Various optimal and suboptimal signal combining and detection techniques are explained in the context of multiple-input multiple-output (MIMO) systems, including successive interference cancellation (SIC) based detection and lattice reduction (LR) aided detection. The techniques are then analyzed using performance analysis tools. The fundamentals of statistical signal processing are also covered, with two chapters dedicated to important background material. With a carefully balanced blend of theoretical elements and applications, this book is ideal for both graduate students and practicing engineers in wireless communications.

JINHO CHOI is currently a Professor in the School of Engineering, and Chair of the Wireless Group, at Swansea University, UK. He is the author of *Adaptive and Iterative Signal Processing in Communications* (Cambridge University Press, 2006) and the recipient of the 1999 Best Paper Award for Signal Processing from EURASIP. A Senior Member of the IEEE, his current research interests include wireless communications and array/statistical signal processing.

Preface

Statistical signal processing is a set of statistical techniques that have been developed to deal with random signals in a number of applications. Since it is rooted in detection and estimation theory, which are well established in statistics, the fundamentals are not changed although new applications have emerged. Thus, I did not have any strong motivation to write another book on statistical signal processing until I was convinced that there was a sufficient amount of new results to be put together with fundamentals of detection and estimation theory in a single book.

These new results have emerged in applying statistical signal processing techniques to wireless communications since 1990. We can consider a few examples here. The first example is smart antenna. Smart antenna is an application of array signal processing to cellular systems to exploit spatial selectivity for improving spectral efficiency. Using antenna arrays, the spatial selectivity can be used to mitigate incoming interfering signals at a receiver or control the transmission direction of signals from a transmitter to avoid any interference with the receivers which do not want to receive the signal. The second example is based on the development of code division multiple access (CDMA) systems for cellular systems. In CDMA systems, multiple users are allowed to transmit their signals simultaneously with different signature waveforms. The matched filter can be employed to detect a desired signal with its signature waveform. This detector is referred to as the single-user detector as it only detects one user's signal. Although this single-user detector is able to provide a reasonable performance, it is also possible to improve the performance to detect multiple signals simultaneously. This detector is called the multiuser detector. The third example is multiple-input multiple-output (MIMO) systems. In MIMO systems, multiple signals are transmitted and multiple signals are received. Thus, it is required to detect multiple signals simultaneously. These new applications promote advances of statistical signal processing. In particular, new and advanced techniques for signal combining and detection have emerged.

This book is intended to provide fundamentals of signal detection and estimation together with new results that have been developed for the new applications mentioned above.

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Finally, I would like to offer very special thanks to my wife, Kila, and children, Seji and Wooji, for their generous support, understanding, and love.

Symbols

General

$j = \sqrt{-1}$

$\mathbb{F}_2$: binary field

$\mathbb{Z}$: set of integer numbers

$\mathbb{R}^n$: real-valued n-dimensional vector space

$\mathbb{C}^n$: complex-valued n-dimensional vector space

$\times$: Cartesian product (if it does not mean the product)

$|\mathcal{A}|$: cardinality of set $\mathcal{A}$ or the number of the elements in $\mathcal{A}$

$\emptyset$: empty set

$\cup$: set union

$\cap$: set intersection

$\setminus$: set difference or set-minus

$\mathcal{A}^c$: the complementary set of set $\mathcal{A}$

$u(x)$: step function

$\delta(x)$: Dirac delta function

Statistics related symbols

$f_X(x)$: pdf of random variable X

$F_X(x)$: cdf of random variable X

$\Pr(\mathcal{A})$: probability of random event $\mathcal{A}$

$\mathcal{E}[X]$: statistical expectation of X

$\mathrm{Var}(X)$: variance of X

$\mathcal{Q}(x)$: Q-function, $\mathcal{Q}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{z^2}{2}} \, \mathrm{d}z$

$\mathcal{N}(\mathbf{x}, \mathbf{R})$: Gaussian probability density function with mean $\mathbf{x}$ and covariance $\mathbf{R}$

$\mathcal{CN}(\mathbf{x}, \mathbf{R})$: circularly symmetric complex Gaussian probability density function with mean $\mathbf{x}$ and covariance $\mathbf{R}$

Vector/Matrix related symbols

$||\cdot||_p$: p-norm

$||\cdot||_\mathrm{F}$: Frobenius norm

$(\cdot)^\mathrm{T}$: transpose

$(\cdot)^\mathrm{H}$: Hermitian transpose

$\det(\cdot)$: determinant of a square matrix

$\mathrm{tr}(\cdot)$: trace of a square matrix

$\mathrm{Diag}(a_1, a_2, \ldots, a_N)$: $N \times N$ diagonal matrix whose elements are $a_1, a_2, \ldots, a_N$

$[\mathbf{a}]_n$: nth element of a vector $\mathbf{a}$
$[\mathbf{A}]_{m,n}$: (m, n)th element of a matrix $\mathbf{A}$
$[\mathbf{A}]_{m_1:m_2,n_1:n_2}$: a submatrix of $\mathbf{A}$ obtained by taking the elements in the m_1th to m_2th columns and the n_1th to n_2th rows
$[\mathbf{A}]_{:,n}$: nth column vector of $\mathbf{A}$
$[\mathbf{A}]_{n,:}$: nth row vector of $\mathbf{A}$

Abbreviations

AR	autoregressive
ARV	array response vector
ASK	amplitude shift keying
AWGN	additive white Gaussian noise
BER	bit error rate
BSC	binary symmetric channel
cdf	cumulative distribution function
CDMA	code division multiple access
CLT	central limit theorem
CRB	Cramer–Rao Bound
CSCG	circularly symmetric complex Gaussian
CVP	closed vector problem
DFE	decision feedback equalizer
DMC	discrete memoryless channel
DMI	direct matrix inversion
DPSK	differential phase shift keying
EGC	equal gain combining
FA	false alarm
GLR	generalized likelihood ratio
GLRT	generalized likelihood ratio test
GSDC	generalized selection diversity combining
iid	independent and identically distributed
ISI	intersymbol interference
LCMV	linearly constrained minimum variance
LLR	log-likelihood ratio
LMS	least mean square
LR	lattice reduction or likelihood ratio
LS	least square
MAC	multiple access channel
MAP	maximum a posteriori probability
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
ML	maximum likelihood
MLE	maximum likelihood estimate

MMSE	minimum mean square error
MRC	maximal ratio combining
MSE	mean square error
MSNR	maximum signal to noise ratio
MUSIC	multiple signal classification
MVDR	minimum variance distortionless response
MVUE	minimum variance unbiased
PAM	pulse amplitude modulation
pdf	probability density function
PEP	pairwise error probability
QAM	quadrature amplitude modulation
QPSK	quadrature phase shift keying
RLS	recursive least square
ROC	receiver operating characteristics
SD	selection diversity
SDR	software defined radio
SLLN	strong law of large numbers
SMI	sample matrix inversion
SIC	successive interference cancellation
SIMO	single-input multiple-output
SISO	single-input single-output
SINR	signal to interference-plus-noise ratio
SNR	signal to noise ratio
SVP	shortest vector problem
ULA	uniform linear array
WLLN	weak law of large numbers
WLS	weighted least square
WSS	wide-sense stationary
ZF	zero-forcing

Contents

Preface — *page* xi
List of symbols — xiii
List of abbreviations — xv

1 Introduction — 1

1.1 Applications in digital communications — 1
1.2 Detection problems — 2
1.3 Combining problems — 3
1.4 Background — 5
1.4.1 Review of probability — 5
1.4.2 Random variables — 6
1.4.3 Random processes — 9

2 Fundamentals of detection theory — 10

2.1 Elements of hypothesis testing — 10
2.1.1 MAP decision rule — 11
2.1.2 Bayesian decision rule — 15
2.1.3 ML decision rule — 17
2.1.4 Likelihood ratio (LR) based decision rule — 18
2.2 Neyman–Pearson lemma — 19
2.3 Symmetric signal detection — 21
2.3.1 Error probability — 22
2.3.2 Bounds on error probability for Gaussian noise — 23
2.3.3 Chernoff bound — 24
2.4 Binary waveform signal detection — 25
2.4.1 Detection of waveform signals — 26
2.4.2 Correlator detector and performance — 27
2.5 M-ary signal detection — 30
2.6 Signal detection in vector space — 31
2.6.1 Karhunen–Loéve expansion — 31
2.6.2 Signal detection in vector space — 33
2.6.3 Pairwise error probability and Bhattacharyya bound — 35

2.7 Signal detection with random parameters 38
2.7.1 LR-based detection with random parameters 38
2.7.2 Signals with random amplitude 39
2.7.3 Signals with random phase 40
2.7.4 Detection of random Gaussian vector 43
2.8 Summary and notes 44
Problems 45

3 Fundamentals of estimation theory 49

3.1 Least square and recursive least square estimation 49
3.1.1 Least square estimation 49
3.1.2 Recursive least square approach 53
3.2 Minimum variance unbiased and best linear unbiased estimation 56
3.3 Maximum likelihood estimation 61
3.4 Cramer–Rao bound 64
3.5 MAP estimation 69
3.6 MMSE estimation 70
3.6.1 MSE cost with linear estimator 71
3.6.2 Conditional mean and Gaussian vectors 72
3.6.3 Geometrical interpretation 73
3.7 Example: channel estimation 75
3.8 Summary and notes 76
Problems 77

4 Optimal combining: single-signal 79

4.1 Signals in space 79
4.2 MMSE combining with known channel vector 82
4.3 Information-theoretical optimality of MMSE combining 84
4.4 Relation with other combining techniques 88
4.4.1 MSNR combining 88
4.4.2 ML combining 89
4.4.3 Maximal ratio and equal gain combining 91
4.4.4 Generalized selection diversity combining 93
4.5 MMSE combining with unknown channel vector 93
4.6 Performance of optimal combining under fading environment 95
4.6.1 Performance of MRC 95
4.6.2 Performance of SD 99
4.6.3 Performance over spatially correlated channels 101
4.7 Summary and notes 103
Problems 103

5	**Array signal processing and smart antenna**	106
	5.1 Antenna arrays	106
	5.1.1 Plane wave model	106
	5.1.2 Array response vector	109
	5.2 AoA estimation	111
	5.2.1 ML approaches	112
	5.2.2 Subspace approaches	116
	5.2.3 Specialized algorithms for ULA	123
	5.3 Beamforming methods	124
	5.3.1 MMSE beamforming	125
	5.3.2 MSINR beamforming	128
	5.3.3 MVDR beamforming	128
	5.3.4 Generalized MVDR beamformer	130
	5.4 Adaptive beamforming	131
	5.4.1 LMS algorithm	132
	5.4.2 Frost's adaptive algorithm	135
	5.5 Smart antenna systems	135
	5.5.1 Interference suppression in cellular systems	135
	5.5.2 Space division multiple access	136
	5.5.3 Adaptive array and switched-beam	137
	5.5.4 Practical issues	137
	5.5.5 Smart antenna for signal transmission	138
	5.6 Summary and notes	141
	Problems	141
6	**Optimal combining: multiple-signal**	143
	6.1 Systems with multiple signals	143
	6.2 Structures of receiver	144
	6.3 Various combining approaches	146
	6.3.1 Matched filtering approach	147
	6.3.2 Zero-forcing approach	148
	6.3.3 MMSE approach	151
	6.3.4 ML and MAP combining	153
	6.4 Properties of MMSE combining	155
	6.4.1 Individual MMSE combining	155
	6.4.2 Biased and unbiased SINR	156
	6.4.3 Joint MMSE combining	157
	6.5 Dimension and suppression capability	159
	6.6 Asymptotic analysis for random signature vectors	161
	6.7 Mutual information with MMSE combiner	167
	6.7.1 Mutual information of joint MMSE	167
	6.7.2 Mutual information of individual MMSE	170

6.8 Summary and notes 172
Problems 174

7 Multiple signal detection in vector space: MIMO detection 176

7.1 SIMO detection 176
7.1.1 ML and MAP detection 177
7.1.2 Soft-decision using LLR 177
7.1.3 Optimality of MMSE combiner 180
7.2 Performance of optimal SIMO detection 182
7.2.1 An exact closed-form for PEP 184
7.2.2 Chernoff bound 185
7.2.3 Approximate PEP based on Gauss–Chebyshev quadrature 186
7.3 Optimal and suboptimal MIMO detection 188
7.3.1 MIMO systems 189
7.3.2 ML and MAP detection 192
7.3.3 Gaussian approximation 196
7.3.4 Linear detection 198
7.4 Space-time codes 200
7.5 Performance of detectors 202
7.5.1 Performance of linear detectors 202
7.5.2 Performance of ML detector 208
7.6 Summary and notes 214
Problems 214

8 MIMO detection with successive interference cancellation 217

8.1 Successive interference cancellation 217
8.2 SIC detectors 219
8.2.1 ZF-SIC detector 220
8.2.2 MMSE-SIC detector 222
8.3 Square-root algorithm for MMSE-SIC detector 224
8.4 Principle of partial MAP detection 229
8.5 Performance analysis of SIC detection 233
8.6 Approaches to mitigate error propagation 235
8.6.1 Ordering 235
8.6.2 SIC-list detection 238
8.6.3 Channel coding 238
8.7 SIC receiver for coded signals 239
8.7.1 Layered transmission for SIC receiver 239
8.7.2 Information-theoretical optimality of MMSE-SIC receiver 242
8.8 Summary and notes 244
Problems 244

9 Lattice-reduction-aided MIMO detection 246

9.1 Lattices and signal constellations 246
9.1.1 Lattices 246
9.1.2 Signal constellation as a subset of lattice 248
9.2 MIMO detection over lattices 249
9.2.1 LR-based extended ML detection 250
9.2.2 LR-based ZF and MMSE detection 252
9.2.3 LR-based ZF-SIC and LR-based MMSE-SIC detection 254
9.2.4 LR-based ML detection 255
9.3 Application to MIMO precoding 257
9.3.1 System model 257
9.3.2 Precoders 258
9.4 Lattice reduction for two-dimensional lattices 261
9.4.1 QR factorization 261
9.4.2 Lattice basis reduction method 264
9.5 LLL algorithm: a lattice reduction algorithm 270
9.5.1 LLL reduced basis 270
9.5.2 LLL algorithm 272
9.5.3 Another version of LLL algorithm 274
9.6 Summary and notes 277
Problems 278

10 Analysis of LR-based MIMO detection 280

10.1 Assumptions for analysis 280
10.2 Analysis of two-dimensional systems 281
10.2.1 Performance analysis 281
10.2.2 Computational complexity 284
10.3 Performance of LR-based SIC detector for higher-dimensional lattices 285
10.3.1 Performance of the ML detector 286
10.3.2 Properties of LLL reduced basis 287
10.3.3 Diversity gain of LR-based SIC detector 291
10.4 Summary and notes 293
Problems 293

Appendix 1 Review of signals and systems 295
Appendix 2 A brief review of entropy, mutual information, and channel capacity 299
Appendix 3 Important properties of matrices and vectors 308
Appendix 4 Lattice theory 314
References 322
Index 328

1 Introduction

Statistical signal processing is a set of tools for dealing with random signals. As a set of tools, statistical signal processing has a broad range of applications from radars and sonars to speech and image processing. There are a number of books on this topic (e.g. (Scharf 1991) and (Orfanidis 1988)). In this book, instead of providing a comprehensive description of statistical signal processing with a broad range of applications, we focus on key approaches for communications. In particular, we attempt to present mainly signal detection and combining techniques in the context of wireless communications.

1.1 Applications in digital communications

The main aim of digital communications is to transmit a sequence of bits over a given channel to a receiver with minimum errors. In implementing digital communication systems, however, there are various constraints to be taken into account. For example, the transmission power is usually limited and the complexity of receiver is also limited. With practical implementation constraints including computational complexity, statistical signal processing plays a crucial role in designing a receiver for digital communications. Although there are a number of different roles that statistical signal processing can play, we confine ourselves to two main topics in this book: one is signal detection and the other is signal combining.

Signal detection has been well established as the main topic in communications. However, advances in multiuser detection have opened up a whole new approach for joint detection (Verdu 1998). In this book, we focus on optimal and suboptimal approaches for joint detection in the context of multiple-input multiple-output (MIMO) communications.

Signal combining is to combine multiple observations and plays a crucial role in both array signal processing and wireless communications. In particular, in wireless communications, signal combining is essential to mitigate fading at the receiver equipped with multiple receive antennas. Furthermore, signal combining can be generalized to mitigate interfering signals as it can provide spatial selectivity in smart antennas. In this book, we discuss signal combining based on this generalized view.

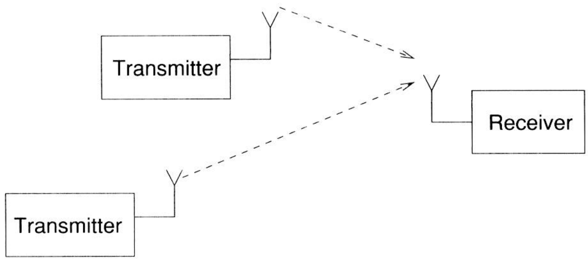

Figure 1.1 Multiuser communication.

1.2 Detection problems

Signal detection (Whalen 1971) (Kay 1998) is an application of statistical hypothesis testing in statistics. In statistical hypothesis testing, there are a finite number of hypotheses. With a set of observations, statistical hypothesis testing attempts to choose a hypothesis that explains observations best under a certain performance criterion. In signal detection, the set of hypotheses is decided by the signal alphabet for transmitted signals. Let $\mathcal{S}$ denote the signal alphabet. Then, the received signal over a memoryless channel at a receiver is given by

$$r = hs + n, \tag{1.1}$$

where $s \in \mathcal{S}$ is the transmitted signal, h is the channel gain, and n is the background noise. The receiver is to detect s from r provided that h is known and the statistical properties of n are also known. In general, the size of $\mathcal{S}$ is finite in digital communications. According to statistical hypothesis testing, signal detection is to choose a signal from $\mathcal{S}$, which can most likely generate r according to a certain criterion. Thus, it is important to define a detection criterion. Various decision criteria, which can be possibly employed for signal detection, have been proposed in statistical hypothesis testing. Some examples are the maximum likelihood (ML), Bayesian, maximum a posteriori probability (MAP) decision criteria. Since the theory of statistical hypothesis testing is well established in statistics, its application to signal detection is straightforward. In Chapter 2, we present an overview of detection theory based on the theory of statistical hypothesis testing.

While detection theory heavily relies on the theory of statistical hypothesis testing, the issues related to implementation with practical constraints are not quite covered by the theory. Most implementation issues are subject to a computational complexity constraint. To illustrate this issue, we can consider the signal detection problem in a multiuser communication system. If there are two users who transmit signals simultaneously as shown in Fig. 1.1, the received signal is given by

$$r = h_1 s_1 + h_2 s_2 + n, \tag{1.2}$$

where h_k and s_k are the channel gain and transmitted symbol from the kth user, respectively. For the signal detection in this case, which is called multiuser detection (Verdu 1998), we need to extend the signal alphabet as $\mathcal{S} \times \mathcal{S}$, where $\times$ denotes the