MAGNETOFLUID DYNAMICS

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Contents

Preface

This work represents the English translation of my book "Magnetodinamica fluidelor" published by Editura Academiei in Romanian in 1969. In its present form this book is however very different from its first edition. First of all it is organized into two well defined parts: the macroscopic and the microscopic theory respectively. Then some new chapters (e.g., chapters 9, 13, 14) have been introduced and the chapters on the theory of MHD generators and thin airfoil theory have been completely altered. All the chapters and the bibliography have been revised and completed. In its present form this work contains some unpublished results.

This book is based on the author's lectures given from 1962 onwards to the fourth and fifth year students of the Fluid Mechanics Section of the Faculty of Mathematics and Mechanics of the Bucharest University. However the book exceeds those lectures by far.

The first chapter postulates the electromagnetic field equations, the expression of force for the continuous media at rest and derives (after Minkowski) their expressions for moving media (the fluids investigated in magnetodynamics are moving continuous media). I wanted to make this introduction since most of the books on magnetofluid dynamics do not specify the conditions for which the electromagnetic force has the expression $\boldsymbol{J}\times\boldsymbol{B}$ *(which is usually utilized), the structural (or constitutive) equations of the field in moving media reduce themselves to* $\mathbf{D}=\varepsilon\mathbf{E}$, $\mathbf{B}=\mu\boldsymbol{H}$, $\boldsymbol{J}=\sigma(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B})$, *the energy equation which has an extremely complex form for moving media reduces itself to the simple form one generally uses, etc.*

After a short introduction in the kinematics of fluids, chapter 2 presents the magnetofluid dynamic equations in integral form and then derive the differential equations for continuous motion. One section is devoted to fluid thermodynamics and the energy equation. By writing the differential equations in dimensionless form one is able to indicate, among others, in what conditions one may use the simplified form of the equations (neglecting the displacement currents, the convection current in Ohm's law and the electric force term). One section deals with the conservation theorems and another with the existence and uniqueness theorems.

The classical problems of flow between parallel plates and through ducts are presented in Chapter 3: the Poiseuille-Hartmann problem (with Hall effect), the Couette problem and the problem of flow through rectangular ducts.

The theoretical model of the MHD generator in the most general conditions (finite electrodes, external magnetic field applied in the electrode region only, the MHD *interaction) is the subject of Chapter 4. The fluid is assumed to be incompressible and the motion to be steady and plane. The mathematical problem consists in integrating a system of second-order partial differential equations (the coefficients being discontinuous functions) with matching conditions along some jump lines and with mixed boundary conditions. Approximate series solutions which can be numerically calculated are given. The Hall effect is also considered.*

The following three chapters are devoted to the theory of thin airfoils: the perfectly conducting fluid theory (Chapter 5), the finite resistivity theory (Chapter 6) and the theory of motion with Hall effect (Chapter 7). Using the linearized theory the general solutions are derived and the lift is calculated pointing out the magnetic field influence. For the particular case of nonconducting fluids the results of classical aerodynamics are recovered.

In Chapter 8 the principal results on the flow of viscous fluids past a flat plate with and without incidence are presented. The integral equations of the problem are asymptotically integrated and the classical solutions are recovered as particular cases.

The next three chapters are devoted to the theory of waves : the first two deal with weak waves and the third with strong (shock) waves. Chapter 9 presents the (linear) analytical theory of perturbations and the solution of the Cauchy problem for perfectly conducting fluids and Chapter 10 the geometrical (non-linear) theory. In the latter one can also find the classical results on the (Friedrichs) wave-front diagrams and the simple wave motion. Finally in Chapter 11 the equations of shock phenomena are derived, their properties are discussed and their solution is presented. One section is devoted to the stability problem.

The second part of the book has the purpose of deriving the microscopic equations of magnetofluid dynamics. The electromagnetic field equations are derived in Chapter 12 from the Hamilton principle and the macroscopic equations are obtained on averaging. Chapter 13 is devoted to the kinetic theory of homogeneous gases, to the derivation of Boltzmann's equation and the conservation equations. In Chapter 14 the conservation equations of (ionized) gas mixtures and Ohm's law are derived. Various types of plasmas are considered.

We have especially insisted upon the mathematical model of magnetofluid dynamics in various specific cases since the corresponding boundary value problems are one of the most complex. Indeed since the magnetic field extends all over the space the problems do not close upon themselves within the domains occupied by the fluid but are connected to other problems (other equations) from the adjacent regions. That is why I attempted to formulate as completely as possible the boundary value problems.

I do not pretend to have presented in this book all the aspects of magnetofluid dynamics (as a matter of fact this would be too great a task for any single research worker). I only intended to write an introduction to this science and to present some of its most significant results. This work also contains some of the author's results like those concerning the MHD generator theory (the whole Chapter 4), those regarding the thin airfoil theory (Chapters 6 and 7 completely and part of Chapter 5), those on the motion of viscous fluids past the flat plate (Chapter 8) and the results on the propagation of small perturbations and the solution of the Cauchy problem (Chapter 9).

I have attempted to make up for some omissions in the work by presenting an extensive bibliography. The references were grouped together according to the chapters, the first papers being noted as they were mentioned in the text; I have then included (in chronological order) some other references relevant to the corresponding chapter. Works of general character are presented (in chronological order) in the General Bibliography and are indicated in the text by the letter G followed by a number. I have adopted a chronological presentation in order to obtain a better image on the progress of the various fields of research.

The references are indicated by one number if the paper quoted belongs to the bibliography of that chapter and by two numbers (the first indicating the chapter and the second the paper) if it belongs to another chapter.

I have written this book following the advice of Prof. C. Iacob and for this I would like to express my gratitude. I would like to thank Drs. N. Marcov, D. Homentcovschi and L. Dinu for their help in writing this book.

The author would welcome any critical observations regarding the book.

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Introduction

Magnetofluid dynamics deals with the study of the motion of electrically conducting fluids in the presence of magnetic fields. In such a problem the magnetic field influences the fluid motion (this influence is expressed mathematically by including the electromagnetic force in the equations of motion) and the fluid motion changes in turn (through Ohm's law) the magnetic field. Therefore an interaction of the electromagnetic field with the fluid motion takes place and this interaction determines the simultaneous consideration of the fluid mechanics equations and the electromagnetic field equations. Then magnetofluid dynamics represents a synthesis of two classical sciences: fluid mechanics and electromagnetic field theory. At the same time, in order to study the motion of compressible fluids, one should take into account their thermodynamic state and thus the object of this new science turns out to be the study of the phenomena related to fluid motion in all their complexity.

From the historical point of view the first investigations in this field are due to Faraday (1836). As an effect of the action of tides the Thames River flows in both directions and thus has salt waters which are electrically conducting. Faraday attempted to detect the electric field induced by the motion of the conducting waters in the Earth's magnetic field by connecting two electrodes situated near the two banks of the river. But due to the very low conductivity of the fluid the results were hardly discernible and Faraday gave up the idea.

A second important stage in the development of this science is represented by the experiments of Hartmann and Lazarus in 1937 who pointed out the magnetic field influence on the motion of a fluid. Mercury flowing through a tube in the presence of a strong magnetic field was used as conducting fluid. Although the experimental results confirmed the theoretical studies no practical reason for continuing these investigations existed at that time.

The first genuine magnetofluid dynamic problems have been posed by astrophysics. So in 1940 the Swedish scientist H. Alfvén proved that new waves, unknown to both fluid mechanics and electromagnetism can be propagated through a conducting fluid in a magnetic field.

But the first important impetus in the development of this science was the discovery of the plasma and its production on the laboratory scale. As it is now well known, the plasma state represents the manner in which matter exists at high temperatures. A plasma is electrically conducting and has the property of maintaining its overall charge neutrality. At sufficiently high temperatures (of the order of thousands of degrees) matter cannot organize itself into electrically neutral entities (even if it has been so organized the entities would be destroyed) since the thermal energy per particle is high enough to break up these entities through frequent collisions. In such conditions the electric field created by various groups cannot capture enough particles to ensure neutralization. Therefore plasma is a mixture of positively charged, negatively charged and maybe neutral particles. In a magnetic field the charged particles of a plasma are oriented after the field lines of force. Plasma is consequently a compressible, continuous (or discrete), electrically conducting medium. It is obvious that the study of the motion of such a medium represents the object of magnetofluid dynamics. Various other fields of investigation (plasma physics, plasma chemistry, etc.) deal with other aspects of this state of matter.

A widespread belief that most of the matter in the Universe is in the plasma state exists at the present time. Especially interstellar, stellar and solar matter is supposed to exist in such a form. It follows that any astrophysical and astronautical investigations should be based on this new science. The cosmical flight of a satellite represents a motion of a body through a conducting medium in the presence of a magnetic field. But we have a magnetoaerodynamic problem even in the hypersonic-velocity aerodynamics since the gas in front of the moving body is heated by friction and becomes a conductor.

Under normal conditions the air is a weakly conducting medium since it is ionized by radioactive substances and cosmic radiation, but its electrical conductivity does not exceed 0.1 mho cm^{-1} (for the conditions in which a satellite flight takes place). The terrestrial magnetic field is also very weak. At large distances its distribution may be assumed to be that of a dipole of magnetic moment 8×10^{25}CGS units situated at the centre of the Earth. However, by an artificial increase of the gas conductivity near the satellite body and an additional magnetic field properly oriented, one can produce magnetodynamic effects and use them even in this case.

The feasibility of two major projects related to the vital energy problem depends on the results of magnetofluid dynamics: they regard the achievement of controlled thermonuclear reactions and the direct conversion of thermal energy to electrical energy by means of the magnetohydrodynamic generator. The former implies investigations on the stability and turbulence of plasmas and the latter implies studies on the motion of ionized gases through ducts.

These are only a few reasons which prove the necessity of further developing the science of magnetofluid dynamics.

Part 1

The macroscopic theory