Second Edition / Teacher's Edition

Algebra and Trigonometry

Functions and Applications

Paul A. Foerster

Addison-Wesley Publishing Company

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Paul A. Foerster has taught mathematics at Alamo Heights High School in San Antonio, Texas since 1961. In that same year he received his teaching certificate from Texas A&M University. His B.S. degree in Chemical Engineering and M.A. degree in Mathematics are from the University of Texas. Among many honors, he was awarded the Presidential Award for Excellence in Mathematics in 1983.

To A. W. Foerster, who helped me to understand the real world,
to Admiral Rickover, who taught me how to write about it, and
to Jo Ann, who helps me to live in it.

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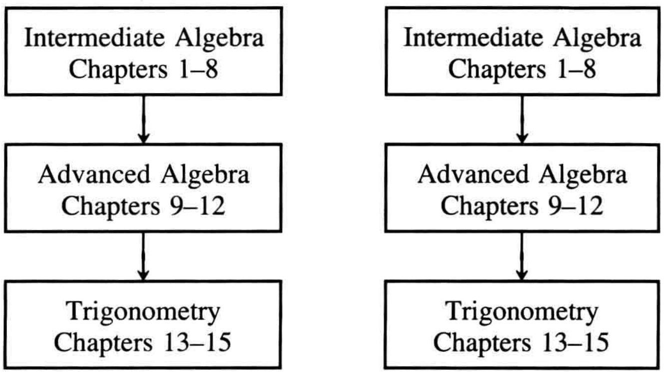
Foreword

Algebra and Trigonometry—Functions and Applications is designed for a course in intermediate algebra, advanced algebra, and trigonometry. The book can be used in two different ways:

1. As an algebra and trigonometry book, with applications,
2. As an applications book, with supporting algebra and trigonometry.

In either case, there are two possible sequences of presentation:



Applications are handled by creating mathematical models of phenomena in the real world. Students must select a kind of function that fits a given situation, and derive an equation that suits the information in the problem. The equation is then used to predict values of y when x is given or values of x when y is given. Sometimes students must use the results of their work to make interpretations about the real world, such as what "slope" means, or why there cannot be people as small as in Gulliver's Travels. The problems require the students to use *many* mathematical concepts in the *same* problem. This is in contrast to the traditional "word problems" of elementary algebra, in which the same one concept is used in many problems.

The second edition differs from the first and revised editions in two significant ways—technology and pedagogy. The computer is assumed to be a normal part of students' classroom experience rather than simply a novelty. For the most part, students are expected to use existing computer programs, notably computer graphic programs, rather than write their own. A disk acompanying the Teacher's Resource Book contains programs written by the author that are sufficient for the purposes of this text. You are, however, urged to seek out commercially-available software that is faster and more user-friendly.

Ideas for using calculators have caused substantial differences in presentation of certain topics. For instance, exponential equations are solved at the beginning of Chapter 6 by iterative methods on the calculators. Logarithms then arise naturally as a quicker way to get the unknown exponent. Presented this way, there is no doubt in the students' minds that a logarithm is an exponent! The calculator thus leads to a better understanding of theoretical concepts and is not simply a way to work old problems quicker.

Pedagogically, there are far more opportunities for review of previous concepts. One major way this review is accomplished is through the "Do These Quickly" problems at the beginning of each problem set. Once the students have learned to work a particular kind of problem, they develop speed as these problems reappear in five-minute exercises that concentrate only on answers. Mathematical-model problems have been spread out so the students must recall how to use linear and quadratic functions while they are working exponential model problems.

The topics in the text itself are largely the same as in the previous editions. Work on data analysis has been added in some problem sets to show students how to decide which kind of function is an appropriate model. Material on statistics, including the normal distribution, appears in the probability chapter.

The text still starts with a brief review of the basic axioms and properties. It moves quickly to topics the students have probably not seen before, at least in the method of presentation. The purpose is two-fold. First, students should not feel as if most of the course is spent reviewing elementary algebra. Second, the presentation emphasizes the role of algebra and trigonometry as the foundation for calculus, rather than as the completion of elementary algebra. By presenting both algebra and trigonometry as the study of classes of functions, students learn the essential unity of the two subjects.

Henry Pollack of Bell Telephone Labs claims that there are just two kinds of numbers: "real" numbers such as encountered in everday life, and "fake" numbers such as encountered in most mathematics classes! Since this book has many problems involving untidy decimals ("real" numbers), a calculator or computer are called for where appropriate. There are also problems that have small-integer answers ("fake" numbers) so that students may gain confidence in their work when they are just learning a new technique.

Since all educators share the responsibility of teaching students to read and write, there are discovery exercises so that students may wrestle with a new concept before it is reinforced by classroom discussion. The students are helped with this reading by the fact that much of the wording came from the mouths of my own students. Special thanks go to Susan Cook, Brad Foster, and Nancy Carnes, whose good class notes supplied input for certain sections. Students Lewis Donzis and David Frey wrote computer programs for some of the problems.

Thanks go to instructors in Florida, Illinois, Pennsylvania, South Dakota, Texas, and Virginia for pilot testing the original materials. Special thanks go to Bob Enenstein and his instructors in California for pilot testing the second edition. The text reflects comments from review and classroom testing by Charley Brown, Sharon Sasch Button, Pat Causey, Loyce Collenback, Bob Davies, Walter DeBill, Rich Dubsky, Michelle Edge, Sandra Frasier, Byron Gill, Pat Johnson, Michael Keeton, Carol Kipps, Bill McNabb, Shirley Scheiner, Ann Singleton, Chuck Straley, Rhetta Tatsch, Joel Teller, Susan Thomas, Kay Thompson, Zalman Usiskin, Jim Wieboldt, Marv Wielard, Mercille Wisakowsky, Martha Zelinka, and Isabel Zsohar. Calvin Butterball and Phoebe Small appear with the kind permission of their parents, Richard and Josephine Andree.

Paul A. Foerster

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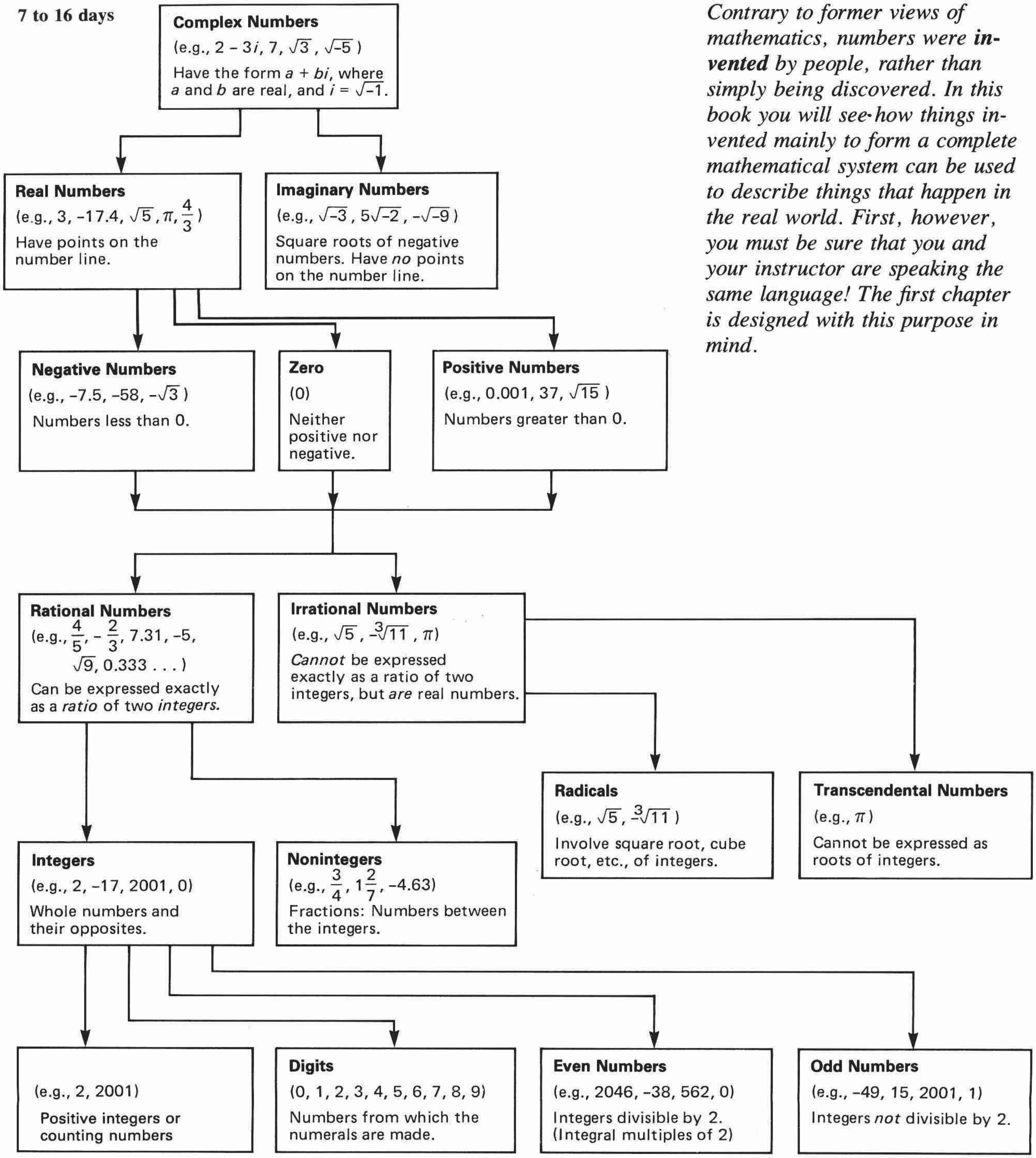
1 Preliminary Information

7 to 16 days

Contrary to former views of mathematics, numbers were ***invented*** *by people, rather than simply being discovered. In this book you will see·how things invented mainly to form a complete mathematical system can be used to describe things that happen in the real world. First, however, you must be sure that you and your instructor are speaking the same language! The first chapter is designed with this purpose in mind.*



Contents

1-1 SETS OF NUMBERS

From previous work in mathematics you should recall the names of different kinds of numbers (positive, even, irrational, etc.). In this section you will refresh your memory so that you will know the exact meaning of these names.

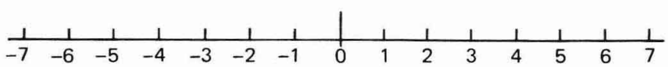
Objective:

Given the name of a set of numbers, provide an example; or given a number, name the sets to which it belongs.

There are two major sets of numbers you will deal with in this course, the *real* numbers and the *imaginary* numbers. The real numbers are given this name because they are used for "real" things such as measuring and counting. The imaginary numbers are square roots of negative numbers. They are useful, too, but you must learn more mathematics to see why.

The real numbers are all numbers which you can plot on a number line (see Figure 1-1). They can be broken into subsets in several ways. For instance, there are positive and negative real numbers, integers and non-integers, rational and irrational real numbers, and so forth. The diagram facing this page shows some subsets of the set of real numbers.

The numbers in the diagram were invented in *reverse* order. The natural (or "counting") numbers came first because mathematics was first used for counting. The negative numbers (those less than zero) were invented so that there would always be answers to subtraction problems. The rational numbers were invented to provide answers to division problems, and the irrational ones came when it was shown that numbers such as $\sqrt{2}$ could not be expressed as a ratio of two integers.



The real number line

Figure 1-1

Time: 1 day

Chalkboard Examples:

The key to success seems to be clear understanding of the word "rational" number. Two features are important:

1. *Rational* means *ratio* of two *integers*.
2. The numeral need not *be* a ratio of two integers, just as long as it *can* be expressed that way. For example, 5, $-\sqrt{49}$, $1\frac{2}{3}$, etc., are rational numbers. Also, not every ratio of two integers is a rational number, for example $\frac{2}{0}$.

Irrational—Can't be expressed as a ratio of two integers.
Transcendental—Can't be expressed even with radicals.

In both cases it is important for the students to realize that these are *real* numbers. For example, one does not even *consider* whether $\sqrt{-49}$ is rational or irrational since it is not a real number to begin with!

Assignment:

Assign all 9 problems.

Problem Notes:

Problem 3 (the table) is tedious to grade. However, it makes an excellent teaching tool if a copy of the table is projected on the screen. After the students have worked the problem, you can quickly check the boxes, a row at a time. You call out the *kind* of number, and they respond with "Yes," or "No. When you hear wrong answers, stop and talk!

Problem 8—Most students readily accept 2.333. . . as equaling $2\frac{1}{3}$, a rational number. If they want a more convincing proof, try the following:

Let $x = 2.333\ldots$
Then $10x = 23.333\ldots$
$\therefore 10x - x = 23.333\ldots - 2.333\ldots$
$9x = 21$
$x = \frac{21}{9} = \frac{7}{3} = 2\frac{1}{3}$, a rational number.

In Section 11-6 they will use convergent geometric series to prove that *any* repeating decimal represents a rational number.

Other operations you will invent, such as taking logarithms and cosines, lead to irrational numbers which go beyond even extracting roots. These are called "transcendental" numbers, meaning "going beyond." When all of these various kinds of numbers are put together, you get the set of *real* numbers. The imaginary numbers were invented because no real number squared equals a negative number. Later, you will see that the real and imaginary numbers are themselves simply subsets of a larger set, called the "complex numbers."

The following exercise is designed to help you accomplish the objectives of this section.

EXERCISE 1-1

1. Write a definition for each of the following sets of numbers. Try to do this *without* referring to the diagram opposite page 1. Then look to make sure you are correct.

a. {integers}	b. {digits}
c. {even numbers}	d. {positive numbers}
e. {negative numbers}	f. {rational numbers}
g. {irrational numbers}	h. {imaginary numbers}
i. {real numbers}	j. {natural numbers}
k. {counting numbers}	l. {transcendental nos.}

2. Write an example of each type of number mentioned in Problem 1.

3. Copy the chart at right. Put a check mark in each box for which the number on the left of the chart belongs to the set across the top.

4. Write another name for {natural numbers}.

5. Which of the sets of numbers in Problem 1 do you suppose was the *first* to be invented? Why?

6. One of the sets of numbers in Problem 1 contains all but one of the others as subsets.
   a. Which one *contains* the others?
   b. Which one is left out?

7. Do decimals such as 2.718 represent *rational* numbers or *irrational* numbers? Explain.

8. Do repeating decimals such as 2.3333 . . . represent *rational* numbers or *irrational* numbers? Explain.

9. What real number is neither positive nor negative?

1. a. whole numbers, positive, negative, and 0
b. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
c. integers divisible by 2
d. numbers greater than 0
e. numbers less than 0
f. numbers expressible as a ratio of two integers
g. numbers not expressible as a ratio of two integers
h. square roots of negative numbers
i. numbers on the number line
j. positive integers
k. positive integers
l. numbers not expressible using only a finite number of the operations $+$, $-$, $\times$, $\div$, or $\sqrt[n]{\ }$ on integers

2. Examples may vary.
a. 71 **g.** $\sqrt{3}$
b. 3 **h.** $\sqrt{-9}$
c. -58 **i.** 56.732
d. 17.23 **j.** 1027
e. $-9\frac{2}{3}$ **k.** 19
l. π
f. $5\frac{3}{8}$

3.
4. {counting numbers}
5. {natural numbers} counting was probably the first thing done with numbers
6. a. {real numbers}
b. {imaginary numbers}
7. rational
8. rational
9. 0

	Integers	Digits	Even Numbers	Positive Numbers	Negative Numbers	Rational Numbers	Irragional Numbers	Imaginary Numbers	Real Numbers	Natural Numbers	Counting Numbers	Transcendental Numbers
a. 5	✓	✓		✓		✓			✓	✓	✓	
b. 2/3				✓		✓			✓			
c. -7	✓				✓	✓			✓			
d. $\sqrt{3}$				✓			✓		✓			
e. $\sqrt{16}$	✓	✓	✓	✓		✓			✓	✓	✓	
f. $\sqrt{-16}$								✓				
g. $\sqrt{-15}$								✓				
h. 44	✓		✓	✓		✓			✓	✓	✓	
i. π				✓			✓		✓			✓
j. 1.765				✓		✓			✓			
k. -10000	✓				✓	✓			✓			
l. $-1\frac{1}{2}$					✓	✓			✓			
m. $-\sqrt{6}$					✓		✓		✓			
n. 0	✓	✓	✓			✓		*	✓			
o. 1	✓	✓		✓		✓			✓	✓	✓	
p. 1/9				✓		✓			✓			

*0 is also an imaginary number.

1-2 THE FIELD AXIOMS

From previous mathematics courses you probably remember names such as, "Distributive Property," "Reflexive Property," and "Multiplication Property of Zero." Some of these properties, called *axioms,* are accepted without proof and are used as a starting point for working with numbers.

Time: 1 day

Chalkboard Examples:
In this section students must recall the meanings "distributive," "commutative," and so forth. One way to introduce the topic is to have the students call out names of all the properties they can remember. let stu-