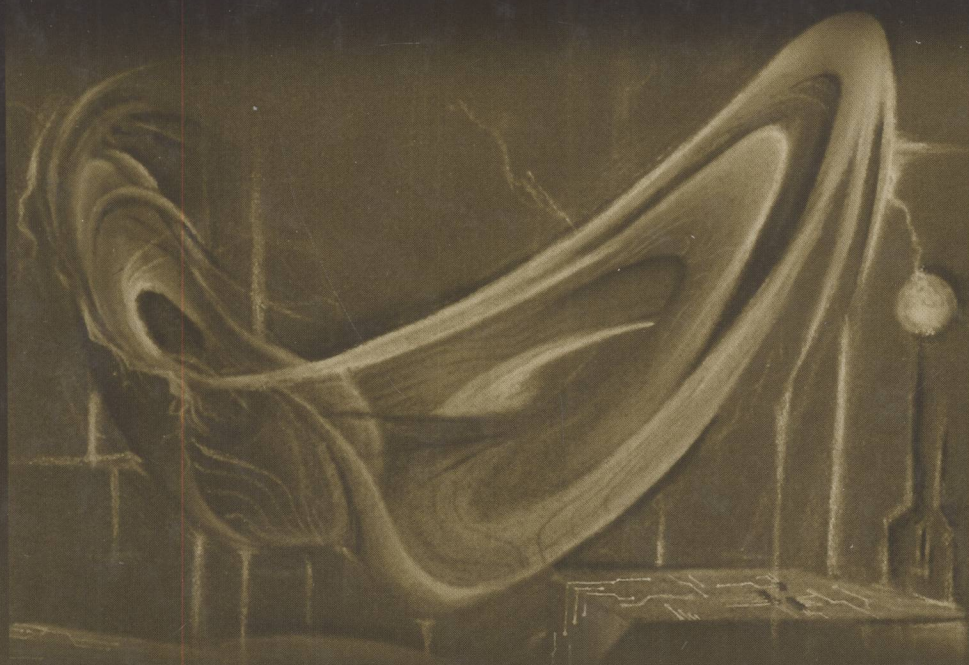


COMPLEX BEHAVIOR OF SWITCHING POWER CONVERTERS



Chi Kong Tse



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COMPLEX
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Preface

Power electronics, as an application-oriented discipline, has been developed to address specific power conversion problems in industrial, commercial, residential and aerospace environments. In the past three decades, motivated by the burgeoning demand of delivering electric power in various specific forms, this branch of electrical engineering has undergone an intense development in many areas of technology, including power devices, control methods, circuit design, computer-aided analysis, passive components, packaging techniques, and so on. The principal focus in power electronics has been to fulfill the functional requirements of the intended applications. Because practical applications are the prime concerns, it often turns out that a particular circuit topology or system implementation has found widespread applications long before it has been thoroughly analyzed. For instance, switching power converters have been used for more than half a century, but analytical models that allow systematic circuit design (e.g., averaged models and sampled-data models) have been available only since the late 1970s.

Power electronics circuits, being nonlinear, exhibit a variety of complex behavior such as sudden change of operating regime, chaotic operation, occasional instability (in certain parameter windows), intermittent subharmonic or chaotic operation, etc. Power electronics engineers are always dealing with these problems in the course of developing power electronics products. Since the engineers' job is to make the circuit work in the expected operating regime, the usual treatment is to find ways to eliminate any unwanted behavior, often in some quick and heuristic manner such as adjusting circuit components and parameters through a trial-and-error procedure. However, as the field of power electronics gains maturity, the quest for better design, functionality and reliability has made it necessary for engineers to understand thoroughly the behavior of the systems being designed under all possible practical conditions. For example, *bifurcation*, a behavior characterized by a sudden change of operating regime when a parameter is varied intentionally or unintentionally, can be catastrophic, leading possibly to unexpected expansion of operating ranges which can damage semiconductor devices. Thus, knowing *when* (under what conditions) and *how* (in what way) a bifurcation occurs should be of fundamental importance. Such knowledge, however, requires appropriate modeling methodology and in-depth analysis.

This book is concerned with the study of complex behavior of switching power converters. The objective is to provide a systematic treatment procedure for observation, identification and diagnosis of the complex behavior

exhibited by switching power converters. The essential techniques for capturing complex behavior on the computer and in the laboratory are explained, along with application examples describing the key procedure for diagnosing complex behavior such as chaos and bifurcation. The target audience includes graduate students, researchers and engineers who work in the field of power electronics and have the need or interest to understand complex behavior in switching power converters. Furthermore, in presenting the techniques of investigation and the various findings, a conscientious effort has been made to emphasize circuit operation rather than mathematical abstraction, and whenever possible, phenomena will be explained in terms of the physical circuit operation with a minimal amount of mathematics. With this, we hope this book can also be useful as a start-up guide for graduate students and researchers who wish to grasp the essentials for analyzing complex behavior in power converters, as well as a readable reference for engineers who wish to understand such complex behavior.

We begin in Chapter 1 with an overview of the complex behavior of switching power converters, outlining some important findings and research methodologies. We will also introduce some salient concepts of nonlinear dynamical systems that are essential to the study of complex behavior in switching converters. In Chapter 2, we introduce specific computer and laboratory techniques for studying complex behavior. In Chapter 3, we describe the key modeling approaches for switching converters which are capable of retaining the salient nonlinear properties and hence can be used to study complex behavior. Our formal investigation of the nonlinear dynamics of switching converters begins in Chapter 4, where we test-drive a discrete-time analysis method on a simple first-order system. The purpose is to illustrate the key procedure involved in the analysis of period-doubling bifurcation, which is a commonly found phenomenon in switching converters. In Chapter 5, we take a detailed look at the basic phenomenology for power electronics circuits, which is characterized by the interaction of smooth and non-smooth bifurcations. In describing this important basic phenomenon, we emphasize the physical mechanism that prevents a switching converter from operating “smoothly.” Specifically, from a circuit operational viewpoint, we explain the mechanism of the so-called *border collision*, and make an attempt to predict its occurrence. In Chapter 6, we move on to a high-order converter, known as the Ćuk converter. The bifurcation behavior of this converter is studied for two different control configurations. Our aim is to highlight the importance of choosing the appropriate models for analysis. This issue is again addressed in Chapters 7 and 8, where two different types of parallel-connected switching converters are treated with different modeling approaches. In Chapter 9, we consider an application of bifurcation analysis to a practical power-factor-correction switching converter where we uncover a possible but rarely known fast-scale instability. In Chapter 10, we investigate the problem of intermittent operation in switching converters. By using an appropriate model that incorporates a mechanism that couples spurious signals into a power converter, we explain

a possible origin for intermittent chaotic or subharmonic operations.

For the successful completion of this book, I am indebted to a number of people, institutions and organizations. First of all, in the course of my research in this field, I have been constantly stimulated, challenged and inspired by my students. Among them, special credits must go to my former graduate students and postdoctoral assistants, William Chan, Octavian Dranga, Herbert Iu and Yufei Zhou, not only for their diligent work which has added materially to this book, but also for their inquiring minds which have prompted me to pay attention to many important but easily overlooked problems. I also wish to express my sincere appreciation to Prof. Soumitro Banerjee, who has read a large part of an earlier version of this book and has made many valuable suggestions and comments. Excellent opportunities and environments for pursuing this writing project were kindly provided by Prof. Hiroshi Kawakami of Tokushima University and Prof. Leon Chua of UC Berkeley while I spent my sabbatical in their laboratories during last fall and spring. Furthermore, I have been particularly fortunate to have the opportunities to discuss research problems with many brilliant researchers and colleagues. Among them, Mario di Bernardo, Ron Chen, Martin Chow, Tetsuro Endo, Yuk-Ming Lai, Francis Lau, Yim-Shu Lee, István Nagy, Shui-Sheng Qiu, Toshimichi Saito, Michael Small, Tetsushi Ueta and Siu-Chung Wong deserve my most grateful thanks. I would also like to thank the editor of this series, Prof. Muhammad Rashid, and the staff of the CRC Press at Boca Raton for their professional and enthusiastic support of this project.

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Hong Kong

C. K. Tse

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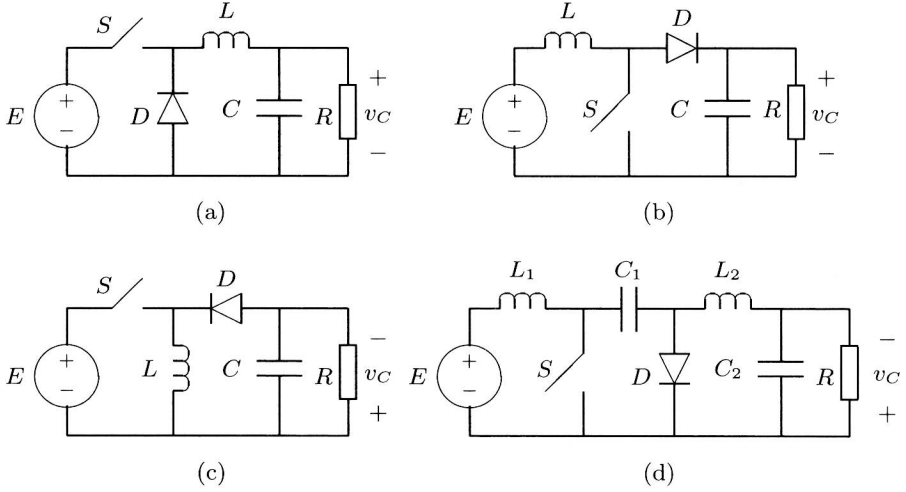
Introduction

Research in *nonlinear systems and complexity* had made remarkable progress in the 1970's and 1980's, leading to discoveries which were not only new, but also revolutionary in the sense that some of our traditional beliefs regarding the behavior of deterministic systems were relentlessly challenged [63, 64, 79, 92]. Most striking of all, simple deterministic systems can behave in a “random-like” fashion and their solution trajectories can deny “long-term predictability” even if the initial conditions are practically known [29, 54, 76, 109]. Such behavior is now termed *chaos*, which underlies the *complexity* and subtle *order* exhibited by real-world systems. Scientists, mathematicians and engineers from a diverging range of disciplines have found remarkably similar complex behavior in their systems. The root cause of such complex behavior has been identified collectively as *nonlinearity*. Precisely, without exception, all systems in the real world are nonlinear. In this book, we are concerned with a particular class of engineering systems, known as power electronics, which by virtue of its rich nonlinearity exhibits a variety of complex behavior.

In this introductory chapter we will take a quick tour of power electronics circuits and dynamical systems. Our aim is to introduce the basic types of switching converters, their salient operating features, modeling approaches and nonlinear behavior. We will also introduce some basic concepts of nonlinear dynamics that are necessary for understanding the complex behavior of switching converters to be described in the later chapters.

1.1 Overview of Power Electronics Circuits

The basic operation of any power electronics circuit involves toggling among a set of linear or nonlinear circuit topologies, under the control of a feedback system [33, 78, 81, 99, 100, 118, 128]. As such, they can be regarded as *piecewise switched* dynamical systems. For example, in simple switching converters, such as the ones shown in Figure 1.1, an inductor (or inductors) is/are “switched” between the input and the output through an appropriate switching element (labelled as S in the figure). The way in which the inductor(s) is/are switched determines the output voltage level and transient behavior. Usually, a semiconductor switch and a diode are used to implement

**FIGURE 1.1**

Examples of simple switching converters. (a) Buck converter; (b) boost converter; (c) buck-boost converter; (d) boost-buck (Ćuk) converter.

such switching. Through the use of a feedback control circuit, the relative durations of the various switching intervals are continuously adjusted. Such feedback action effectively controls the transient and steady-state behaviors of the circuit. Thus, both the circuit topology and the control method determine the dynamical behavior of a power electronics circuit.

1.1.1 Switching Power Converters

Most power converters are constructed on the basis of the simple converters shown in Figure 1.1 [128]. Typically, the switch and the diode are turned on and off in a cyclic and complementary manner. The switch is directly controlled by a pulse-width modulated signal which is derived from a feedback circuit. The diode turns on and off depending upon its terminal condition. When the switch is closed, the diode is reverse biased and hence open. Under this condition, the inductor current ramps up. When the switch is turned off, the diode is forward biased and behaves as a short circuit. This causes the inductor current to ramp down. The process repeats cyclically. The system can therefore be plainly described by a set of state equations, each responsible for one particular switch state. For the operation described above, we have two state equations:

$$\dot{\mathbf{x}} = \mathbf{A}_1 \mathbf{x} + \mathbf{B}_1 E \quad \text{switch on and diode off} \quad (1.1)$$

$$\dot{\mathbf{x}} = \mathbf{A}_2 \mathbf{x} + \mathbf{B}_2 E \quad \text{switch off and diode on} \quad (1.2)$$

where \mathbf{x} is the state vector usually consisting of all capacitor voltages and inductor currents, the \mathbf{A} 's and \mathbf{B} 's are the system matrices, and E is the input voltage. Furthermore, because the conduction of the diode is determined by its own terminal condition, there is a possibility that the diode can turn itself off even when the switch is off. This happens when the diode current becomes zero and is not permitted to reverse its direction. In the power electronics literature, this operation has been termed *discontinuous conduction mode*, as opposed to *continuous conduction mode* where the switch and the diode operate strictly in a complementary fashion.* Clearly, we have another state equation for the situation where both switch and diode are off.

$$\dot{\mathbf{x}} = \mathbf{A}_3\mathbf{x} + \mathbf{B}_3E \quad \text{switch off and diode off.} \quad (1.3)$$

In practice, the choice between continuous and discontinuous conduction modes of operation is often an engineering decision. Continuous conduction mode is more suited for high power applications, whereas discontinuous conduction mode is limited to low power applications because of the relatively high device stresses. On the other hand, discontinuous conduction mode gives a more straightforward control design and generally yields faster transient responses. Clearly, a number of factors determine whether the converter would operate in continuous or discontinuous conduction mode. For instance, the size of the inductance determines how rapidly the current ramps up and down, and hence is a determining factor for the operating mode. We will postpone the detailed discussion of the operating modes to Chapter 3.

We now examine the control of switching converters. First, as in all control systems, a control input is needed. For switching converters, the usual choice is the *duty cycle*, d , which is defined as the fraction of a repetition period, T , during which the switch is closed, i.e.,

$$d = \frac{t_c}{T} \quad (1.4)$$

where t_c is the time duration when the switch is held closed. In practice, the duty cycle is continuously controlled by a feedback circuit that aims to maintain the output voltage at a fixed level even under input and load variations. In the steady state, the output voltage is a function of the duty cycle and the input voltage. For the buck converter operating in continuous conduction mode, for example, the volt-time balance for the inductor requires that the following be satisfied in the steady state:

$$(E - V_C)DT = V_C(1 - D)T \Rightarrow V_C = DE \quad (\text{buck converter}) \quad (1.5)$$

where uppercase letters denote steady-state values of the respective variables. Likewise, for the other converters shown in Figure 1.1 operating in continuous

*For simplicity, we omit details of the other operating modes which can possibly happen in the Ćuk converter [143].