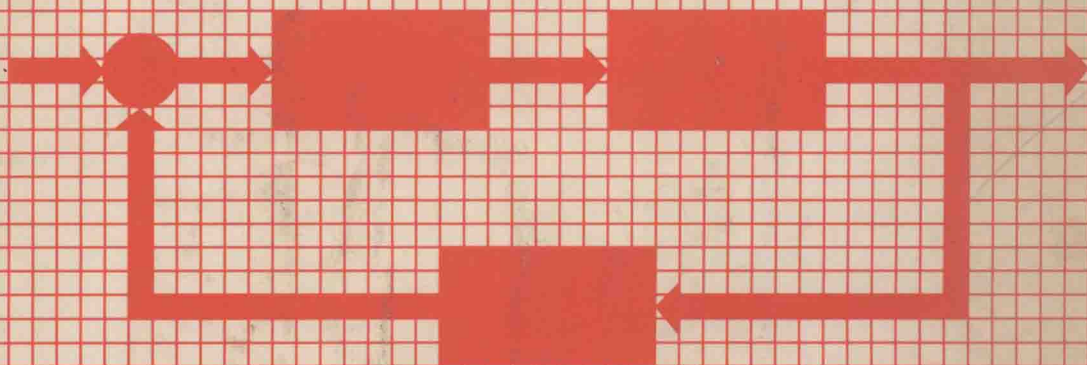


Tutorial Guides in Electronic Engineering

15

Control Engineering



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North-Holland Reinhold (International)

Control Engineering

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Control Engineering

TUTORIAL GUIDES IN ELECTRONIC ENGINEERING

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This series is aimed at first- and second-year undergraduate courses. Each text is complete in itself, although linked with others in the series. Where possible, the trend towards a 'systems' approach is acknowledged, but classical fundamental areas of study have not been excluded. Worked examples feature prominently and indicate, where appropriate, a number of approaches to the same problem.

A format providing marginal notes has been adopted to allow the authors to include ideas and material to support the main text. These notes include references to standard mainstream texts and commentary on the applicability of solution methods, aimed particularly at covering points normally found difficult. Graded problems are provided at the end of each chapter, with answers at the end of the book.

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Preface

The appearance of this book in the series 'Tutorial Guides in Electronic Engineering' is a reflection of the importance attached to control in electronics and electrical engineering curricula. Yet control engineering is essentially interdisciplinary in nature, and plays a fundamental role in many other areas of technology. I have therefore tried to make this text equally relevant to readers whose main interest lie outside electronics, by concentrating on general systems characteristics rather than on specific implementations.

I have restricted myself to the 'classical' approach to single-input, single-output systems, since I feel this is the most appropriate subject matter for a first course in control. However, the Tutorial Guide style, with its detailed treatment of simple design examples, should also render the text useful to practising engineers who need to revise and apply dimly remembered material – or even to those whose training did not include control.

The reader is assumed to be familiar with complex numbers, phasors, and elementary calculus. Apart from these topics, the mathematical requirements are few, although prior knowledge of simple first- and second-order linear differential equations would be useful.

Where possible I have tried to indicate how computer-based tools can reduce the labour involved in control system design, although limitations of space have precluded detailed description. However, CAD software or other computer-based approaches can only be as effective as the understanding and skill of the user. In the chapters dealing with aspects of design I have tried to develop such understanding by dealing with a limited number of examples in depth, rather than giving a cursory treatment of a wider range of material. Nevertheless, the examples have been chosen to illustrate most of the major classical techniques of feedback control, including some of the distinctive features of digital implementations.

My approach has been strongly influenced by the Open University course T391 Control Engineering and its successor T394, and it is a pleasure to record my debt to other members of those course teams. I am particularly grateful for many hours of discussion with Chris Dillon, who has read and commented on draft chapters with great perception, and whose ideas have contributed substantially to the final version. Thanks are also due to series editor Professor Kel Fidler for support and guidance.

Grateful acknowledgement is also given to The Open University Press for the use of Figures 3.21, 3.22, 4.21, 4.22, 8.14, 8.16 and 8.19 (© The Open University Press, 1984, 1984, 1978, 1978, 1986, 1986 and 1986 respectively).

Contents

<i>Preface</i>	vii
1 Systems, objectives and strategies	1
Introduction	1
Control strategies	2
2 General characteristics of feedback	7
Modelling a feedback loop	7
Sensitivity of closed-loop gain to changes in parameters	10
Disturbance rejection	12
Linearization about an operating point	13
3 Modelling dynamic systems	17
The modelling approach	17
A first-order differential equation model	18
An alternative description of system behaviour: frequency response	25
An integrator model	32
A second-order lag model	36
Higher-order models	42
Time delays	46
System analysis and system identification	46
4 The frequency response approach to control system design	50
Closing the loop	50
The Nichols chart	55
Stability	62
Integrating action	64
The proportional + integral controller	66
A design example	69
Non-unity feedback systems	75
Computers as a design aid	76
A note of caution	78
5 The s-plane and transient response	83
The Laplace transform approach	83
Poles and zeros	88
Calculating system response	91
Standard models and the s -plane	99
Higher-order systems and dominance	102

6 The root-locus technique	111
First- and second-order root loci	111
An alternative approach	115
Sketching simple root loci	123
Root locus in analysis and design	128
7 Steady-state performance	140
An intuitive approach	140
The transform approach	145
8 Controllers and compensators	150
Three-term controllers	151
Compensators	153
Digital controllers and compensators	164
<i>Appendix 1: Polar plots</i>	174
<i>Appendix 2: The Routh-Hurwitz criterion</i>	176
<i>Further reading</i>	179
<i>Answers to numerical problems</i>	180
<i>Index</i>	183

Systems, Objectives and Strategies

1

-
- ☐ To introduce the concept of control in an engineering context, and to indicate the wide variety of control tasks in a large engineering system.
 - ☐ To describe the three common control strategies – open loop, feedforward, and feedback (or closed-loop control).
-

Objectives

Introduction

The word 'control' is used in many different contexts. We talk of quality control, financial control, command and control, production control, and so on – terms which cover an enormous range of activities. Yet all these types of control, if they are to be successful, have certain features in common. One is that they all presuppose the existence of a *system* whose behaviour we wish to influence, and the freedom to take actions which will force it to behave in some desirable way. For example, for the manager of a large chemical plant the system of interest may be the entire plant, as illustrated in Fig. 1.1. The *inputs* to the system, which we assume the manager can influence, are the various flows of energy and raw materials into the plant; the *outputs* are not only the finished products but also the waste, environmental effects, and so on. Note that there are also *disturbance inputs*, which the manager cannot control, such as market fluctuations, changes in the environment etc, and these will also affect the plant outputs. For another engineer in the same plant, however, the system of interest might be one particular reaction vessel and specifically, the design of a control system to maintain the

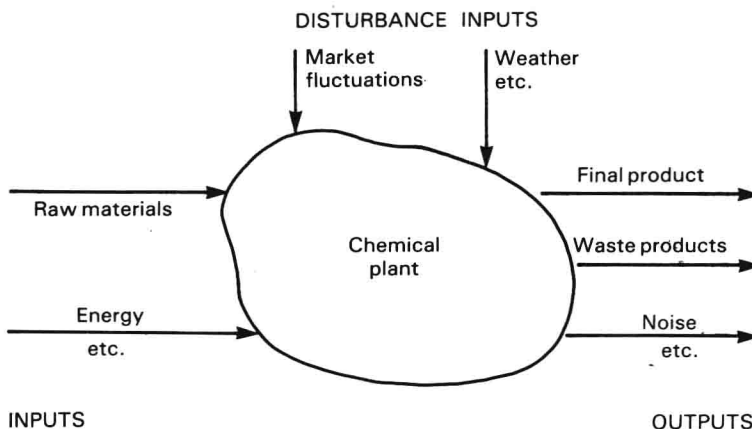


Fig. 1.1 A chemical plant considered as a single system.

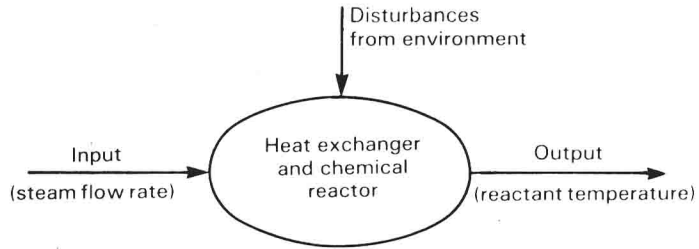


Fig. 1.2 A reactor subsystem of the chemical plant.

In general, a system can be defined loosely as the set of interconnected elements which are of interest for some specific purpose.

reactants at a constant temperature by adjusting the flow of steam to a heater, as represented in Fig. 1.2. From a control point of view there is a single input (steam flow rate) and a single output (reactor temperature); in addition, there will again be disturbances caused by various outside factors such as unwanted fluctuations in the steam supply or changes in the ambient temperature.

No control system can be designed without a clear specification of control *objectives*. For a chemical plant as a whole, the ultimate control objective might be to produce a final product meeting quality specifications, while minimizing costs. For the temperature control system the objective might be to remain within a certain temperature range under specified operating conditions. In each case, however, there will be limitations or *constraints* to what can be achieved; not only limits to the physical capabilities of the equipment being used, but also, for example, economic, legal, and safety constraints.

Control engineering can perhaps be summed up as the design and implementation of automatic control systems to achieve specified objectives under given constraints. For a complex system, the overall objectives and constraints will need to be translated into performance specifications for the various subsystems – ultimately into control system specifications for low-level subsystems, such as individual chemical reactors in the chemical plant example.

Control engineering as a discipline is characterized by a common approach to a great variety of control tasks, and by a set of mathematical tools which have proved to be generally applicable. Computers are used widely to implement control schemes, and an increasing knowledge of information technology and software engineering is therefore being demanded of control engineers. Nevertheless, the fundamental requirement is still a thorough understanding of the dynamics of individual control systems. This book will concentrate on those strategies, models and techniques vital to this understanding.

Control Strategies

This text will deal with single-input, single-output systems only. Such systems can be represented by a block diagram such as Fig. 1.3, which shows a single-input, single-output process to be controlled.

In the introduction to this chapter the terms input and output were used very generally to signify any flow of information, energy or material into or out of a system. From now on, however, the terms will be used more precisely. In Fig. 1.3, for example, $u(t)$, the 'input' to the process, represents the variable which is

The term 'process' is used generally to mean any system to be controlled. The term 'plant' is often also used with exactly the same meaning.

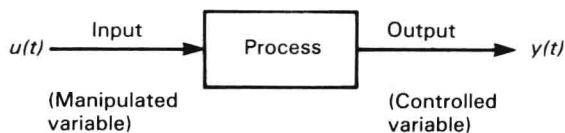


Fig. 1.3 A single-input, single-output process.

adjusted in order to bring about the control action: it is often known as the *manipulated variable*. Similarly $y(t)$, the ‘output’ of the process, is the variable which the engineer wishes to control in order to fulfil the desired objectives: not surprisingly, it is referred to as the *controlled variable*. Hence in the temperature control system mentioned above the ‘input’ (manipulated variable) was the rate of flow of steam and the ‘output’ (controlled variable) was the temperature – even though to a chemical engineer the various reactants and products might be perceived as the system inputs and outputs! To a control engineer, inputs and outputs are defined so as to represent a *signal flow* through the control system, a concept which will become clearer in subsequent chapters.

Returning to Fig. 1.3, then, we can express the goal of all controllers – whether automatic systems or human operators – as attempting to achieve a desired output behaviour $y(t)$ by applying an appropriate control action $u(t)$ to the process. Automatic controllers or control systems do this by using information about the process and the particular operating conditions to determine an appropriate $u(t)$ for a given situation, as represented by Fig. 1.4. Such information might include externally supplied data about operating conditions – such as the desired and current values of $y(t)$, the rate at which $y(t)$ is changing, and so on – but also ‘built-in’ information which takes into account how the process is likely to behave in response to a particular control action.

The latter, ‘built-in’ information is derived from a *model* of the process which can be used to predict the variation of $y(t)$ for a given applied $u(t)$. Models of process behaviour are vital if a control system is to be designed which will automatically generate an appropriate control action, and the sort of models commonly used by control engineers will be described in some detail in Chapter 3. First, however, let us examine a number of general approaches or *control strategies* which can be adopted. All require the process to be modelled, but the general characteristics of each strategy can be described without making any particular assumptions about the type of model employed.

The first strategy, known as *open-loop control*, is illustrated in Fig. 1.5. The

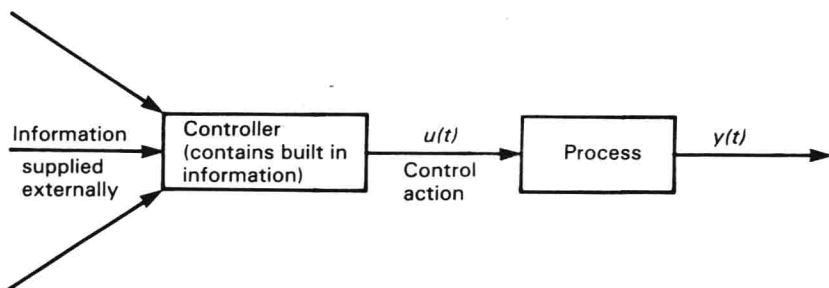


Fig. 1.4 The general control problem.

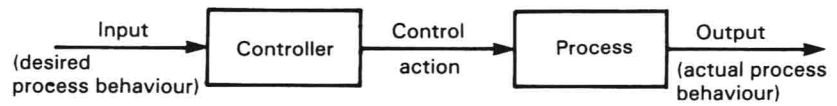


Fig. 1.5 The open-loop strategy.

The precise form of the model used will depend on many factors, including the objectives of the control system. The modelling process will be discussed in detail in Chapter 3.

controller can be thought of as using an 'inverse' model of the process, together with externally-supplied information about the desired output, to determine control action. A simple example should make this clearer. Figure 1.6 shows an open-loop motor speed control system. Suppose that we have a model relating the motor armature voltage to the resulting motor speed. Using such a speed/voltage relationship, we can attempt to design an open-loop controller such that the armature voltage generated in response to a given desired speed (as defined by the position of a control knob, for example) is just what is required, in theory, to produce that particular motor speed. If the motor speed/voltage relationship is modelled by a constant – say $G \text{ rad s}^{-1} \text{ V}^{-1}$ – then the effect of the controller and power amplifier combined must be to produce $1/G$ volts for each rad s^{-1} of demanded speed. In this simple case the open-loop controller attempts to implement the exact inverse model, $1/G$.

There are various drawbacks to this type of control strategy, however. If the load on the motor changes, the speed will alter even if the demanded speed and hence the armature voltage is held constant. Furthermore, the characteristics of the motor may vary with time – for example, the speed obtained for a given voltage when the motor is cold may be very different from that when the lubricating oil in the bearings has reached its normal operating temperature. Open-loop control cannot compensate for either *disturbances* to the system (such as a varying load) or changes in plant parameters (such as varying friction in the bearings).

One way of compensating for disturbances is to measure them and make corresponding changes to the control action, as illustrated in general terms in Fig. 1.7. Here one input to the *controller* represents the desired behaviour of the process in some way. The control action taken by the controller is determined not only by using a model of how the process behaves, but also by taking into account the measured disturbances. In the case of the temperature control system of Fig. 1.2, for example, disturbances to the steam supply system might conceivably be measured and the value used to open or close a supply valve as appropriate, again using a model of the reaction vessel to determine the compensating control action.

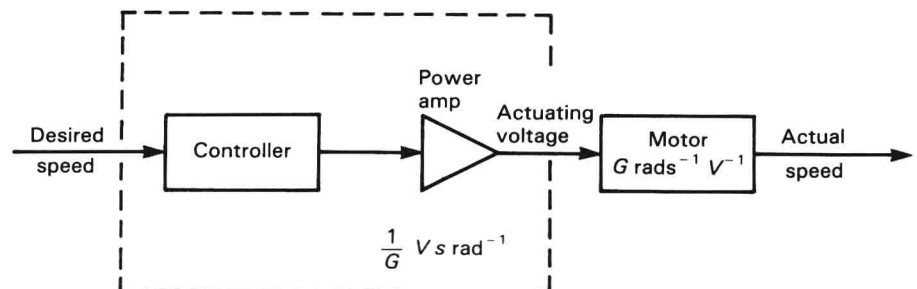


Fig. 1.6 An open-loop speed control system.

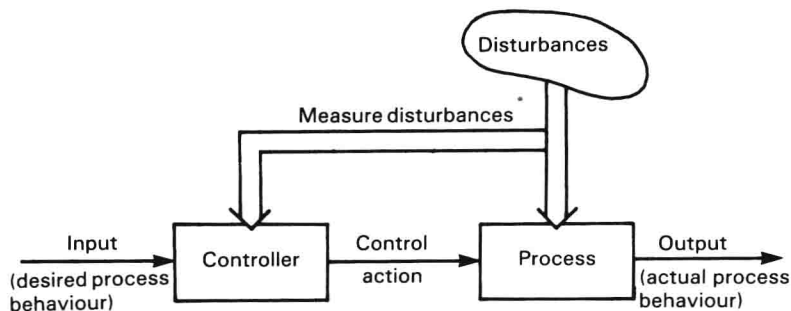


Fig. 1.7 The feedforward strategy.

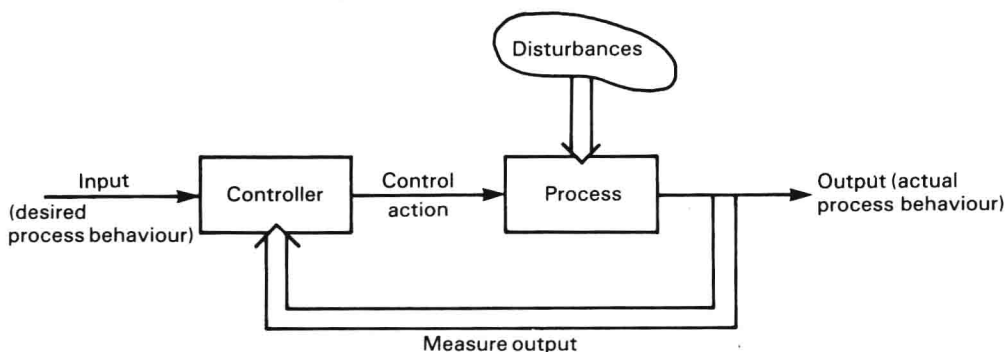


Fig. 1.8 The closed-loop (feedback) control strategy.

This type of control attempts to compensate for disturbances before they have any effect on the system output, and is known as *feedforward control*. Feedforward control can be a very effective strategy if the disturbances have a known effect and can be easily measured. If there are too many disturbances, however, or they cannot easily be measured, then feedforward control is not effective. Furthermore, feedforward control cannot compensate for any changes in the plant characteristics which cannot be measured and treated as a disturbance.

The most common control strategy is *feedback* or *closed-loop control*, illustrated in Fig. 1.8. Here the process output is monitored, and control actions are taken to counteract deviations from required behaviour. In the case of the temperature control system, therefore, the reactant temperature is measured, and if this differs from the desired value the rate of flow of steam is increased or decreased as appropriate to return the temperature towards the desired value. In the case of the motor speed control system, the speed is measured, and the applied voltage modified as required. The effect of feedback is to compensate for *any* discrepancy in the controlled variable, whatever its cause. This type of control strategy is used in such everyday applications as water tank level control using a ball valve, or room temperature control with a thermostat. It is such an important strategy in control engineering that it forms the major subject matter of this book.

Note the distinction between feedforward and feedback, despite the apparent similarities of Figs 1.7 and 1.8.

Feedforward involves measuring disturbances directly, whereas feedback measures the controlled variable, and compensates for disturbances only after their *effects* on the controlled variable have taken place. In practice, feedback and feedforward are often combined in a single system.

Summary

Control systems are designed to achieve specified objectives within a given set of constraints. The three common control strategies are open-loop, feedforward and closed-loop control. These strategies are often combined within a single control system. Each strategy requires some model of the process to be controlled.

General Characteristics of Feedback

2

- To present a simple description of a closed-loop control system, and hence analyse some of the main properties of feedback.
- To introduce the concepts of steady-state error, disturbances and disturbance rejection.
- To show how component characteristics may be linearized about an operating point.

Objectives

In this chapter some of the broad features of feedback control will be analysed using very simple models of control system components. A detailed analysis of closed-loop behaviour must wait until more sophisticated mathematical models have been discussed in Chapter 3. However, it is possible to provide partial answers immediately to such questions as ‘how accurate is feedback control?’ or ‘how well can it counteract disturbances?’

Modelling a Feedback Loop

Figure 2.1 shows a closed-loop motor speed control system in generalized form, using the symbols almost universally adopted. Here, a voltage corresponding to a measure of the actual speed Ω is compared with a reference voltage, r . The difference between these two voltages is then amplified and applied to the motor, hence generating a control action tending to maintain the speed at a value determined by the reference input.

Let us assume that the motor is modelled by a gain G , the constant of proportionality relating the input voltage to output speed. That is,

$$\Omega = v \times G$$

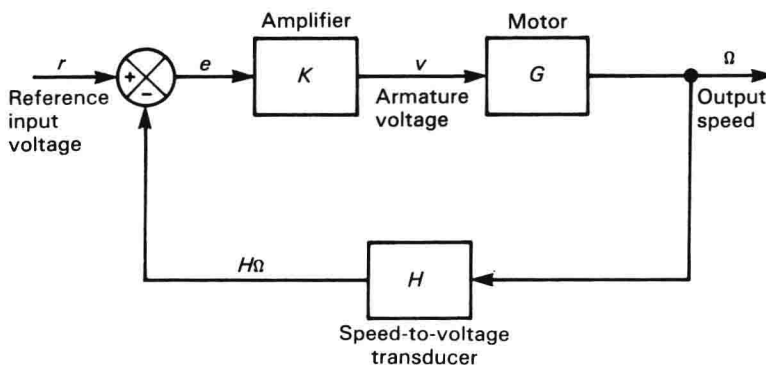


Fig. 2.1 A generalized control loop.

In this chapter the term *steady-state* is used to indicate that the entire system has settled down to constant values of all variables. As will be seen in later chapters, the term *steady-state* can have a more general meaning – the restricted meaning employed here might be termed *static*, in the sense of unchanging with time.

A wide range of commercial controllers are available for process control applications. Until recently most controllers were either analogue electronic circuits, or mechanical (e.g. pneumatic) devices. Now it is becoming increasingly common for a computer to be programmed to act as the controller, and even stand-alone controllers are usually based on digital electronics.

This implies that any change in voltage is reflected immediately in a change in motor speed. No motor can respond instantaneously to a change in applied voltage, of course, so clearly this model is greatly over-simplified. Nevertheless, assuming that the system settles down eventually to some *steady-state* value of motor speed for a given applied voltage, then the above expression can be used to model the *steady-state* condition.

$$G = \frac{\text{steady-state output}}{\text{steady-state input}} = \frac{\Omega_{ss}}{v_{ss}}$$

The entire analysis of this chapter is subject to this important assumption.

Similarly we can model the velocity transducer by another gain. This time, we assume that the transducer produces an output voltage proportional to the speed

$$\text{transducer output voltage} = \text{motor speed} \times H$$

The output voltage from the transducer is compared with a *reference input* voltage r , which is an expression of the desired motor speed. The difference between these two signals – often known as the *error* signal – becomes the input to the amplifier, and the amplifier gain K determines what armature voltage should be applied to the motor. If the speed is too low, the voltage is therefore increased, and vice versa.

The simplest – and very common – form of general closed-loop controller is known as a *proportional controller*, and corresponds to a constant gain, K , acting on the error signal. In the system of Fig. 2.1 the amplifier may be thought of as a proportional controller, producing a control action proportional to the error signal.

Let us assume that the motor control system of Fig. 2.1 has reached a steady-state, with the motor running at a constant speed Ω in response to a reference input r volts. In order to assess the performance of the closed-loop system we need to derive a relationship between r and Ω in the absence of any disturbances.

From the figure we can write down immediately

$$e = r - H\Omega$$

$$\text{and } \Omega = KGe$$

$$\begin{aligned} \text{Hence } \Omega &= KG(r - H\Omega) \\ &= KGr - KGH\Omega \end{aligned}$$

Rearranging gives

$$\begin{aligned} \Omega(1 + KGH) &= KGr \\ \text{or } \Omega &= \frac{KG}{1 + KGH} r \end{aligned}$$

Hence the expression $\frac{KG}{1 + KGH}$ corresponds to the *closed-loop-gain* of the complete feedback system – that is, the factor relating the output (speed) to the reference input in the steady state. The quantity KG is often referred to as the forward path gain, while KGH is known as the loop gain. Note that the closed loop gain can therefore be written as

$$\frac{\text{forward path gain}}{1 + \text{loop gain}}$$

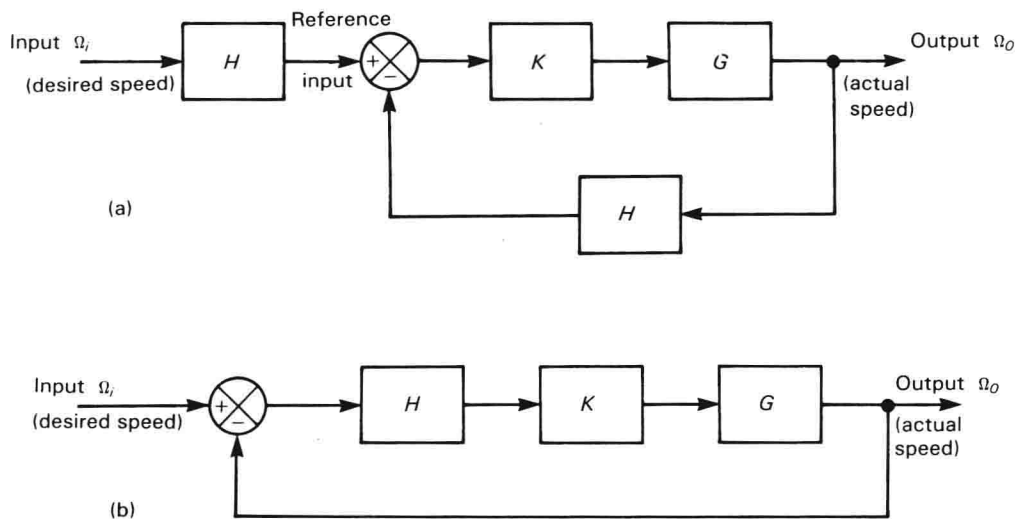


Fig. 2.2 The unity feedback model.

It is often more convenient to work with a modified version of Fig. 2.1 in order to obtain an expression directly relating the actual speed to the *desired* motor speed, rather than to a reference input voltage. In this case we can imagine an input 'desired speed' variable, which is then multiplied by a gain exactly equivalent to that of the transducer, in order to give an appropriate reference input voltage, as shown in Fig. 2.2(a). This procedure models the fact that in general the comparison between desired and actual values of a controlled variable will be made in terms of signals representing these measures – an analogue voltage, for example – and not the numerical values themselves. The additional scaling factor H is introduced to reflect this.

Now, it makes no difference whether the gain H is applied before or after the comparator, so long as it is applied to both the signals being compared. Figure 2.2(a) can therefore be re-drawn in the equivalent form of Fig. 2.2(b). This is known as the *unity feedback* form of the closed-loop system, and is an extremely useful concept in the modelling process. Remember, however, that we are assuming here that G and H are pure gains, modelling the steady-state condition. Special procedures are necessary when transducer *dynamics* need to be taken into account, as will be discussed in later chapters.

One further simplification of the unity feedback model can be made. At the design stage it is often convenient to assume that $H = 1$ in Fig. 2.2(b). This allows design calculations – such as determining an appropriate value of controller gain K – to be carried out more simply. When the system is implemented, an appropriately modified value of K can be used, reflecting the various scaling factors involved in the practical system.

The preceding general analysis can now be used to illustrate some of the major features of feedback control. Let us begin by investigating the steady-state error, defined as the difference between desired and actual output when the output has reached a steady, constant value. The unity feedback model of Fig. 2.2(b) makes it particularly easy to relate closed-loop static gain to steady-state error. The error may be calculated easily from the closed-loop gain.