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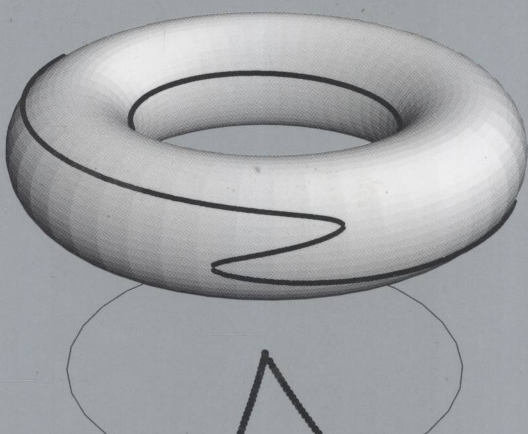
GEOMETRIC FUNDAMENTALS OF ROBOTICS

J.M. Selig

Second Edition



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To Kathy

Preface

This book is an extended and corrected version of an earlier work, "Geometrical Methods in Robotics" published by Springer-Verlag in 1996. I am extremely glad of the opportunity to publish this work which contains many corrections and additions. The extra material, two new chapters and several new sections, reflects some of the advances in the field over the past few years as well as some material that was missed in the original work.

As before this book aims to introduce Lie groups and allied algebraic and geometric concepts to a robotics audience. I hope that the power and elegance of these methods as they apply to problems in robotics is still clear. By now the pioneering work of Ball is well known. However, the work of Study and his colleagues is not so widely appreciated, at least not in the English speaking world. This book is also an attempt to bring at least some of their work to the attention of a wider audience.

In the first four chapters, a careful exposition of the theory of Lie groups and their Lie algebras is given. All examples used to illustrate these ideas, except for the simplest ones, are taken from robotics. So, unlike most standard texts on Lie groups, emphasis is placed on a group that is not semi-simple—the group of proper Euclidean motions in three dimensions. In particular, the continuous subgroups of this group are found, and the elements of its Lie algebra are identified with the surfaces of the lower Reuleaux pairs. These surfaces were first identified by Reuleaux in the latter half of the 19th century. They allow us to associate a Lie algebra element to every basic mechanical joint. The motions allowed by the joint are then just the one-parameter subgroups generated by the

Lie algebra element. A detailed study of the exponential map and its derivative is given for the rotation and rigid body motion groups.

Chapter 5 looks at some geometrical problems that are basic to robotics and the theory of mechanisms. Having developed in the previous chapter the description of robot kinematics using exponentials of Lie algebra elements, these ideas are used to generalise and simplify some standard results in kinematics. The chapter looks at the kinematics of 3-joint wrists and 3-joint regional manipulators.

Some of the classical theory of ruled surfaces and line complexes is introduced in Chapter 6. This material also benefits from the Lie algebra point of view. For robotics, the most important ruled surfaces are the cylindrical hyperboloid and the cylindroid. A full description of these surfaces is given.

In Chapter 7, the theory of group representations is introduced. Once again, the emphasis is on the group of proper Euclidean motions. Many representations of this group are used in robotics. A benefit of this is that it allows a concise statement and proof of the 'Principle of Transference', a result that, until recently, had the status of a 'folk theorem' in the mechanism theory community.

Ball's theory of screws underlies much of the work in this book. Ball's treatise was written at the turn of the twentieth century, just before Lie's and Cartan's work on continuous groups. The infinitesimal screws of Ball can now be seen as elements of the Lie algebra of the group of proper Euclidean motions. In Chapter 8, on screw systems, the linear subspaces of this Lie algebra are explored. The Gibson–Hunt classification of these systems is derived using a group theoretic approach.

Clifford algebra is introduced in Chapter 9. Again, attention is quickly specialised to the case of the Clifford algebra for the group of proper Euclidean motions. This is something of an esoteric case in the standard mathematical literature, since it is the Clifford algebra of a degenerate bilinear form. This algebra is a very efficient vehicle for carrying out computation both in the group and in some of its geometrical representations. Moreover, it allows us to define the Study quadric, an algebraic variety that contains the elements of the group of proper Euclidean motions.

Chapter 10 explores this Clifford algebra in more detail. It is shown how points, lines and planes can be represented in this algebra, and how geometric operations can be modelled by algebraic operations in the algebra. The results are used to look at the kinematics of six-joint industrial robots and prove an important theorem concerning designs of robots that have solvable inverse kinematics.

The Study quadric is more fully explored in Chapter 11, where its subspaces and quotients are examined in some depth. The intersection theory of the variety is introduced and used to solve some simple enumerative problems like the number of postures of the general 6-R robot.

Chapters 12, 13 and 14 cover the statics and dynamics of robots. The dual space to the Lie algebra is identified with the space of wrenches, that is, force-torque vectors. This facilitates a simple description of some standard problems in robotics, in particular, the problem of gripping solid objects. The group theory helps to isolate the surfaces that cannot be completely immobilised without friction. They turn out to be exactly the surfaces of the lower Reuleaux pairs.

In order to deal with the dynamics of robots, the inertia properties of rigid bodies must be studied. In standard dynamics texts, the motion of the centre of mass and the rotation about the centre of mass are treated separately. For robots, it is more convenient to use a six-dimensional notation, which does not separate the rotational and translational motion. This leads to a six-by-six inertia matrix for a rigid body and also allows a modern exposition of some ideas due to Ball, namely conjugate screws and principal screws of inertia. The standard theory of robot dynamics is presented in two ways, first as a simple Newtonian-style approach, and then using Lagrangian dynamics. The Lagrangian approach leads to a simple study of small oscillations of the end-effector of a robot and reintroduces what Ball termed harmonic screws. The neat formalism used means that the equations of motion for a simple robot can be studied quite easily. This advantage is used to look at the design of robots with a view to simplifying their dynamics. Several approaches to this problem are considered.

The dynamics of robots with end-effector constraints and the dynamics of robots with star structures is also investigated. This allows the description of the dynamics of parallel manipulators and some simple examples of these are presented.

In Chapter 15 some deeper applications of differential geometry are explored. Three applications are studied: the mobility of overconstrained mechanisms, the control of robots along geodesic paths, and hybrid control.

The original book was never intended as an encyclopedic account of "robot geometry", but over the last few years this field has expanded so much that it is no longer even feasible to catalogue the omissions. The criterion for selecting material for this book is still a reliance on the methods outlined in the first few chapters of the book, essentially elementary differential geometry.

However, one omission that I would like to mention is the field of robot vision. A central problem in robot vision is to find the rigid motion undergone by the camera using information derived from the images. There are many other interesting geometric problems in this area, see Kanatani [61] for example. I feel that this area is so large and with very specific problems that it deserves separate treatment.

I would like to thank the many people who pointed out errors in the original book, in particular Charles Wampler, Andreas Ruf and Ross McAree. I met Pertti Lounesto shortly before his untimely death in 2002. Naturally he found an error in the chapter on Clifford algebra in the original book, but this is

almost a source of pride for me. His plans to apply his considerable knowledge and skill to mathematical problems in robotics were tragically cut short.

It is also with sadness that I report that Ken Hunt and Joe Duffy both passed away in 2002. Both made substantial contributions to the fields of robotics and kinematics and both will be greatly missed.

London 2003

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Geometric Fundamentals of Robotics

Contents

1
Introduction

1.1 Theoretical Robotics?

In May 2000 there was a meeting at the National Science Foundation in Arlington Virginia on "The Interplay between Mathematics and Robotics". Many leading experts in the U.S. discussed the importance of mathematics in robotics and also the role that robotic problems could play in the development of mathematics. The experts gave a broad overview of the problems they saw as important and worth studying. Their list was long and touched on many branches of mathematics and many areas in robotics.

Robotics is a practical discipline. It grew out of engineers' ability to build very sophisticated machines that combine computer control with electro-mechanical actuators and sensors. Any theory in the subject must take account of what is practically possible with real machines. Nevertheless, there is clearly a place for a theoretical side to the subject.

Of course, by definition theory is always useless, otherwise it wouldn't be theory! But surely all disciplines recognise the need for sound theoretical underpinnings. The question really is whether the theoretical underpinnings of robotics are distinct or just a part of the general theory used in the disciplines that make up robotics. One cannot sensibly separate say, a theory of robot mechanisms from the general theory of mechanisms and linkages. However, there is something special about robotics and that is the central importance of the group of rigid body motions $SE(3)$. That is not to say that theory not involving this group is not robotics nor that other disciplines can't profitably use this group. Its just that I see this as a major theme running through much of robotics: The links of a robot are not really rigid, but to a first approximation they are. The motions allowed by the joints of the robot are rigid body motions. The payload