

Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

8260021

31

Berc Rustem

Projection Methods
in Constrained Optimisation
and Applications
to Optimal Policy Decisions



Springer-Verlag
Berlin · Heidelberg · New York

8260021

Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

31



E8260021

Berc Rustem

Projection Methods
in Constrained Optimisation
and Applications
to Optimal Policy Decisions



Springer-Verlag
Berlin Heidelberg New York 1981

Series Editors

A. V. Balakrishnan · M. Thoma

Advisory Board

L. D. Davisson · A. G. J. MacFarlane · H. Kwakernaak

J. L. Massey · Ya. Z. Tsyarkin · A. J. Viterbi

Author

Dr. Berc Rustem

Control Section

Dept. of Electrical Engineering

Imperial College of Science and Technology

Exhibition Road

London. SW7 2BT – England



ISBN 3-540-10646-4 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-10646-4 Springer-Verlag New York Heidelberg Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, re-printing, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks.

Under § 54 of the German Copyright Law where copies are made for other than private use a fee is payable to 'Verwertungsgesellschaft Wort', Munich.

© Springer-Verlag Berlin Heidelberg 1981

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2061/3020-543210

Lecture Notes in Control and Information Sciences

Edited by A. V. Balakrishnan and M. Thoma

Vol. 1: Distributed Parameter Systems: Modelling and Identification

Proceedings of the IFIP Working Conference, Rome, Italy, June 21–26, 1976

Edited by A. Ruberti
V, 458 pages. 1978

Vol. 2: New Trends in Systems Analysis

International Symposium, Versailles, December 13–17, 1976

Edited by A. Bensoussan and J. L. Lions
VII, 759 pages. 1977

Vol. 3: Differential Games and Applications

Proceedings of a Workshop, Enschede, Netherlands, March 16–25, 1977

Edited by P. Hagedorn, H. W. Knobloch, and G. J. Olsder
XII, 236 pages. 1977

Vol. 4: M. A. Crane, A. J. Lemoine

An Introduction to the Regenerative Method for Simulation Analysis
VII, 111 pages. 1977

Vol. 5: David J. Clements, Brian D. O. Anderson

Singular Optimal Control: The Linear Quadratic Problem
V, 93 pages. 1978

Vol. 6: Optimization Techniques

Proceedings of the 8th IFIP Conference on Optimization Techniques, Würzburg, September 5–9, 1977
Part 1

Edited by J. Stoer
XIII, 528 pages. 1978

Vol. 7: Optimization Techniques

Proceedings of the 8th IFIP Conference on Optimization Techniques, Würzburg, September 5–9, 1977
Part 2

Edited by J. Stoer
XIII, 512 pages. 1978

Vol. 8: R. F. Curtain, A. J. Pritchard

Infinite Dimensional Linear Systems Theory
VII, 298 pages. 1978

Vol. 9: Y. M. El-Fattah, C. Foulard

Learning Systems:
Decision, Simulation, and Control
VII, 119 pages. 1978

Vol. 10: J. M. Maciejowski

The Modelling of Systems with Small Observation Sets
VI, 241 pages. 1978

Vol. 11: Y. Sewaragi, T. Soeda, S. Omatu

Modelling, Estimation, and Their Applications for Distributed Parameter Systems
VI, 269 pages. 1978

Vol. 12: I. Postlethwaite, A. G. J. McFarlane

A Complex Variable Approach to the Analysis of Linear Multivariable Feedback Systems
IV, 177 pages. 1979

Vol. 13: E. D. Sontag

Polynomial Response Maps
VIII, 168 pages. 1979

Vol. 14: International Symposium on Systems Optimization and Analysis

Rocquencourt, December 11–13, 1978;
IRIA LABORIA

Edited by A. Bensoussan and J. Lions
VIII, 332 pages. 1979

Vol. 15: Semi-Infinite Programming

Proceedings of a Workshop, Bad Honnef, August 30 – September 1, 1978
V, 180 pages. 1979

Vol. 16: Stochastic Control Theory and Stochastic Differential Systems

Proceedings of a Workshop of the „Sonderforschungsbereich 72 der Deutschen Forschungsgemeinschaft an der Universität Bonn“ which took place in January 1979 at Bad Honnef
VIII, 615 pages. 1979

Vol. 17: O. I. Franksen, P. Falster, F. J. Evans

Qualitative Aspects of Large Scale Systems
Developing Design Rules Using APL
XII, 119 pages. 1979

Vol. 18: Modelling and Optimization of Complex Systems

Proceedings of the IFIP-TC 7 Working Conference
Novosibirsk, USSR, 3–9 July, 1978

Edited by G. I. Marchuk
VI, 293 pages. 1979

Vol. 19: Global and Large Scale System Models

Proceedings of the Center for Advanced Studies (CAS) International Summer Seminar

Dubrovnik, Yugoslavia, August 21–26, 1978
Edited by B. Lazarevic

VIII, 232 pages. 1979

Vol. 20: B. Egardt

Stability of Adaptive Controllers
V, 158 pages. 1979

Vol. 21: Martin B. Zarrop

Optimal Experiment Design for Dynamic System Identification
X, 197 pages. 1979

For further listing of published volumes please turn over to inside of back cover.

ABSTRACT



This work is concerned with projection methods in constrained optimisation and the application of projection techniques to policy optimisation problems. The constrained optimisation problem of minimising a nonlinear function of n variables subject to equality and inequality constraints is also known as the nonlinear programming problem. Projection methods for constrained minimisation involve projections of descent directions. The basic idea underlying these methods is the principle of projections which may be considered to be the generalisation of the fact that in n -dimensional Euclidean space the shortest vector from a point to a subspace is orthogonal to the subspace.

In effect, this work consists of two parts. The first part, Chapters 1-4, is concerned with the development of projection techniques for different aspects of constrained optimisation. Chapter 1 provides a unified approach to the derivation of projection techniques and reviews existing methods. The application of projection techniques to the computation of a feasible point of a linearly constrained region is discussed in Chapter 2. In Chapter 3 a projection method is discussed for the quadratic programming problem of minimising a quadratic objective function subject to inequality constraints. This method requires an initial feasible point. In Chapter 4 a method is discussed for solving the nonlinear programming problem. This method requires solutions to quadratic minimisation subproblems.

The second part of the work is concerned with the application of projection techniques to computational problems in policy optimisation. A fundamental problem in the optimisation of policy decisions is the specification of a suitable objective function. In Chapter 5 an iterative method is given for specifying objective functions. In Chapter 6 policy optimisation algorithms, based on extensions of projection methods in Chapter 1, are discussed. These algorithms minimise quadratic objective functions subject to large nonlinear models treated as equality constraints.

ACKNOWLEDGEMENTS



I would like to thank Dr. Heather Liddell for her valuable guidance and great patience.

I am also grateful to Dr. Alan Frieze for helpful discussions and valuable comments and to Drs. Kumaraswamy Velupillai, Sean Holly for providing the economics background of Chapter 5.

Finally, I wish to thank Miss Claudia Schamberger for patiently typing the manuscript.

CONVENTIONS AND SYMBOLS

Assumptions, Definitions, Lemmas, Theorems etc. are given a number preceded by the chapter number. Equations and algorithms are given a number, preceded by the section and chapter numbers in which they occur.

The end of a proof or a particular train of thought is denoted by \square . The symbol \triangleq denotes 'defined equal to'.

Since each chapter deals with a different aspect of constrained optimisation (e.g. feasible point algorithms, quadratic programming, nonlinear programming, etc.) the symbols used in each chapter have been slightly adapted to the needs of that chapter. The usage of commonly used symbols is given below.

Symbol

\underline{a}	coefficient vector of the linear term of the quadratic function
\underline{b}	right hand side vector of a system of linear equalities and/or inequalities
\underline{d}	descent direction
E^n	Euclidean n-space
F	policy maker's admissible region for \underline{U} and \underline{Y}
$F(\underline{Y}, \underline{U}) = \underline{0}$	Equations of the econometric model
$f(\underline{x})$	nonlinear objective function, $\underline{x} \in E^n$

G	Hessian matrix
$G(\underline{U})$	reduced form of $J(\underline{Y}, \underline{U})$ with \underline{Y} eliminated
$\underline{g}(\underline{U})$	reduced form equations of the econometric model
$\underline{g}(\underline{x})$	vector of inequality and/or equality constraints
H	inverse Hessian matrix
$\underline{h}(\underline{x})$	vector of equality constraints
$I(\underline{x})$	index set of active constraints at \underline{x}
$J(\underline{Y}, \underline{U})$	quadratic performance index for optimal control
$L(\underline{x}, \underline{\lambda})$	Lagrangian function
N	matrix of constraint normals
$P, P[\underline{H}], \bar{P}, \hat{P}$	projection operators
q, \bar{q}	quadratic functions approximating $f(\underline{x})$
\hat{q}	\bar{q} with approximate Hessian
R	the feasible region
$S(\underline{x})$	index set of the satisfied constraints at \underline{x}
\underline{U}	vector of controls (policy instruments)
$V(\underline{x})$	index set of the violated constraints at \underline{x}
\underline{x}_u	the point reached along an unconstrained direction
\underline{x}_p	the projection of \underline{x}_u
\underline{Y}	vector of outputs (endogenous variables)

α	steplength factor along an unconstrained direction
γ	change in the gradient vector of the objective function
$\underline{\theta}(\underline{x})$	residual vector of a system of linear equalities and/or inequalities evaluated at \underline{x}
$\underline{\lambda}$	the multiplier vector
τ	steplength along a projected direction
$\nabla_z(\cdot)$	gradient vector of (\cdot) with respect to z (z is omitted wherever the argument is obvious)

CONTENTS

	Page
Conventions and Symbols	XIII
Chapter 1 NONLINEARLY CONSTRAINED OPTIMISATION TECHNIQUES BASED ON PROJECTIONS	
1.1 Introduction	1
1.1.1 Fundamental Concepts and Results	3
1.1.2 Projection Methods for Constrained Minimisation	7
1.1.3 Methods Based on Computing Bases for $\bar{\Omega}$	16
1.1.4 Methods Based on Computing Bases for Ω_0	23
1.2 Extensions of Projection Algorithms for solving the Linear Equality Constrained Problem	29
1.2.1 Extensions for Nonlinear Constraints	30
1.2.2 Active Set Strategies	38
1.3 Review and Original Contributions	40

Chapter 2 PROJECTION METHODS FOR COMPUTING
FEASIBLE POINTS OF LINEARLY
CONSTRAINED REGIONS

2.1	Introduction	43
2.1.1	The Feasible Region	44
2.1.2	The Linear Dependence of Constraint Normals	48
2.1.3	A Projection Algorithm	52
2.2	Projection Algorithms, Redundancy and Degeneracy	61
2.2.1	Degeneracy, Redundant Constraints, Infeasibility and Computational Considerations	61
2.2.2	Projection Algorithms for Computing Feasible Points of a Linearly Constrained Region	67
2.3	Concluding Remarks	74

Chapter 3 AN ALGORITHM FOR POSITIVE DEFINITE
QUADRATIC PROGRAMMING

3.1	Introduction	76
3.1.1	The Unconstrained Minimum	77
3.1.2	The Quadratic Programming Problem	78
3.2	Projection Operators	79
3.2.1	Preliminaries	79
3.2.2	Projections in E^n	80
3.3	Motivation for a Positive Definite Quadratic Programming Algorithm	82
3.3.1	The Quadratic Objective Function	82
3.3.2	Inequality Constraints and the Unconstrained Optimum	90
3.4	The Algorithm	96
3.4.1	A Positive Definite Quadratic Programming Algorithm	97
3.4.2	The Positive Semi-Definite and Indefinite Cases	100
3.4.3	Extensions to Nonlinear Constraints	101

3.5	Recurrance Relations	110
3.5.1	Updating N^* and Related Operators	111
3.5.2	Updating $(N_m^T H N_m)^{-1}$	120
3.5.3	Linear Dependence	123
3.6	Convergence	125
3.7	Concluding Remarks	130
	Appendix: Numerical Results	131

Chapter 4 OPTIMISATION WITH LINEAR AND NONLINEAR CONSTRAINTS

4.1	Introduction	135
4.1.1	Projection Algorithms for Nonlinear Programming	137
4.1.2	Approximations to the Inverse Hessian and the Computation of Constrained Descent Directions	144
4.2	Convergence of the Algorithm	156
4.2.1	Stepsize Strategies	156

4.2.2	Convergence Proofs:	
	Exact Second Derivatives	167
4.2.3	Convergence Proofs:	
	Approximate Second	
	Derivatives	180
4.3	Concluding Remarks	218
Chapter 5	THE ITERATIVE SPECIFICATION OF OBJECTIVE FUNCTIONS IN ECONOMIC POLICY OPTIMISATION : AN APPLICATION OF PROJECTION METHODS	
5.1	Introduction	219
5.2	Preliminaries	220
5.3	The Conceptual Algorithm	228
5.4	Computational Considerations for the Implementation of the Algorithm	233
5.5	Conclusions	238
	Appendix: A Numerical Example	240
A.1	The Structure of the Objective Function	243
A.2	First Respecification	245

A.3	Second Respecification	249
A.4	Third Respecification	252
Chapter 6	POLICY OPTIMISATION ALGORITHMS FOR NONLINEAR ECONOMETRIC MODELS	
6.1	Introduction	258
6.2	Policy Optimisation and Simulations	260
6.3	A Newton-Type Algorithm	263
6.4	A Quasi-Newton Algorithm	274
6.5	Concluding Remarks	282
	Appendix: Numerical Results	283
Chapter 7	SUGGESTIONS FOR FURTHER RESEARCH	289
	REFERENCES FOR CHAPTER 1	291
	REFERENCES FOR CHAPTER 2	299
	REFERENCES FOR CHAPTER 3	301
	REFERENCES FOR CHAPTER 4	304
	REFERENCES FOR CHAPTER 5	309
	REFERENCES FOR CHAPTER 6	311
	REFERENCES FOR CHAPTER 7	315

CHAPTER 1

NONLINEARLY CONSTRAINED OPTIMISATION TECHNIQUES BASED ON PROJECTIONS

1.1 INTRODUCTION

This work is concerned with projection methods for constrained optimisation and the application of projection techniques to policy optimisation problems.

The principle of projections may be considered to be the generalisation of the fact that in n -dimensional Euclidean space the shortest vector from a point to a subspace is orthogonal to the subspace. Projections have a wide application in mathematics and especially in systems theory. For example, in the analysis of distributed systems, the method of orthogonal projections is applied by Mikhlin (1964) to solve the Dirichlet problem. Another important application arises in the derivation of the least-squares, minimum variance and recursive estimation (Kalman filtering) techniques in statistical estimation (see, e.g. Davis (1977), Luenberger (1969, Chapter 4)). Such techniques have found wide application in engineering and econometrics (see, e.g. Rustem and Velupillai (1978, 1979)). In all these applications a suitable normed linear vector space such as a Hilbert or Euclidean space is chosen and the projection of vectors in this space onto a