A STORY OF

LIGHT

A Short Introduction to Quantum Field Theory of Quarks and Leptons

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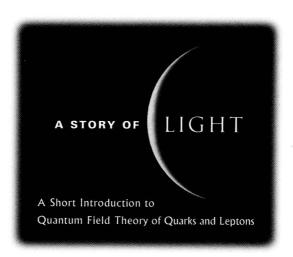
An Introduction to Quantum Field Theory of Quarks and Leptons

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For Eema, Grace and Leilani

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Prologue

The relativistic quantum field theory, or quantum field theory (QFT) for short, is the theoretical edifice of the standard model of elementary particle physics. One might go so far as to say that the standard model is the quantum field theory. Having said that as the opening statement of this book, we must be mindful that both quantum field theory and the standard model of elementary particle physics are topics that are not necessarily familiar to many individuals. They are subject areas that are certainly not familiar to those outside the specialty of elementary particle physics, and in some cases not too well grasped even by those in the specialty.

The Standard Model of elementary particle physics is a term that has come into prominence as it became the paradigm of particle physics for the last three decades. In brief, the standard model aims to understand and explain three of the four fundamental forces — the electromagnetic, strong nuclear and weak nuclear — that define the dynamics of the basic constituents of all known matter in the universe. As such, it consists of two interrelated parts: the part

¹The fourth force of nature, gravity, does not come into play in the scale of the mass of elementary particles and is not included in the standard model. Attempts

that deals with the question of what are the basic building blocks of matter and the second part concerned with the question of what is the theoretical framework for describing the interactions among these fundamental constituents of matter.

A century after the original discovery of quantum of light by Max Planck in 1900 and its subsequent metamorphosis into photon, the zero-mass particle of light, by Albert Einstein in 1905, we have come to identify the basic constituents of matter to be quarks and leptons — the up, down, strange, charm, top and bottom, for quarks, and the electron, muon, tauon, electron-type neutrino, muon-type neutrino, and tauon-type neutrino, for leptons. The three forces are understood as the exchange of "quanta" of each force — photons for the electromagnetic force, weak bosons for the weak nuclear force, and gluons for the strong nuclear force. These particles, some old, such as photons and electrons and some relatively new, such as the top and bottom quarks or the tauons and their associated neutrinos, represent our latest understanding of what are the basic constituents of known matter in the universe.

There are scores of books available which discuss the basic particles of matter, at every level of expertise. For a general readership, we can mention two books that contain no or very little mathematics, *Quarks and Gluons* by myself and *Facts and Mysteries in Elementary Particle Physics* by Martinus Veltman.²

The theoretical framework for the three forces or interactions is quantum field theory, that is, the relativistic quantum field theory. Each force has its own form, and again, some old and some new. Quantum electrodynamics, QED for short, was fully developed by the end of the 1940s and is the oldest — and more significantly, the only truly successful quantum field theory to date — of the family. Quantum chromodynamics, QCD, is the framework for the strong nuclear force that is mediated by exchanges of gluons. It was initiated

to merge gravity with the standard model have spawned such ideas as the grand unified theory, supersymmetry, and supersting, the so-called theory of everything. These topics are not discussed in this book.

² Quarks and Gluons by M. Y. Han, World Scientific (1999); Facts and Mysteries in Elementary Particle Physics by Martinus Veltman, World Scientific (2003).

in the 1960s and has been continually developed since, but it is far from becoming a completely successful quantum field theory yet. The theory for the weak nuclear force, in its modern form, was also started in the 1960s, and in the 1970s and 1980s, it was merged with quantum electrodynamics to form a unified quantum field theory in which the two forces — the electromagnetic and weak nuclear — were "unified" into a single force referred to as the electroweak force. Often this new unified theory is referred to as the quantum flavor dynamics, QFD. Thus, the quantum field theory of the standard model consists of two independent components — quantum chromodynamics and quantum flavor dynamics, the latter subsuming quantum electrodynamics.

Despite the abundant availability of books, at all levels, on basic building blocks of matter, when it comes to the subject of relativistic quantum field theory, while there are several excellent textbooks at the graduate level, few resources are available at an undergraduate level. The reason for this paucity is not difficult to understand. The subject of quantum field theory is a rather difficult one even for graduate students in physics. Unless a graduate student is interested in specializing into elementary particle physics, in fact, most graduate students are not required to take a course in quantum field theory. It is definitely a highly specialized course. Quantum field theory thus remains, while a familiar term, a distant topic. Many have not had the opportunity to grasp what the subject is all about, and for those with some rudimentary knowledge of physics at an undergraduate level beyond the general physics, the subject lies well beyond their reach.

The main purpose of this book is to try to fill this gap by bringing out the conceptual understanding of the relativistic quantum field theory, with minimum of mathematical complexities. This book is not at all intended to be a graduate level textbook, but represents my attempt to discuss the essential aspects of quantum field theory requiring only some rudimentary knowledge of the Lagrangian and Hamiltonian formulation of Newtonian mechanics, special theory of relativity and quantum mechanics.

There is another theme in this book and it is this. Throughout the course of development of quantum field theory, from the original quantum electrodynamics in which the Planck–Einstein photon is deemed as the natural consequence of field quantization to the present-day development of the gauge field theory for quarks and leptons, the theories of electromagnetic field have been — and continue to be — a consistently useful model for other forces to emulate. In this process of emulating theories of electromagnetic field, the concept of particles and fields would go through three distinct phases of evolution: separate and distinct concepts in classical physics, the particle-wave duality in quantum mechanics, and finally, particles as the quanta of quantized field in quantum field theory. As we elaborate on this three-stage evolution, we will see that the photon has been — and continues to be — the guiding light for the entire field of relativistic quantum field theory, the theoretical edifice of the standard model of elementary particle physics.

1

Particles and Fields I: Dichotomy

One may have wondered when first learning Newtonian mechanics, also called the classical mechanics, why the concept of a field, the force field of gravity in this case, is hardly mentioned. One usually starts out with the description of motion under constant acceleration — the downward pull of gravity with the value of $9.81\,\mathrm{m/s^2}$. Even when the universal law of gravity is discussed, for example, to explain the Kepler's laws, we do not really get into any detailed analyses of the force field of gravity.

In classical mechanics the primary definition of matter is the point mass, and the emphasis is on the laws of motion for point masses under the influence of force. The focus is on the laws of motion rather than the nature of force field, which is not really surprising when we consider the simplicity of the terrestrial gravitational force field — uniform and in one parallel direction, straight down toward the ground. A point mass is an abstraction of matter that carries mass and occupies one position at one moment of time and this notion of a point mass is diagonally opposite from the notion of a field, which, by definition, is an extended concept, spread out over a region of space.

As we proceed from the study of classical mechanics to that of classical electromagnetism, we immediately notice a big change; from

day one it is all about fields. First the electric field, then the magnetic field, and then the single combined entity, the electromagnetic field. No sooner than the Coulomb's law is written down, one defines the electric field and its spatial dependence is determined by Gauss' Law. Likewise, Ampere's Law determines the magnetic field and finally the laws of Faraday and Maxwell lead to the spatial as well as temporal dependence of electromagnetic field.

This dichotomy of the concept of point particle and that of field is in fact as old as the history of physics. From the very beginning, back in the 17th century, there were two distinct views of the physical nature of light. Newton advocated the particle picture — the corpuscular theory of light — whereas Christian Huygens advanced the wave theory of light. For some time — for almost a century and half — these two opposing views remained compatible with what was then known about light — refraction, reflection, lenses, etc. Only when in 1801 Thomas Young demonstrated the wave nature of light by the classic double-slit interference experiment, with alternating constructive and destructive interference patterns, the wave theory triumphed over the particle theory of light.

One might have wondered why the notion of field did not play a prominent role in the initial formulation of Newtonian mechanics, especially since both the gravitational force law and the Coulomb's law obey the identical inverse square force law:

$$F = G \frac{m_1 m_2}{r^2} \quad \text{for gravity}$$

and

$$F = k \frac{q_1 q_2}{r^2}$$
 for Coulomb's law

where G and k are the respective force constants, m is mass and q is the electric charge.

The disparity is simply a practical matter of scale. At the terrestrial level, in our everyday world, the inverse square law really does not come into play; the curvature of the surface of the earth is approximated by a flat ground and the gravitational force lines directed toward the center of the earth become, in this approximation, parallel lines pointing downward. In this scale of things, the field aspect of gravity is just too simple to be taken into account. There is no need to bring in any analyses of the gravitational field in the flat surface approximation.

On the contrary, with electric and magnetic forces, we notice and measure in the scale of tabletop experiments the spatial and temporal variations of these fields. The gradients, divergences and curls, to use the language of differential vector calculus, of the electric and magnetic fields come into play in the scale of the human-sized world and this is why the study of electromagnetism always starts off with the definition of electric and magnetic fields.

This well-defined dichotomy of particles and fields, diagonally opposite concepts in classical physics, would evolve through many twists and turns in the twentieth century physics of relativity and quantum mechanics, ending up eventually with the primacy of the concept of field over that of particle in the framework of quantum field theory.

The process of evolution of the concepts of particles and fields have taken a quite disparate path. The Newtonian mechanics has evolved through several steps, some quite drastic. First, there was the Lagrangian and Hamiltonian formulation of mechanics. One of the most important outcomes of this formalism is the definition of what is called the canonically conjugate momentum and this would pave the way for the transition from classical mechanics to quantum mechanics. Quantum field theory could not have developed had it not been the idea of canonically conjugate momentum defined within the Lagrangian and Hamiltonian formalism. As quantum mechanics is merged with special theory of relativity, the culmination of the particle view was reached in the form of relativistic quantum mechanical wave equations, such as the Klein-Gordon and Dirac equations. wherein the wavefunction solutions of these equations provide the relativistic quantum mechanical description of a particle. (More on these equations in later chapters.)

In contradistinction to this development of particle theory, the field view of classical electromagnetism remained almost totally unmodified. The equation of motion for charged particles in an electromagnetic field is naturally accommodated in the Lagrangian and Hamiltonian formalism. In the Lagrangian formulation of classical mechanics, Maxwell's equations find a natural place by being one of the few examples of what is called the velocity-dependent potentials (more on this in the next chapter). The very definition of the canonically conjugate momentum for charged particles to be the sum of mechanical momentum and the vector potential of the electromagnetic field, discovered back in the 19th century, is in fact the foundation for quantum electrodynamics of the 20th century.

The contrast between the mechanics of particles and the field theory of electromagnetic fields becomes sharper when dealing with the special theory of relativity. The errors of Newtonian mechanics at speeds approaching the speed of light are quite dramatic, and of course, the very foundation of mechanics had to be drastically modified by the relativity of Einstein. Maxwell's equations for the electromagnetic field, on the other hand, required no modifications whatsoever at high speeds; the equations are valid for all ranges of speeds involved, from zero to all the way up to the speed of light. At first, this may strike as quite surprising, but the fact of the matter is that Maxwell's equations lead directly to the wave equations for propagating electromagnetic radiation — light itself. Maxwell's theory of the electromagnetic field is already fully relativistic and hence need no modifications at all.

The development of relativistic quantum mechanics demonstrates quite dramatically the primacy of the classical field concept over that of particles. To cite an important example, in relativistic quantum mechanics, the first and foremost wave equation obeyed by particles of any spin, both fermions of half-integer spin and bosons of integer spin, is the Klein–Gordon equation. Fermions must also satisfy the Dirac equation in addition to the Klein–Gordon equation (more on this in later chapters).

For a vector field $\phi_{\mu}(x)$ $[\mu = 0, 1, 2, 3]$ for spin one particles with mass m, the Klein–Gordon equation is¹

$$(\partial_{\lambda}\partial^{\lambda}+m^2)\phi_{\mu}(x)=0$$

¹Notations and the natural unit system are given in Appendices 1 and 2.

where

$$\partial_{\lambda}\partial^{\lambda} = \frac{\partial^2}{\partial t^2} - \nabla^2.$$

For the special case of mass zero particles, of spin one, the Klein–Gordon equation reduces to

$$\partial_{\lambda}\partial^{\lambda}\phi_{\mu}(x) = 0.$$

The classical wave equation for the electromagnetic four-vector potential $A_{\mu}(x)$, on the other hand, in the source-free region is

$$\partial_{\lambda}\partial^{\lambda}A_{\mu}(x) = 0.$$

An equation for a zero-mass particle of spin one (photon) in relativistic quantum mechanics turns out to be none other than the classical wave equation for the electromagnetic field of the 19th century that predates both relativity and quantum physics!

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