STUDENT SOLUTIONS MANUAL

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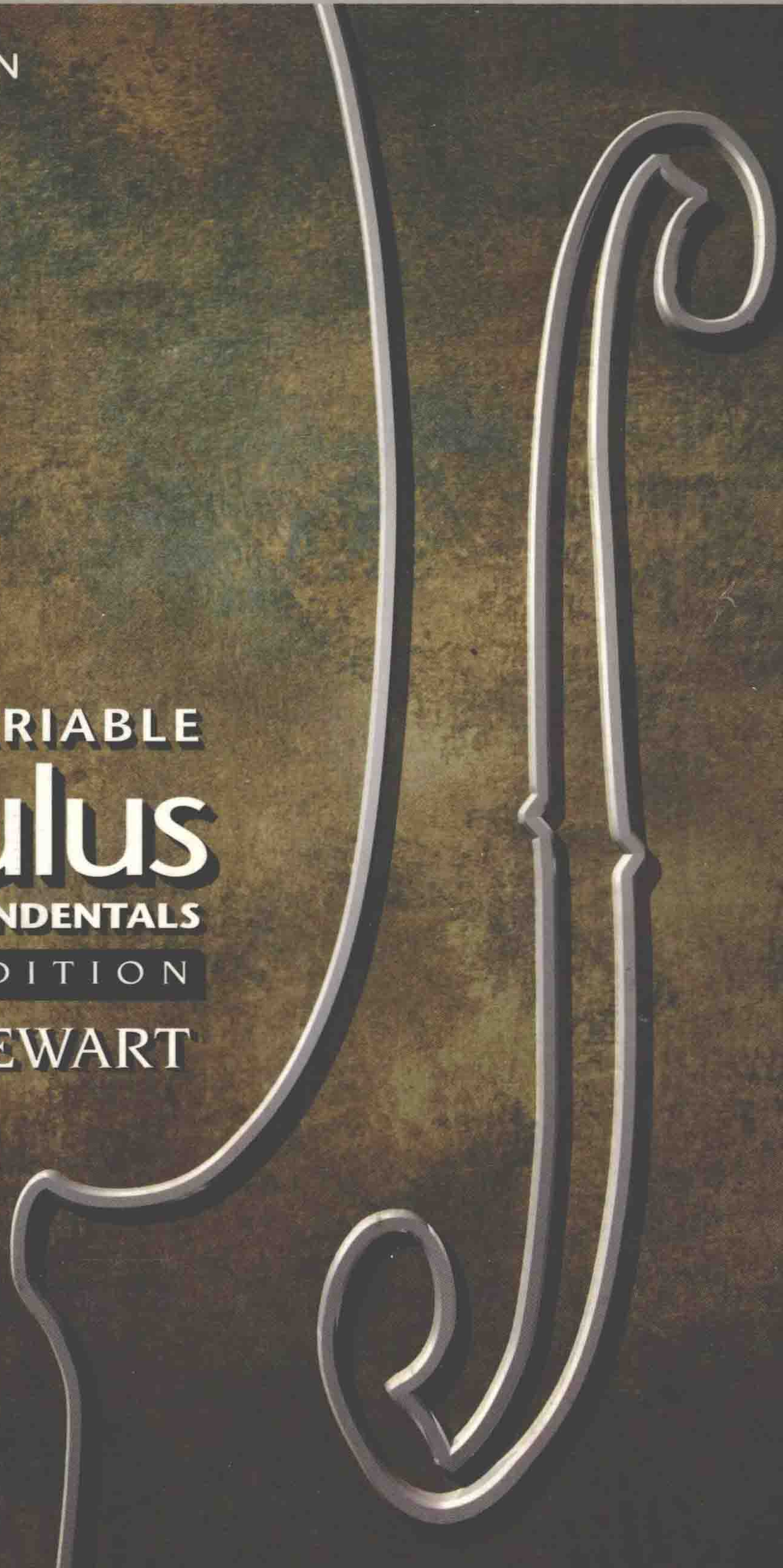

SINGLE VARIABLE

Calculus

EARLY TRANSCENDENTALS

FOURTH EDITION

JAMES STEWART

Student Solutions Manual for Stewart's

SINGLE VARIABLE CALCULUS
Early Transcendentals

FOURTH EDITION

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Preface

This *Student Solutions Manual* contains strategies for solving and solutions to selected exercises in the text *Single Variable Calculus: Early Transcendentals, Fourth Edition*, by James Stewart. It contains solutions to the odd-numbered exercises in each section, the review sections, the True-False Quizzes, and the Problem Solving sections, as well as solutions to all the exercises in the Concept Checks.

We use some non-standard notation in order to save space. If you see a symbol which you don't recognize, refer to the Table of Abbreviations and Symbols on page iv.

This manual is a text supplement and should be read along *with* the text. You should read all exercise solutions in this manual because many concept explanations are given and then used in subsequent solutions. All concepts necessary to solve a particular problem are not reviewed for every exercise. If you are having difficulty with a previously covered concept, refer back to the section where it was covered for more complete help.

A significant number of today's students are involved in various outside activities, and find it difficult, if not impossible, to attend all class sessions; this manual should help meet the needs of these students. In addition, it is our hope that this manual's solutions will enhance the understanding of all readers of the material and provide insights to solving other exercises.

We appreciate feedback concerning errors, solution correctness or style, and manual style. Any comments may be sent directly to `jcole@an.cc.mn.us`, or in care of the publisher: Brooks/Cole Publishing Company, 511 Forest Lodge Road, Pacific Grove, CA 93950.

We would like to thank Andrew Bulman-Fleming, for typesetting the manuscript; Brian Betsill, Stephanie Kuhns, and Kathi Townes, of TECH-arts, for their production services; and Carol Ann Benedict, of Brooks/Cole Publishing Company, for her patience and support. All of these people have provided invaluable help in creating this manual.

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Abbreviations and Symbols

CD	concave downward
CU	concave upward
D	the domain of f
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\stackrel{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\stackrel{j}{=}$	indicates the use of Formula j in the Table of Integrals in the back endpapers.
$\stackrel{\text{s}}{=}$	indicates the use of the substitution $\{u = \sin x,\, du = \cos x\, dx\}$.
$\stackrel{\text{c}}{=}$	indicates the use of the substitution $\{u = \cos x,\, du = -\sin x\, dx\}$.

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1 Functions and Models

1.1 Four Ways to Represent a Function

In exercises requiring estimations or approximations, your answers may vary slightly from the answers given here.

1. (a) The point $(-1, -2)$ is on the graph of f, so $f(-1) = -2$.

(b) When $x = 2$, y is about 2.8, so $f(2) \approx 2.8$.

(c) $f(x) = 2$ is equivalent to $y = 2$. When $y = 2$, we have $x = -3$ and $x = 1$.

(d) Reasonable estimates for x when $y = 0$ are $x = -2.5$ and $x = 0.3$.

(e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-3 \le x \le 3$. The range of f consists of all y-values on the graph of f. For this function, the range is $-2 \le y \le 3$.

(f) As x increases from -1 to 3, y increases from -2 to 3. Thus, f is increasing on the interval $[-1, 3]$.

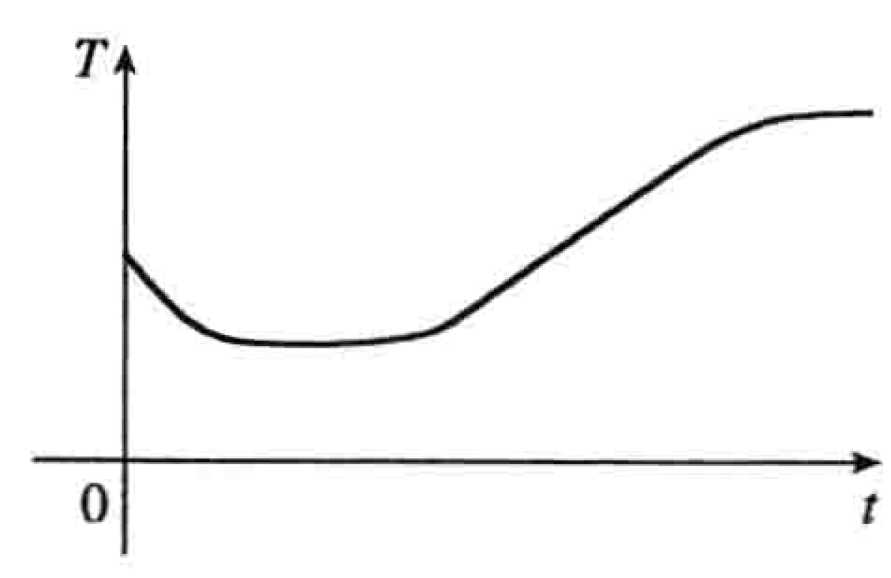
3. From Figure 1 in the text, the lowest point occurs at about $(t, a) = (12, -85)$. The highest point occurs at about $(17, 115)$. Thus, the range of the vertical ground acceleration is $-85 \le a \le 115$. In Figure 11, the range of the north-south acceleration is approximately $-325 \le a \le 485$. In Figure 12, the range of the east-west acceleration is approximately $-210 \le a \le 200$.

5. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-2, 2]$.

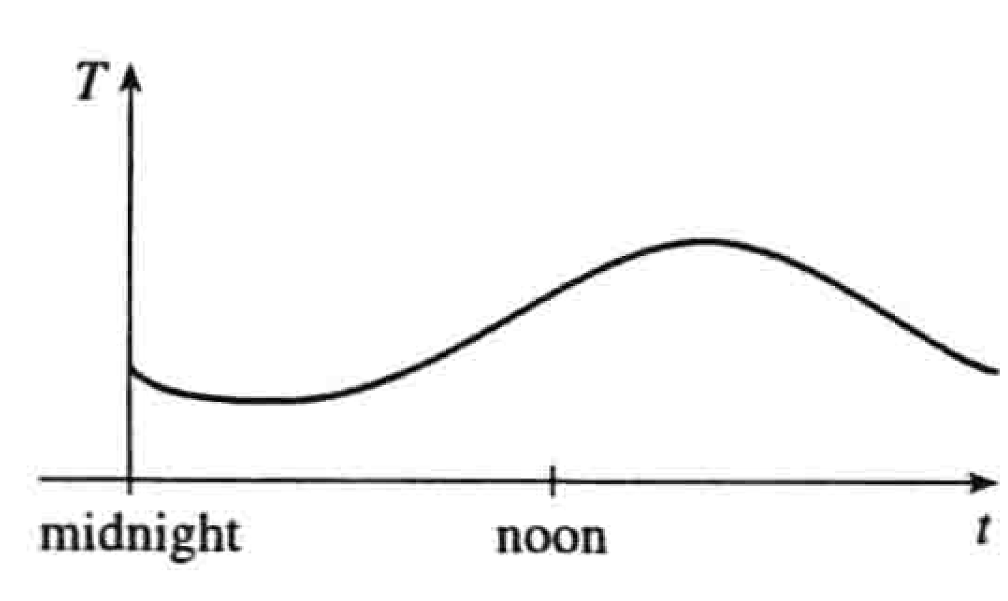
7. No, the curve is not the graph of a function since for $x = -1$ there are infinitely many points on the curve.

9. The person's weight increased to about 160 pounds at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 120 pounds for the next 5 years, then increased rapidly to about 170 pounds. The next 30 years saw a gradual increase to 190 pounds. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.

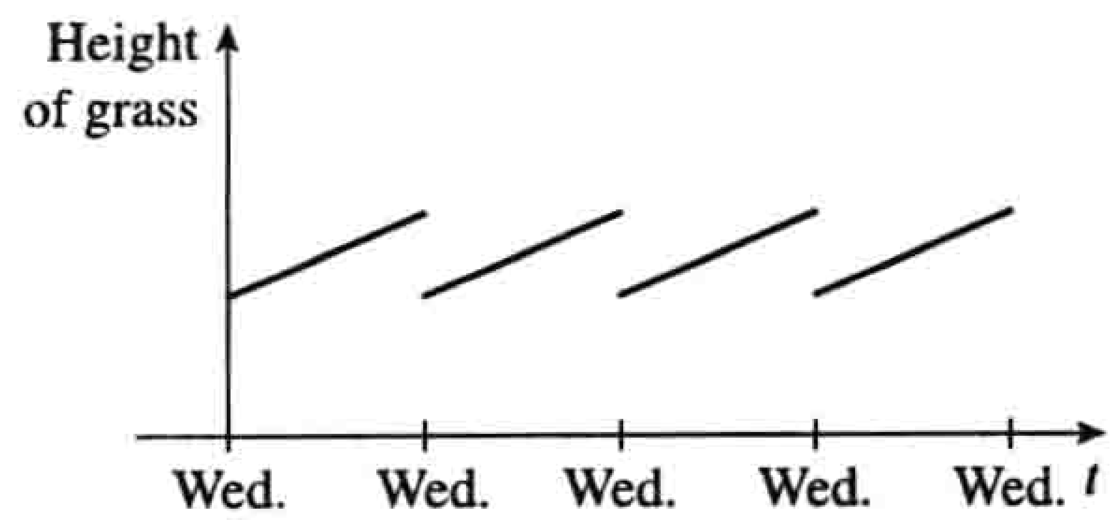
11. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



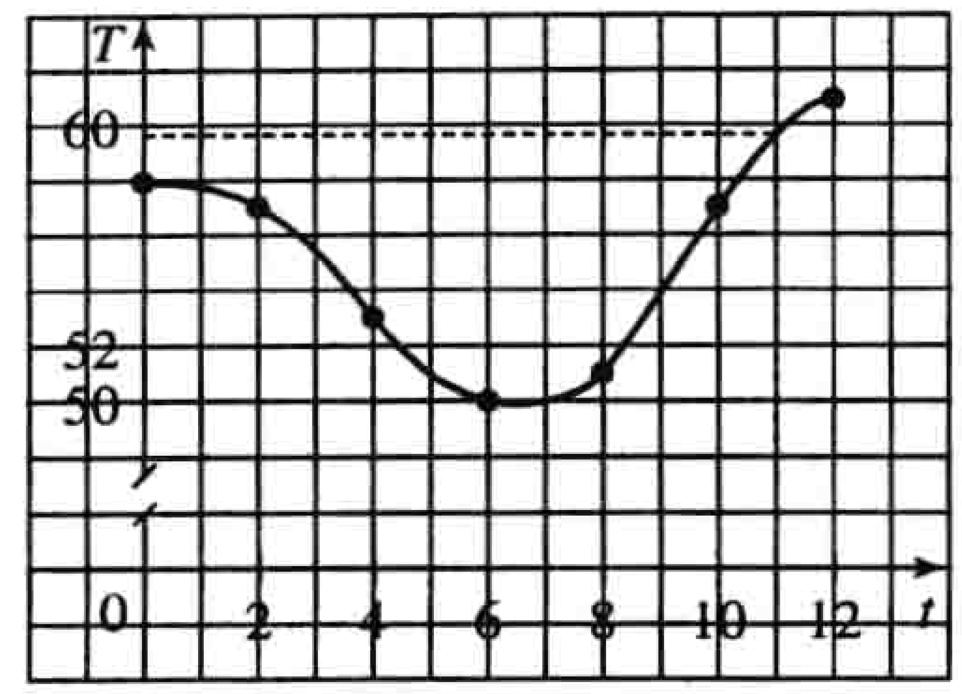
13. Of course, this graph depends strongly on the geographical location!



15.



17. (a)



(b) $T(11) \approx 59°\text{F}$

19. $f(x) = 2x^2 + 3x - 4$, so $f(0) = 2(0)^2 + 3(0) - 4 = -4$,
$f(2) = 2(2)^2 + 3(2) - 4 = 10$, $f(\sqrt{2}) = 2(\sqrt{2})^2 + 3(\sqrt{2}) - 4 = 3\sqrt{2}$,
$f(1+\sqrt{2}) = 2(1+\sqrt{2})^2 + 3(1+\sqrt{2}) - 4 = 2\left(1 + 2 + 2\sqrt{2}\right) + 3 + 3\sqrt{2} - 4 = 5 + 7\sqrt{2}$,
$f(-x) = 2(-x)^2 + 3(-x) - 4 = 2x^2 - 3x - 4$,
$f(x+1) = 2(x+1)^2 + 3(x+1) - 4 = 2\left(x^2 + 2x + 1\right) + 3x + 3 - 4 = 2x^2 + 7x + 1$,
$2f(x) = 2\left(2x^2 + 3x - 4\right) = 4x^2 + 6x - 8$, and
$f(2x) = 2(2x)^2 + 3(2x) - 4 = 2\left(4x^2\right) + 6x - 4 = 8x^2 + 6x - 4$.

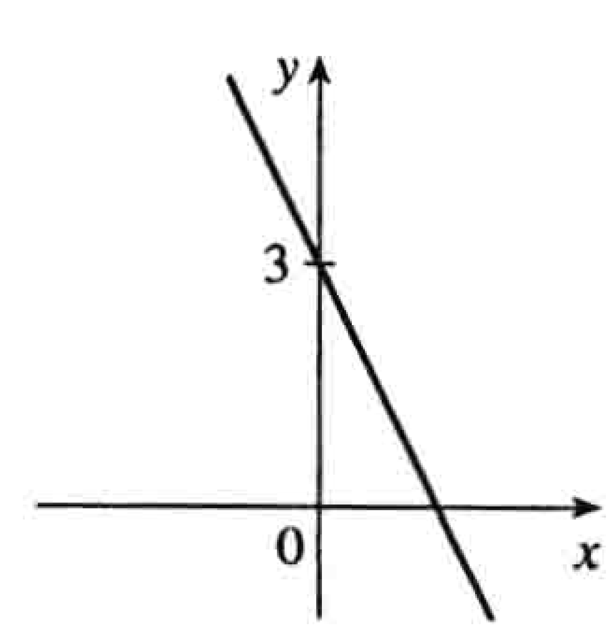
21. $f(x) = x - x^2$, so $f(2+h) = 2 + h - (2+h)^2 = 2 + h - 4 - 4h - h^2 = -\left(h^2 + 3h + 2\right)$,
$f(x+h) = x + h - (x+h)^2 = x + h - x^2 - 2xh - h^2$, and
$$\frac{f(x+h) - f(x)}{h} = \frac{x + h - x^2 - 2xh - h^2 - x + x^2}{h} = \frac{h - 2xh - h^2}{h} = 1 - 2x - h.$$

23. $f(x) = \dfrac{x+2}{x^2 - 1}$ is defined for all x except when $x^2 - 1 = 0 \quad \Leftrightarrow \quad x = 1$ or $x = -1$, so the domain is $\{x \mid x \neq \pm 1\}$.

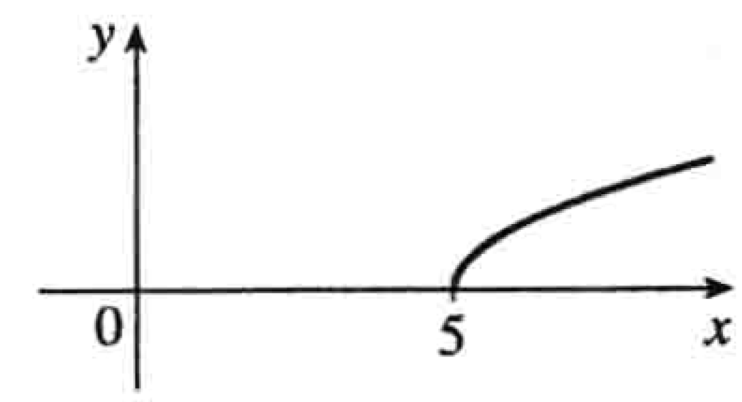
25. $g(x) = \sqrt[4]{x^2 - 6x}$ is defined when $0 \le x^2 - 6x = x(x-6) \quad \Leftrightarrow \quad x \ge 6$ or $x \le 0$, so the domain is $(-\infty, 0] \cup [6, \infty)$.

27. $f(t) = \sqrt[3]{t-1}$ is defined for every t, since every real number has a cube root. The domain is the set of all real numbers, $\mathbb{R}$.

29. $f(x) = 3 - 2x$. Domain is $\mathbb{R}$.



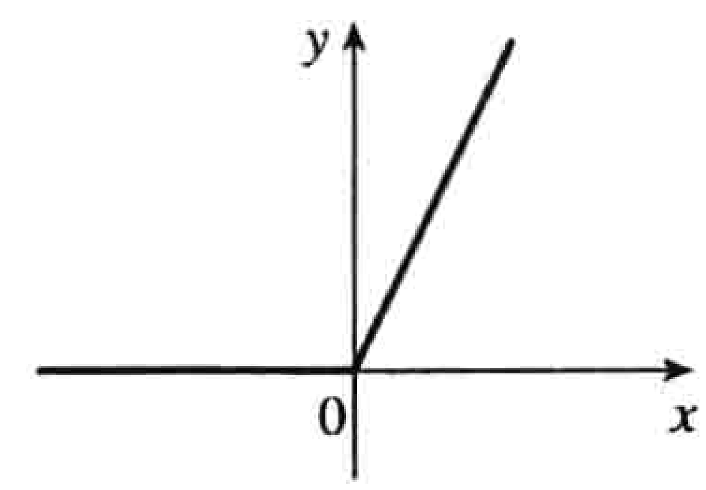
31. $g(x) = \sqrt{x-5}$ is defined when $x - 5 \geq 0$ or $x \geq 5$, so the domain is $[5, \infty)$. Since $y = \sqrt{x-5} \Rightarrow y^2 = x - 5 \Rightarrow x = y^2 + 5$, we see that g is the top half of a parabola.



33. $G(x) = |x| + x$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ we have

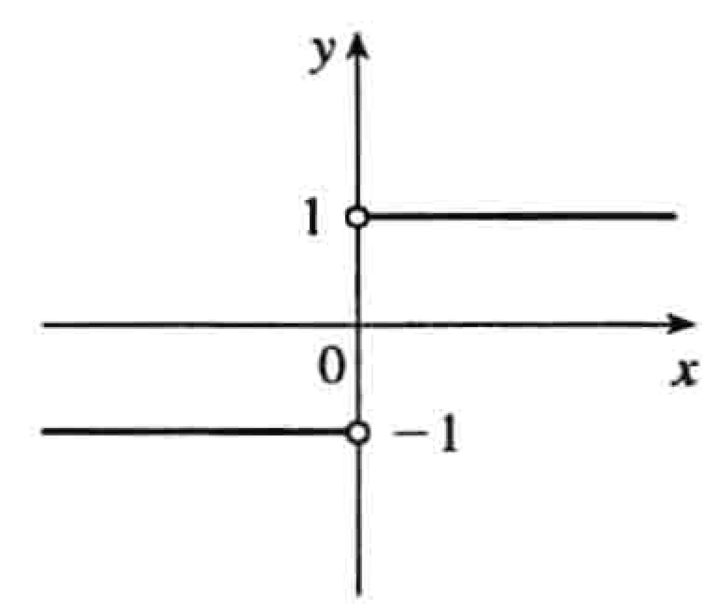
$$G(x) = \begin{cases} x + x & \text{if } x \geq 0 \\ -x + x & \text{if } x < 0 \end{cases} = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Domain is $\mathbb{R}$. Note that the negative x-axis is part of the graph of G.



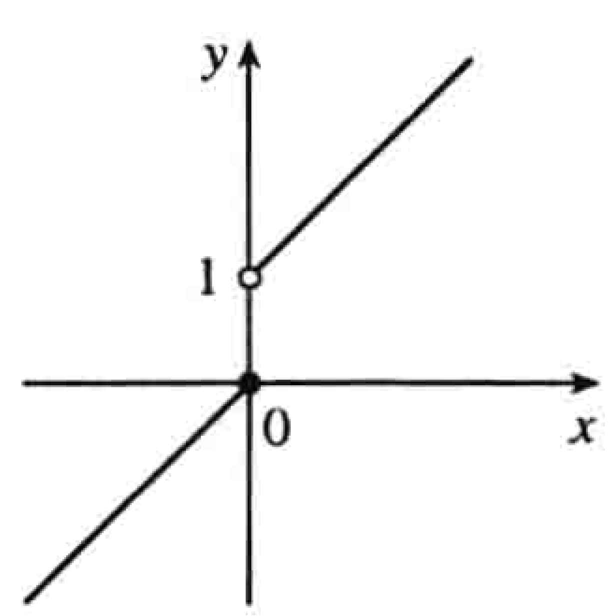
35. $f(x) = \dfrac{x}{|x|} = \left\{ \begin{array}{ll} x/x & \text{if } x > 0 \\ x/(-x) & \text{if } x < 0 \end{array} \right\} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

Note that we did not use $x \geq 0$, because $x \neq 0$. Hence, the domain of f is $\{x \mid x \neq 0\}$.



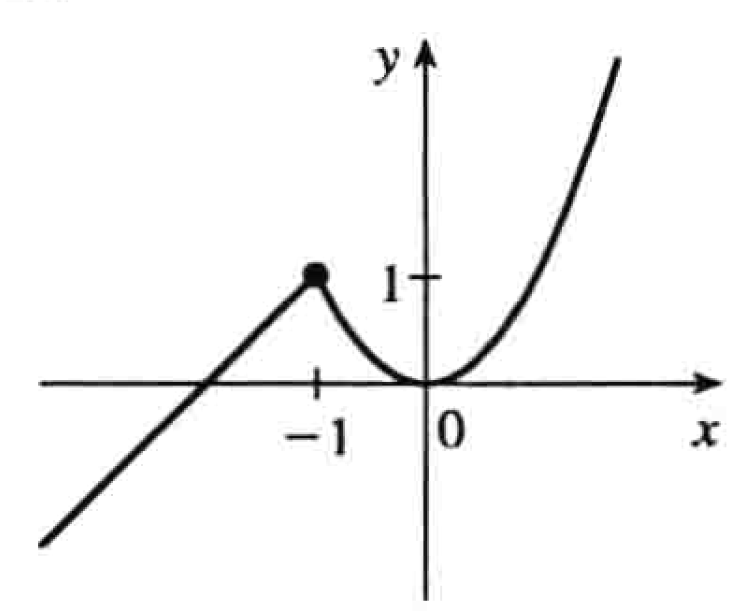
37. $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

Domain is $\mathbb{R}$.



39. $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

Domain is $\mathbb{R}$.



41. Recall that the slope m of a line between the two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and an equation of the line connecting those two points is $y - y_1 = m(x - x_1)$. The slope of this line segment is $\dfrac{-6 - 1}{4 - (-2)} = -\dfrac{7}{6}$, so an equation is $y - 1 = -\frac{7}{6}(x + 2)$. The function is $f(x) = -\frac{7}{6}x - \frac{4}{3}$, $-2 \leq x \leq 4$.

43. We need to solve the given equation for y. $x + (y-1)^2 = 0 \Rightarrow (y-1)^2 = -x \Rightarrow y - 1 = \pm\sqrt{-x} \Rightarrow y = 1 \pm \sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Hence, we want $f(x) = 1 - \sqrt{-x}$, $x \leq 0$.

45. For $-1 \leq x \leq 2$, the graph is the line with slope 1 and y-intercept 1, that is, the line $y = x + 1$. For $2 < x \leq 4$, the graph is the line with slope $-\frac{3}{2}$ and x-intercept 4, so $y = -\frac{3}{2}(x - 4) = -\frac{3}{2}x + 6$. So the function is

$$f(x) = \begin{cases} x + 1 & \text{if } -1 \leq x \leq 2 \\ -\frac{3}{2}x + 6 & \text{if } 2 < x \leq 4 \end{cases}$$

47. Let the length and width of the rectangle be L and W. Then the perimeter is $2L + 2W = 20$ and the area is $A = LW$. Solving the first equation for W in terms of L gives $W = \dfrac{20 - 2L}{2} = 10 - L$. Thus, $A(L) = L(10 - L) = 10L - L^2$. Since lengths are positive, the domain of A is $0 < L < 10$. If we further restrict L to be larger than W, then $5 < L < 10$ would be the domain.

49. Let the length of a side of the equilateral triangle be x. Then by the Pythagorean Theorem, the height y of the triangle satisfies $y^2 + \left(\frac{1}{2}x\right)^2 = x^2$, so that $y = \frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle, $A = \frac{1}{2}$ (base) (height), we obtain $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, with domain $x > 0$.

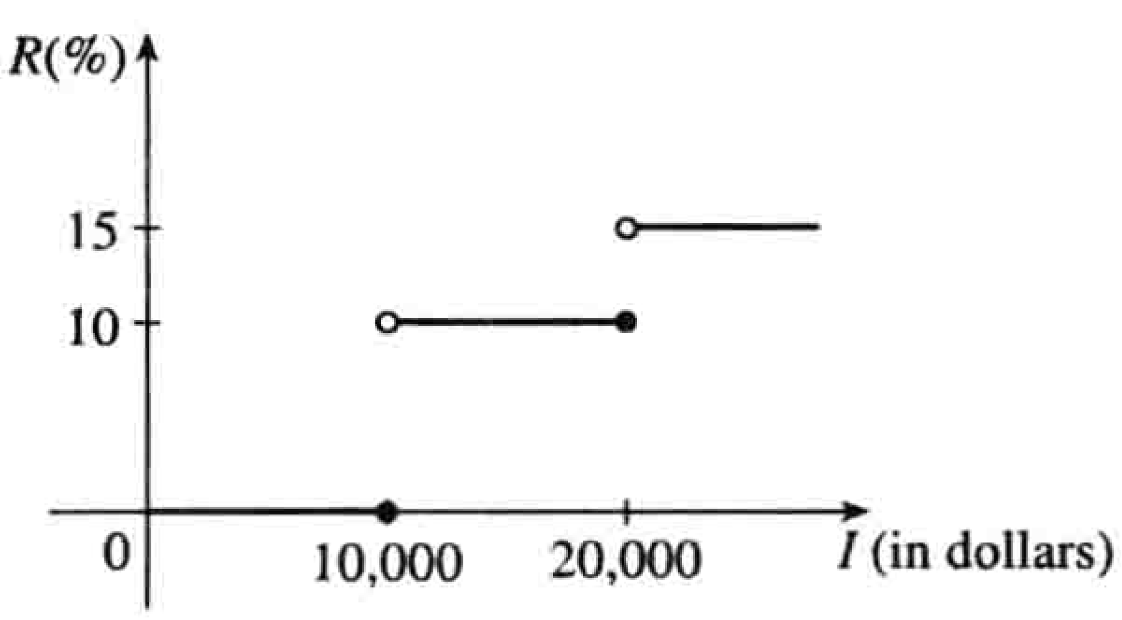
51. Let each side of the base of the box have length x, and let the height of the box be h. Since the volume is 2, we know that $2 = hx^2$, so that $h = 2/x^2$, and the surface area is $S = x^2 + 4xh$. Thus, $S(x) = x^2 + 4x\left(2/x^2\right) = x^2 + 8/x$, with domain $x > 0$.

53. The height of the box is x and the length and width are $L = 20 - 2x$, $W = 12 - 2x$. Then $V = LWx$ and so

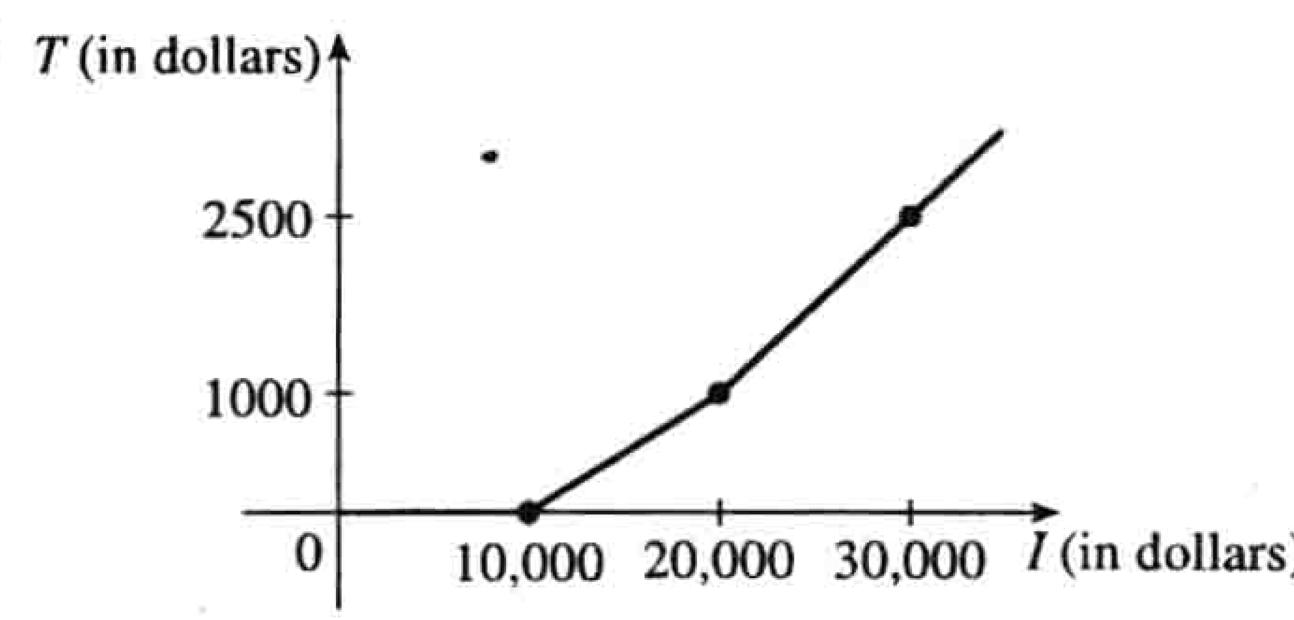
$$V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x\left(60 - 16x + x^2\right)$$
$$= 4x^3 - 64x^2 + 240x$$

The sides L, W, and x must be positive. Thus, $L > 0 \Leftrightarrow 20 - 2x > 0 \Leftrightarrow x < 10$; $w > 0 \Leftrightarrow 12 - 2x > 0 \Leftrightarrow x < 6$; and $x > 0$. Combining these restrictions gives us the domain $0 < x < 6$.

55. (a)



(c)



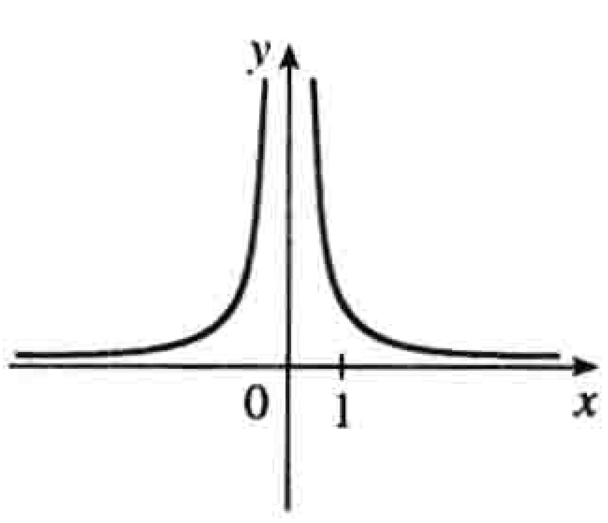
(b) On \$14,000, tax is assessed on \$4000, and $10\%(\$4000) = \$400$.
On \$26,000, tax is assessed on \$16,000, and $10\%(\$10{,}000) + 15\%(\$6000) = \$1000 + \$900 = \$1900$.

57. (a) Because an even function is symmetric with respect to the y-axis, and the point $(5, 3)$ is on the graph of this even function, the point $(-5, 3)$ must also be on its graph.

(b) Because an odd function is symmetric with respect to the origin, and the point $(5, 3)$ is on the graph of this odd function, the point $(-5, -3)$ must also be on its graph.

59. $f(-x) = (-x)^{-2} = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2}$
$= x^{-2} = f(x)$

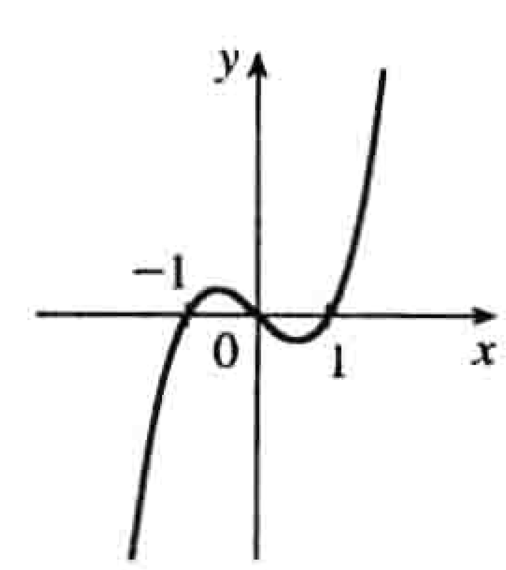
so f is an even function.



61. $f(-x) = (-x)^2 + (-x) = x^2 - x$. Since this is neither $f(x)$ nor $-f(x)$, the function f is neither even nor odd.

63. $f(-x) = (-x)^3 - (-x) = -x^3 + x$
$= -\left(x^3 - x\right) = -f(x)$

so f is odd.



1.2 Mathematical Models

1. (a) $f(x) = \sqrt[5]{x}$ is a root function.

(b) $g(x) = \sqrt{1 - x^2}$ is an algebraic function because it is a root of a polynomial.

(c) $h(x) = x^9 + x^4$ is a polynomial of degree 9.

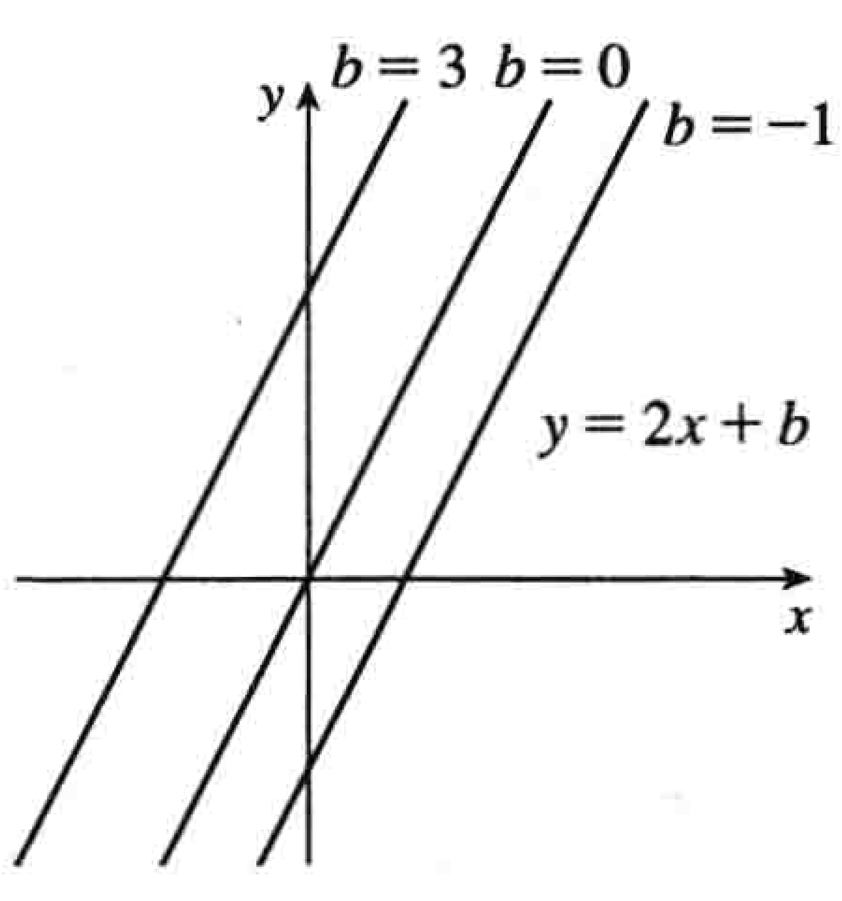
(d) $r(x) = \dfrac{x^2+1}{x^3+x}$ is a rational function because it is a ratio of polynomials.

(e) $s(x) = \tan 2x$ is a trigonometric function.

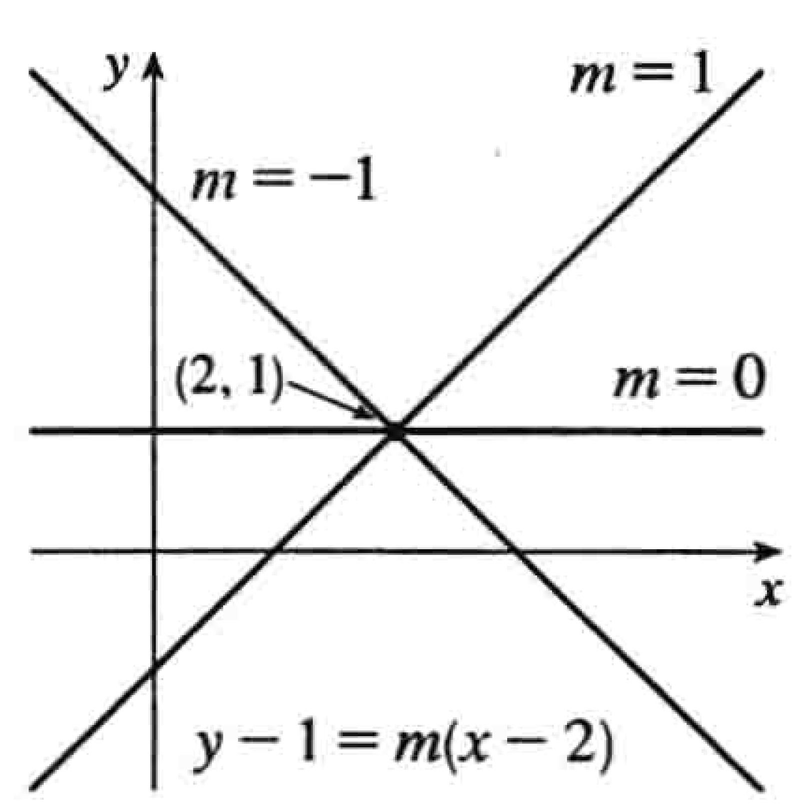
(f) $t(x) = \log_{10} x$ is a logarithmic function.

3. We notice from the figure that g and h are even functions (symmetric with respect to the y-axis) and that f is an odd function (symmetric with respect to the origin). So (b) $[y = x^5]$ must be f. Since g is flatter than h near the origin, we must have (c) $[y = x^8]$ matched with g and (a) $[y = x^2]$ matched with h.

5. (a) An equation for the family of linear functions with slope 2 is $y = f(x) = 2x + b$, where b is the y-intercept.

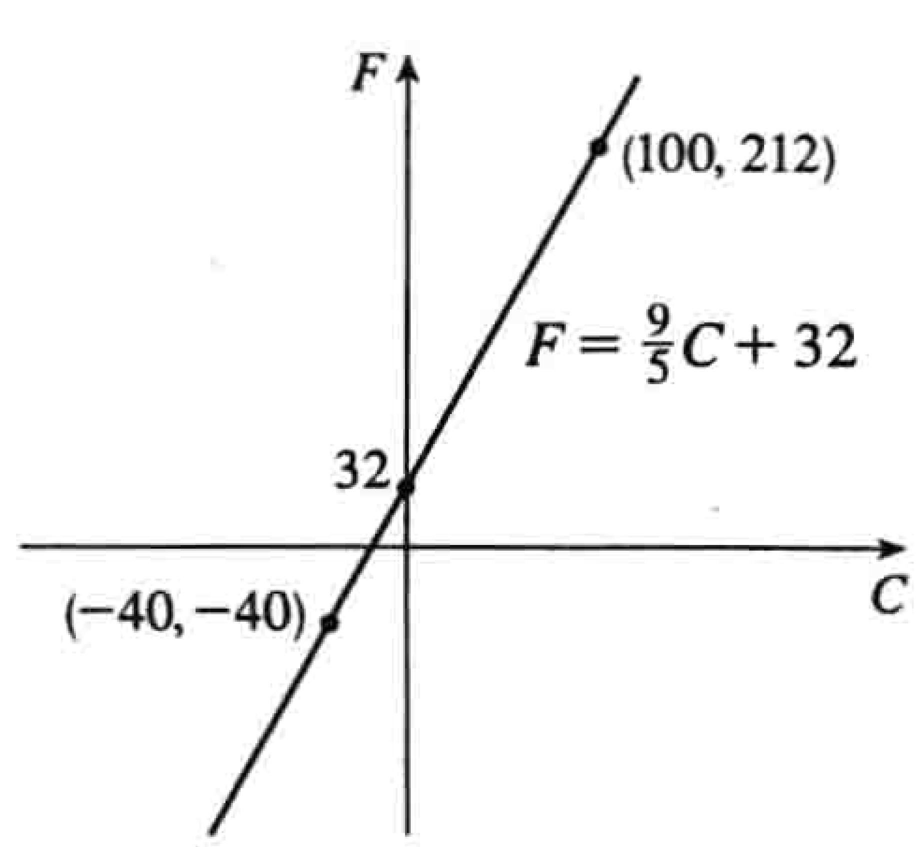


(b) $f(2) = 1$ means that the point $(2, 1)$ is on the graph of f. We can use the point-slope form of a line to obtain an equation for the family of linear functions through the point $(2, 1)$. $y - 1 = m(x - 2)$, which is equivalent to $y = mx + (1 - 2m)$ in slope-intercept form.



(c) The slope m must equal 2, so the equation in part (b), $y = mx + (1 - 2m)$, becomes $y = 2x - 3$. It is the *only* function that belongs to both families.

7. (a)



(b) The slope of $\frac{9}{5}$ means that F increases $\frac{9}{5}$ degrees for each increase of 1°C. (Equivalently, F increases by 9 when C increases by 5 and F decreases by 9 when C decreases by 5.) The F-intercept of 32 is the Fahrenheit temperature corresponding to a Celsius temperature of 0.

9. (a) Using N in place of x and T in place of y, we find the slope to be $\dfrac{T_2 - T_1}{N_2 - N_1} = \dfrac{80-70}{173-113} = \dfrac{10}{60} = \dfrac{1}{6}$. So a linear equation is $T - 80 = \frac{1}{6}(N - 173) \;\Leftrightarrow\; T - 80 = \frac{1}{6}N - \frac{173}{6} \;\Leftrightarrow\; T = \frac{1}{6}N + \frac{307}{6}$ $\left[\frac{307}{6} = 51.1\overline{6}\right]$.

(b) The slope of $\frac{1}{6}$ means that the temperature in Fahrenheit degrees increases one-sixth as rapidly as the number of cricket chirps per minute. Said differently, each increase of 6 cricket chirps per minute corresponds to an increase of 1°F.

(c) When $N = 150$, the temperature is given approximately by $T = \frac{1}{6}(150) + \frac{307}{6} = 76.1\overline{6}$°F ≈ 76°F.

11. (a) We are given $\dfrac{\text{change in pressure}}{\text{10 feet change in depth}} = \dfrac{4.34}{10} = 0.434$. Using P for pressure and d for depth with the point $(d, P) = (0, 15)$, we have $P - 15 = 0.434(d - 0) \;\Leftrightarrow\; P = 0.434d + 15$.

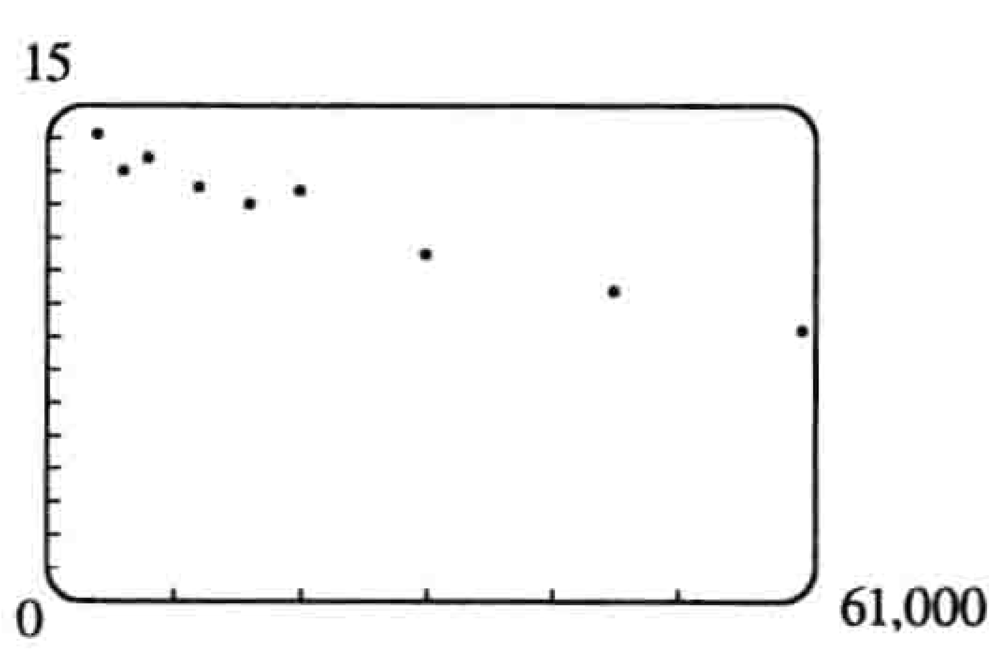
(b) When $P = 100$, then $100 = 0.434d + 15 \;\Leftrightarrow\; 0.434d = 85 \;\Leftrightarrow\; d \approx 195.85$ feet. Thus, the pressure is 100 lb/in^2 at a depth of approximately 196 feet.

13. (a) The data appear to be periodic and a sine or cosine function would make the best model. A model of the form $f(x) = a\cos(bx) + c$ seems appropriate.

(b) The data appear to be decreasing in a linear fashion. A model of the form $f(x) = mx + b$ seems appropriate.

Some values are given to many decimal places. These are the results given by several computer algebra systems — rounding is left to the reader.

15. (a)

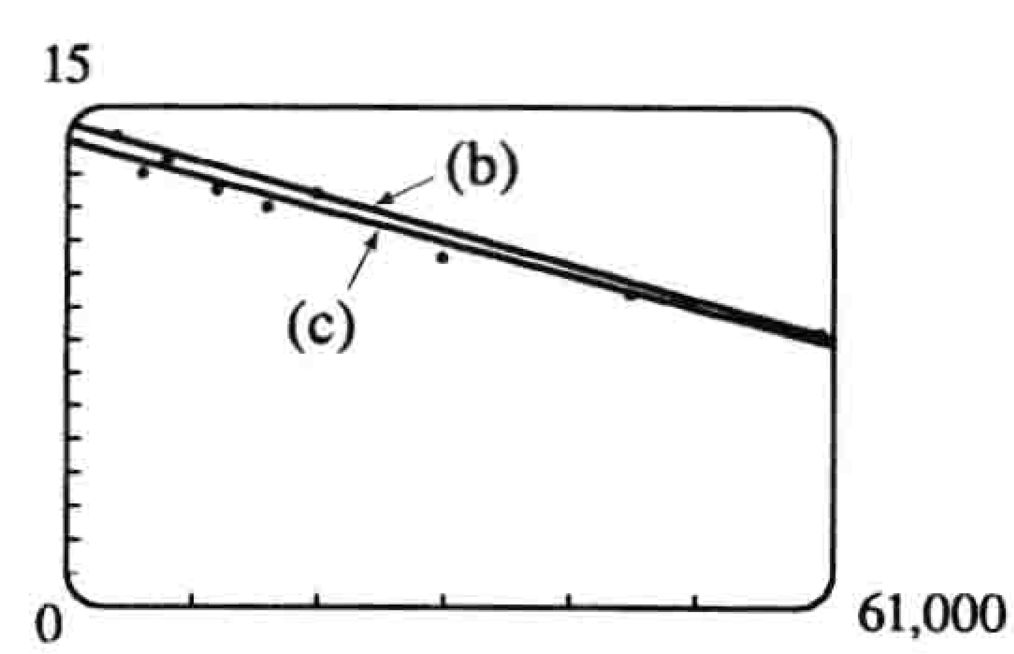


A linear model does seem appropriate.

(b) Using the points (4000, 14.1) and (60,000, 8.2), we obtain

$$y - 14.1 = \frac{8.2 - 14.1}{60{,}000 - 4000}(x - 4000) \text{ or, equivalently,}$$

$y \approx -0.000105357x + 14.521429$.



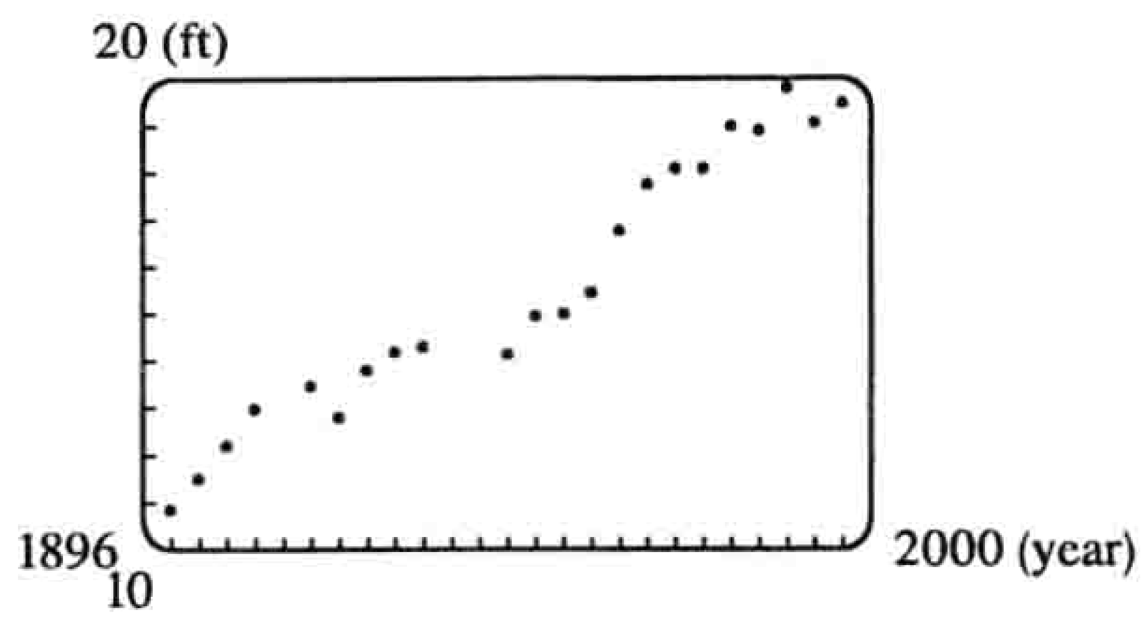
(c) Using a computing device, we obtain the least squares regression line $y = -0.0000997855x + 13.950764$.

(d) When $x = 25{,}000$, $y \approx 11.456$; or about 11.5 per 100 population.

(e) When $x = 80{,}000$, $y \approx 5.968$; or about a 6% chance.

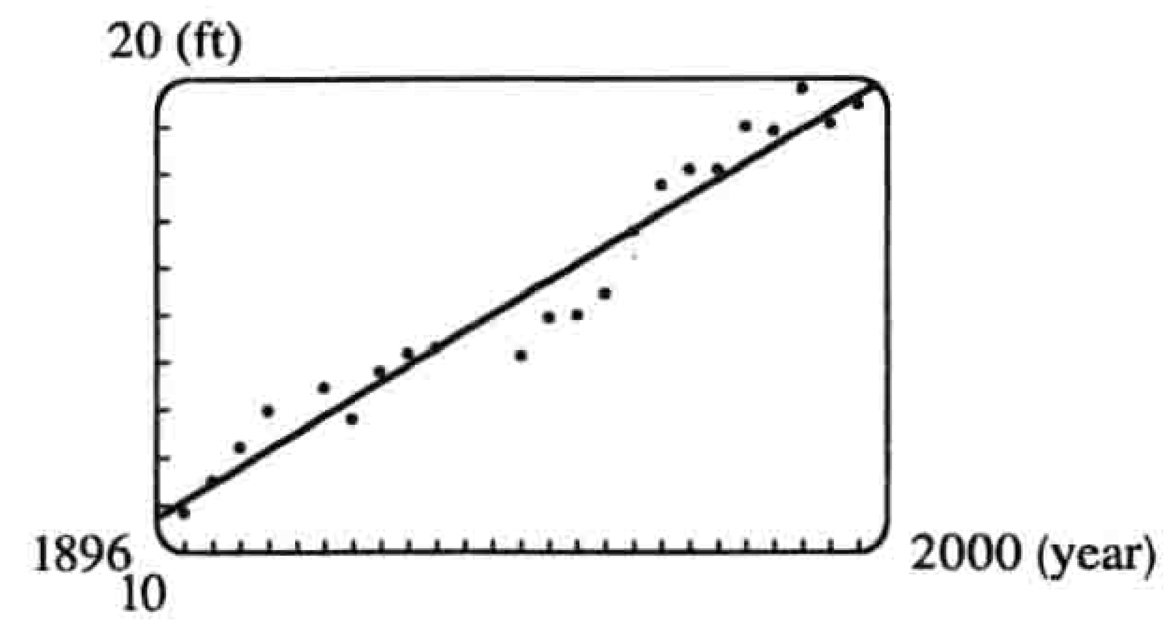
(f) When $x = 200{,}000$, y is negative, so the model does not apply.

17. (a)



A linear model does seem appropriate.

(b)

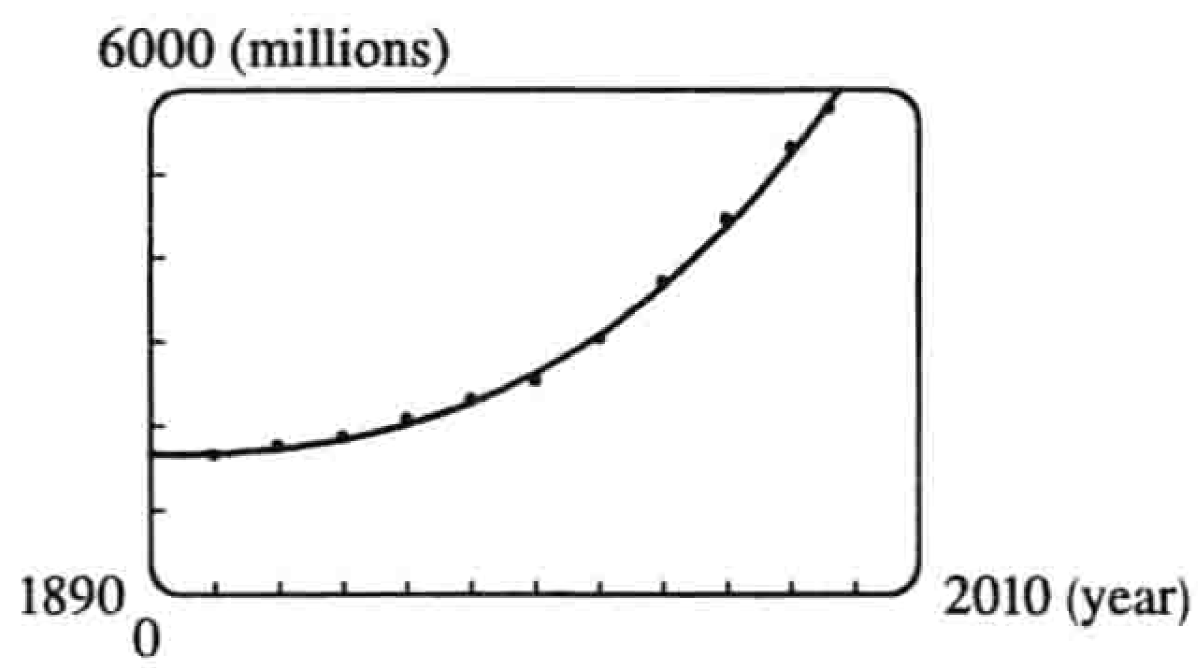


Using a computing device, we obtain the least squares regression line $y = -158.2403249x + 0.089119747$, where x is the year and y is the height in feet.

(c) When $x = 2000$, $y \approx 20.00$ ft.

(d) When $x = 2100$, $y \approx 28.91$ ft. This would be an increase of 9.49 ft from 1996 to 2100. Even though there was an increase of 8.59 ft from 1900 to 1996, it is unlikely that a similar increase will occur over the next 100 years.

19.



Using a computing device, we obtain the cubic function $y = ax^3 + bx^2 + cx + d$ with $a = 0.00232567051876$, $b = -13.064877957628$, $c = 24{,}463.10846422$, and $d = -15{,}265{,}793.872507$. When $x = 1925$, $y \approx 1922$ (millions).

1.3 New Functions from Old Functions

1. (a) If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

(b) If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

(c) If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

(d) If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

(e) If the graph of f is reflected about the x-axis, its equation becomes $y = -f(x)$.

(f) If the graph of f is reflected about the y-axis, its equation becomes $y = f(-x)$.

(g) If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

(h) If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.

3. (a) (graph 3) The graph of f is shifted 4 units to the right and has equation $y = f(x - 4)$.

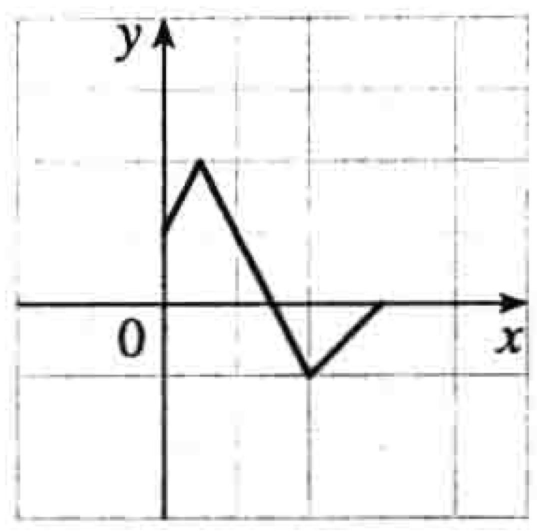
(b) (graph 1) The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.

(c) (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

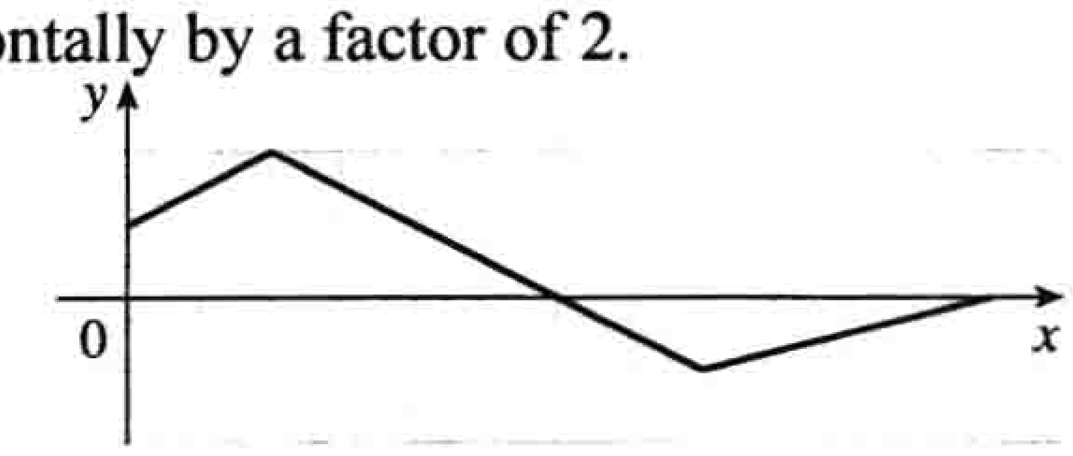
(d) (graph 5) The graph of f is shifted 4 units to the left and reflected about the x-axis. Its equation is $y = -f(x + 4)$.

(e) (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.

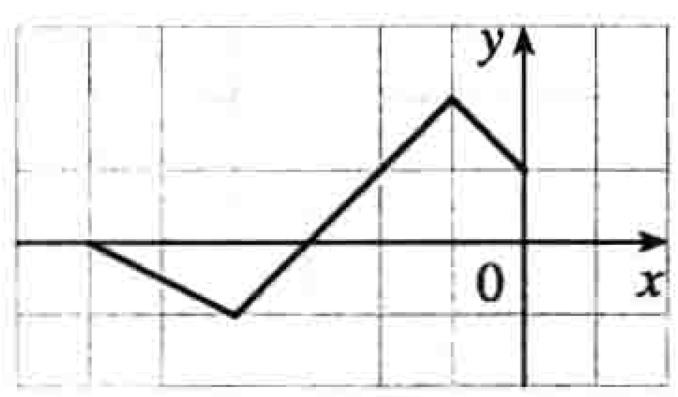
5. (a) To graph $y = f(2x)$ we shrink the graph of f horizontally by a factor of 2.



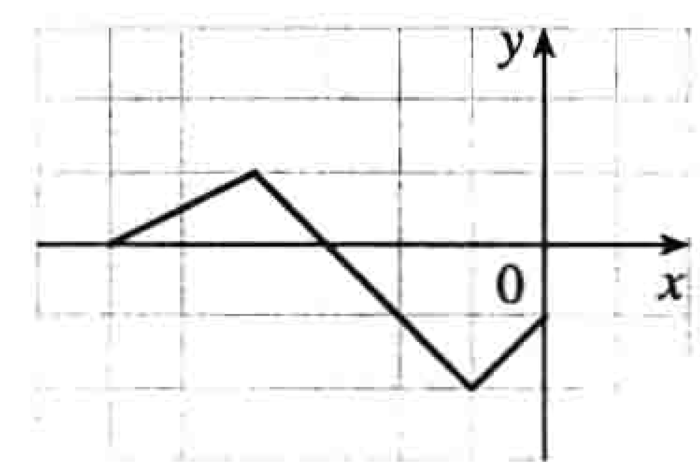
(b) To graph $y = f\left(\frac{1}{2}x\right)$ we stretch the graph of f horizontally by a factor of 2.



(c) To graph $y = f(-x)$ we reflect the graph of f about the y-axis.



(d) To graph $y = -f(-x)$ we reflect the graph of f about the y-axis, then about the x-axis.



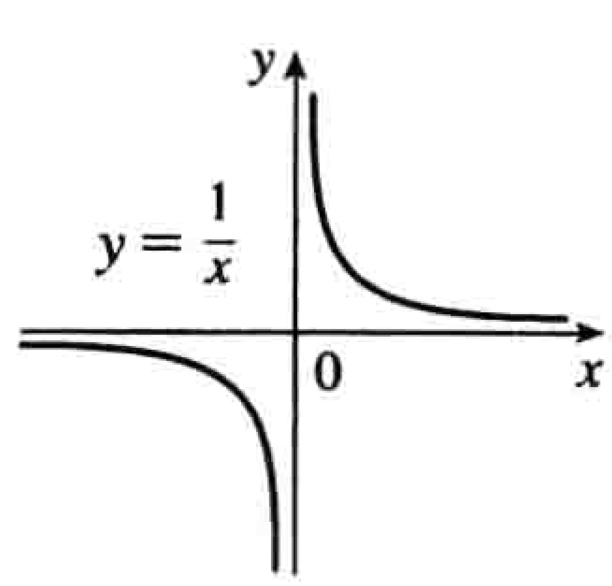
7. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x-axis, and shifted downward 1 unit. Thus, a function describing the graph is

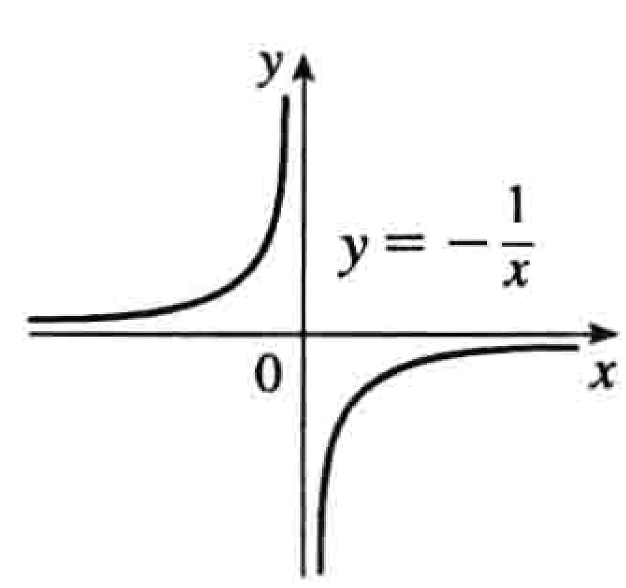
$$y = \underbrace{-1 \cdot}_{\substack{\text{reflect}\\ \text{about}\\ x\text{-axis}}} \underbrace{f(x+4)}_{\substack{\text{shift}\\ \text{4 units}\\ \text{left}}} \underbrace{-1}_{\substack{\text{shift}\\ \text{1 unit}\\ \text{down}}}$$

This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1$$
$$= -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1$$

9. $y = -1/x$: Start with the graph of $y = 1/x$ and reflect about the x-axis.





11. $y = \tan 2x$: Start with the graph of $y = \tan x$ and compress horizontally by a factor of 2.