

THE APPLIED DYNAMICS OF OCEAN SURFACE WAVES

CHIANG C. MEI

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Preface

A substantial growth of knowledge in the dynamics of ocean surface waves has been witnessed over the past 20 years. While many advances have been stimulated by purely scientific inquiry in geophysics, the pace of progress has also been quickened by the increase in large engineering projects both offshore and along the coast. A major construction project now demands not only careful estimates of wave conditions near the site but also reliable predictions of the effects on and of the construction itself. With a view to bringing together scientific and engineering aspects of ocean waves, educational and research programs have naturally been established in a number of universities and industries.

This book is the outgrowth of my lecture notes for a two-semester course taught at M.I.T. since 1974 to graduate students in civil and ocean engineering, with occasional participants from physical oceanography. The aim of the book is to present selected theoretical topics on ocean-wave dynamics, including basic principles and applications in coastal and offshore engineering, all from the deterministic point of view. The bulk of the material deals with the linearized theory which has been well developed in the research literature. The inviscid linearized theory is covered in Chapters One to Five and again in Seven. Frictional effects caused directly or indirectly by viscosity are treated in Chapters Six, Eight, and Nine. A special effect of breaking waves on beaches is examined in Chapter Ten. Chapters Nine and Ten focus on the secondary effects of nonlinearity. The cases where nonlinearity is of primary importance are the subjects of Chapters Eleven and Twelve, for shallow and deep waters, respectively. The last chapter (Thirteen) is on wave-induced stresses in a porous but deformable seabed, which is a problem vital to offshore engineering. In the construction of a gravity platform, the cost of the foundation alone can be as high as 40% of the total. Under the influence of waves, the strength of a porous seabed is affected to varying degrees by fluid in the pores. Hence hydrodynamics is an essential part of the problem. In this chapter a well-known fluid-dynamic reasoning is applied to a soil model which includes fluid and

solid phases. I hope the material will stimulate further interaction among researchers in different disciplines.

Most parts of this book have been used either for my own lectures or for self-paced reading by the students. Since contributions by mathematical scientists have always been prominent in this field, the use of certain analytical techniques which may be less familiar to many potential readers cannot be avoided. Therefore, considerable space is devoted to the informal explanation and demonstration of those techniques not customarily discussed in a course on advanced calculus. The derivations of most of the results are given in detail in order to reduce possible frustrations to those who are still acquiring the requisite skills. A few exercises are included; nearly all of them demand some effort. For additional exercises, I have usually suggested term papers based on the student's own survey of literature.

Studies on waves in general, and on water waves in particular, have always been enriched by cross fertilization among diverse fields of science and engineering, including physics, mathematics, oceanography, electrical engineering, and others. A conscientious effort has been made in this book to reflect this fact which I hope will induce more engineers and scientists to join their talents for further challenges of the sea.

Several important areas which are either beyond my own experience or have been treated in other books are not included here. The mechanisms of wave generation by wind and many aspects of resonant interactions have been admirably surveyed by Phillips (1977) and by LeBlond and Mysak (1978). On the statistical *description* of random sea waves, a detailed discussion of the basic aspects may be found in Price and Bishop (1974). For the statistical *mechanics* of sea waves one should consult Phillips (1977) and West (1981). The rapid advance on steep waves, spearheaded by M. S. Longuet-Higgins, is of obvious interest to engineers and oceanographers alike; the numerous papers by him and his associates on the subject cannot be matched for clarity and thoroughness. Waves due to advancing bodies belong to the realm of ship hydrodynamics; the definitive treatises by Stoker (1957), Wehausen and Laitone (1960), and Newman (1977), and all the past proceedings of the Naval Hydrodynamics Symposium should be consulted. Wave-induced separation around small bodies is at the heart of force prediction for offshore structures; it is a subject where experiments play the leading role and has been expertly covered in a recent book by Sarpkaya and Issacson (1981). Storm surges are also omitted.

In a book containing many mathematical expressions, freedom from error can be strived for but is hard to achieve. I shall be grateful to readers who wish to inform me of any oversights that remain.

CHIANG C. MEI

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I gratefully acknowledge the kindness of the following colleagues for sending me photographs of certain figures: Dr. J. R. Houston of Waterways Experiment Station, Vicksburg (Figs. 11.3–11.5, Chapter Four, and Fig. 10.5, Chapter Five), Dr. J. Lau, formerly of Florida State University (Plate 1, Chapter Nine), and Dr. H. C. Yuen of TRW, Inc. (Figs. 6.4 and 6.5, Chapter Twelve).

I also take this opportunity to express my gratitude and admiration for Professor T. Y. T. Wu of Cal Tech whose approach to engineering science I try to emulate in this book. By his open-mindedness and exemplary dedication to both basics and practice, the late Professor A. T. Ippen of the Civil Engineering Department at M.I.T. inspired all those around him to contribute to engineering, in whatever style. In addition, my colleagues at the same department, Professors D. R. F. Harleman, P. S. Eagleson, and J. F. Kennedy, who is now at the University of Iowa, have rendered timely help which either exposed me to new challenges or enabled me to do what I enjoyed.

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C. C. M.

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Introduction

Many types of waves involving different physical factors exist in the ocean. As in the elementary problem of a spring-mass system, all waves must be associated with some kind of restoring force. It is therefore convenient to make a crude classification of ocean waves according to the restoring force, as shown in Table 1.1.

Wind waves and swell, generated by local and distant storms are the most directly experienced by mankind. Occurring less frequently but with occasionally disastrous consequences are the tsunamis which usually refer to long-period [$\sim O(1 \text{ h})$] oscillations caused by large submarine earthquakes or landslides. Within the same broad range of time scales, waves can also exist as a result of human activities (ship motion, explosion, and so on). Since these waves are the most prominent on the water surface and their main restoring force is gravity, they are called the *surface gravity waves*. The shorter term, *surface waves*, is often used if the exclusion of surface capillary waves is understood.

Important in the science of oceanography are the internal gravity waves along the thermoclines which are horizontal layers of sharp density stratification beneath the sea surface. The associated wave motion is generally not pronounced on the surface except for some indirect signs of its presence. These waves contribute to the process of mixing and affect the eddy viscosity of ocean currents. Storm surges are the immediate consequence of local weather and can inflict severe damages to human life and properties by inundating the coast.

In nature, several restoring forces can be in effect at the same time, hence the distinction between various waves listed in Table 1.1 is not always very sharp.

This book will be limited to wave motions having time scales such that compressibility and surface tension at one extreme and earth rotation at the other are of little direct importance. Furthermore, the vertical stratification of sea water is assumed to be small enough within the depth of interest. Therefore, we shall only be concerned with the surface gravity waves, that is, wind waves, swell, and tsunamis. Discussions of all other waves listed in Table 1.1

Table 1.1 Wave Type, Physical Mechanism, Activity Region.

Wave Type	Physical Mechanism	Typical Period ^a	Region of Activity
Sound	Compressibility	10^{-2} – 10^{-5} s	Ocean interior
Capillary ripples	Surface tension	$< 10^{-1}$ s	Air–water interface
Wind waves and swell	Gravity	1–25 s	
Tsunami	Gravity	10 min–2 h	
Internal waves	Gravity and density stratification	2 min–10 h	
Storm surges	Gravity and Earth rotation	1–10 h	Near coastline
Tides	Gravity and Earth rotation	12–24 h	Entire ocean layer
Planetary waves	Gravity, Earth rotation and variation of latitude or ocean depth	$O(100 \text{ days})$	

^aIn seconds (s), minutes (min), hours (h), and days.

can be found in the oceanographic treatises by Hill (1962) and LeBlond and Mysak (1978).

In this chapter we first review the basic equations of fluid motion and some general deductions for inviscid, irrotational flows. Linearized equations for infinitesimal waves are then derived. After introducing the general notions of propagating waves, we examine the properties of simple harmonic progressive waves on constant depth. An elementary discussion of group velocity will be given from both kinematic and dynamic points of view.

1.1 REVIEW OF BASIC FORMULATION FOR AN INCOMPRESSIBLE FLUID OF CONSTANT DENSITY

1.1.1 Governing Equations

In a wide variety of gravity wave problems, the variation of water density is insignificant over the temporal and spatial scales of engineering interest. The fundamental conservation laws are adequately described by the following Navier–Stokes equations:

$$\text{mass:} \quad \nabla \cdot \mathbf{u} = 0, \quad (1.1)$$

$$\text{momentum:} \quad \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla \left(\frac{P}{\rho} + gz \right) + \nu \nabla^2 \mathbf{u}, \quad (1.2)$$