Monetary Policy Rules

Edited by John B. Taylor

The University of Chicago Press

Chicago and London

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of Economic Research
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John B. Taylor

Introduction

John B. Taylor

This book has two broad goals. The first goal is to present econometric evidence on which type of monetary policy rule is likely to be both efficient and robust when used as a guideline for the conduct of monetary policy in the United States. The second goal is to settle several current monetary policy issues—such as the effects of uncertainty about potential GDP growth or the role of the exchange rate in the setting of interest rates—that are most naturally addressed within a framework of monetary policy rules.

To achieve these two goals, a number of economists who are actively engaged in research on monetary policy put their econometric policy evaluation methods to use in order to investigate various monetary policy rules. The economists then came together at a conference in the Florida Keys to discuss their results with policymakers and other economists. This volume—including nine papers, comments on the papers, and discussions from the conference—is the outcome of that effort. Many researchers at universities, central banks, and private financial institutions around the world are now using modern econometric policy evaluation methods to analyze monetary policy rules. We are fortunate that many of them—over 30 individuals are represented in the volume—were able to participate in the project.

A Variety of Models and a Uniform Methodology

The research reported in this volume represents a wide variety of models. The models differ in size: from 3 equations to 98 equations. They differ in degree of openness; some are closed economy models, some are small open econ-

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omy models, and some are large open economy models. The models also differ in degree of forward looking assumed, in the method of establishing a good microeconomic foundation for the equations, and in the goodness of fit to the data. Some models are estimated with formal econometric methods and fit the historical data tightly. Others are calibrated using rules of thumb or information from other studies, and they give rough approximations to historical data.

To get a feel for the differences between the models, consider some key features of the nine papers. The models developed by Bennett McCallum and Edward Nelson, by Julio Rotemberg and Michael Woodford, and by Robert King and Alex Wolman have a microfoundation built around a *representative agent framework* in which a household maximizes utility over time. The representative agent approach is attractive because it automatically builds in people's responses to policy and because it allows policy to be evaluated using the utility function of the representative agent. These models tend to be smaller than many of the other models in the volume, and they give rough approximations of the quarterly time series in the United States.

Like the models using a representative agent framework, the model used by Nicoletta Batini and Andrew Haldane and the four models used by Andrew Levin, Volker Wieland, and John Williams assume that agents have *rational expectations.* However, the microeconomic foundations for these models are separate decision rules for a household's consumption or for a firm's investment and production, rather than explicit dynamic optimization of a representative agent. These decision rules are motivated by rational behavior and frequently have the same variables as the equations in the explicitly derived models. These rational expectations models are generally more detailed, and they fit the data better than the representative agent models.

The models used by Laurence Ball, by Glenn Rudebusch and Lars Svensson, and by Arturo Estrella and Frederic Mishkin are *non–rational expectations models.* In order to achieve better empirical accuracy (Rudebusch and Svensson) or to focus on other issues such as exchange rates (Ball) or measurement error (Estrella and Mishkin), these models do not build in agents' responses to future policy decisions as the rational expectations models do—whether representative agent models or not. These non–rational expectations models make the simplifying assumption that the parameters will not change when policy changes.

In contrast to these model-based policy evaluation models, my own paper in the volume uses a historical methodology to evaluate policy rules. This approach is similar to that used by Milton Friedman and Anna Schwartz in their monetary history of the United States or to that of Christina Romer and David Romer in their analysis of Federal Open Market Committee decision making. Rather than testing policy rules in a structural model, this paper looks at different historical periods to see if different policy rules result in different

macroeconomic outcomes. Moreover, the paper uses a general monetary theory rather than a tightly specified model to interpret the historical data.

Despite these differences, the papers in the volume share an important common methodology that defines the state of the art in monetary policy evaluation research. First, each of the models is a dynamic, stochastic, general equilibrium model. The relevance of expectations of the future and events of the past to current decisions gives the models a dynamic feature. Shocks to preferences, to technology, or simply to decision rules make the models stochastic. The term "general equilibrium" applies because the models pertain to the whole economy, not to an individual sector of the economy.

Second, each of the models incorporates some form of temporary nominal rigidity, usually a variant of staggered wage or price setting, which results in a short-run trade-off between inflation and output or unemployment. With stochastic shocks, the short-run inflation-output relationship can be characterized as a trade-off between the variance of inflation and the variance of output, but none of the models has a long-run trade-off between the level of inflation and unemployment. Several of the papers in the volume break new ground in modeling price rigidities. For example, the paper by King and Wolman derives a firm's pricing rule by analyzing how the firm would maximize its present discounted value in a setting where there are monopolistic competition and infrequent price adjustment opportunities. It is interesting to note that the optimal decision rules resemble the staggered price-setting equations studied by Levin, Wieland, and Williams. King and Wolman show that staggered price setting increases the costs of inflation, an issue that has not been raised in earlier calculations of the welfare costs of inflation. Because of their explicit derivation one can calculate the welfare costs of steady inflation with their model.

Third, for each model the variances can be computed directly or through stochastic simulation, and the measure of economic performance depends on the variance of inflation around the target inflation rate, the variance of real output around a measure of potential or full-employment output, and, in some cases, the variance of unanticipated inflation or the variance of the interest rate. It is possible to feed these variances into an objective function that is a weighted average of the variances, and in some of the papers (Rotemberg-Woodford and King-Wolman), the objective function is the same as the utility function of the representative household.

These common features can be illustrated by noting that all the models can be written in the following general form:

$$(1) \qquad y_t = A(L,g)y_t + B(L,g)i_t + u_t.$$

This equation is the reduced-form solution to the model. The vector y_t contains the endogenous variables. The scalar i_t is the short-term nominal interest rate. The vector u_t is a serially uncorrelated random variable with covariance matrix Σ. The matrices $A(L,g)$ and $B(L,g)$ are polynomials in the lag operator L. These matrix polynomials depend on the parameter vector g, which consists of all the parameters in the policy rule. The policy rule itself can be written as

$$i_t = G(L)y_t, \tag{2}$$

where $G(L)$ is a vector polynomial in the lag operator L. Making the parameter vector g explicit in this notation emphasizes that reduced-form parameters in A and B depend on the parameters of the policy rule, an important common feature of these models. For the Ball model, the Rudebusch-Svensson model, and the Estrella-Mishkin model, none of which are rational expectations models, the above equation for y_t is the model itself. For the Rotemberg-Woodford, McCallum-Nelson, and King-Wolman models, there are forward-looking expectations variables that enter through the Euler equations of the representative agent's optimizing problem; these have been solved out using a rational expectations solution method to get the reduced-form equation for y_t. For the Batini-Haldane model and the four models considered by Levin, Wieland, and Williams (Federal Reserve, Fuhrer-Moore, MSR, and linearized Taylor multicountry), there are also forward-looking variables that have been solved out to get the reduced-form equation.

Substitution of the policy rule for i_t into the reduced-form equation for y_t above results in a vector autoregression in y_t. The steady state stochastic distribution of y_t is a function of the parameter vector g of the policy rule. Hence, for any choice of parameters in g one can evaluate an objective function that depends on the steady state distribution of y_t. For example, if the loss function is a weighted average of the variance of inflation and the variance of real output, then the two diagonal elements of the covariance matrix corresponding to inflation and real output are used. Using this approach, the papers in the volume present simulation evidence that helps determine the optimal policy rule.

I believe that there is much to be learned from these simulations, not only from the tables and charts presented in the nine papers, but also from the comments on the papers (many of which also contain new results) and the discussions about the papers. Here I can only summarize some key results. Rather than reviewing each paper and comment separately I will try to organize the summary around the following key issues: (1) robustness of policy rules, (2) usefulness of simple policy rules compared with complex rules, (3) role of the exchange rate, (4) role of inflation forecasts, (5) importance of information lags, (6) uncertainty about potential GDP or the natural rate of unemployment, and (7) implications of the historical evidence.

Robustness of Policy Rules

A number of the papers in this volume propose specific monetary policy rules. Some of these rules are modifications of policy rules that have been proposed in earlier research. Others would involve more substantial changes. Regardless of the specific form, each rule is proposed because, according to the model used in the research, the rule results in good macroeconomic performance. But how robust are the proposed rules? How would the rule proposed by one researcher stand up to scrutiny by other researchers using different models and methods? To answer these questions we asked researchers who participated in the conference to investigate the other researchers' proposals for policy rules using their own models. We did not specify what model (whether large or small, rational or nonrational) should be used. That decision was left up to the researchers. In the end, nine models were used in this robustness exercise. The models, all described in the conference papers published in this volume, are

1. Ball model
2. Batini-Haldane model
3. McCallum-Nelson model
4. Rudebusch-Svensson model
5. Rotemberg-Woodford model
6. Fuhrer-Moore model
7. MSR (small Federal Reserve model)
8. FRB/US (large Federal Reserve model)
9. TMCM (Taylor multicountry model)

The last four of these models (6 through 9) are used in the paper by Levin, Wieland, and Williams, which is a robustness study itself as the title indicates.

Of course, these nine models do not include all possible models that could be used for a robustness study. For example, as part of their comment on the Levin, Wieland, and Williams paper, Lawrence Christiano and Christopher Gust analyze several monetary policy rules using a type of model much different from those used in the other papers. The short-run monetary nonneutralities in the Christiano-Gust model are based on limited participation in financial markets rather than on temporary price and wage rigidities. Christiano and Gust report deterministic simulations and a stability analysis that tend to favor money supply rules over interest rate rules. Note also that the King and Wolman paper was not included in the robustness analysis because the authors believe that their type of model is in an early stage of development, and they are hence not ready to make an empirical identification of business cycle determinants in the way that the robustness analysis requires.

Five different policy rules were selected for the robustness exercise. These rules are of the form

$$i_t = g_\pi \pi_t + g_y y_t + \rho i_{t-1}, \tag{3}$$

where i is the nominal interest rate, π is the inflation rate, and y is real GDP measured as a deviation from potential GDP. (The intercept term is ignored here.) The coefficients defining the five policy rules are shown in table 1.

Rules I and II have the interest rate reacting to the lagged interest rate with a response coefficient of one. Rule I has a high weight on inflation compared

Table 1 **Five Conference Rules**

Rule	g_π	g_y	ρ
I	3.0	0.8	1.0
II	1.2	1.0	1.0
III	1.5	0.5	0.0
IV	1.5	1.0	0.0
V	1.2	0.06	1.3

to the weight on output, and rule II has a smaller weight on inflation compared to output. These two rules are referred to as interest-rate-smoothing rules and are the type of rule favored in the simulations in the Levin, Wieland, and Williams paper, though not necessarily with these coefficient values on inflation and output. As I show below these rules sometimes result in more interest rate volatility than rules that do not involve a reaction to the lagged interest rate. Rule III is the simple rule that I proposed in 1992 after considering the policy evaluation results from a number of multicountry models. Rule IV is much like rule III except there is a coefficient of 1.0 rather than 0.5 on real output. The simulation results of several researchers, including Laurence Ball and John Williams, indicate that the interest rate should respond about twice as aggressively to output than the 0.5 response coefficient in the simple rule that I proposed. Rule V is the rule favored in the paper by Rotemberg and Woodford in this volume. This rule is distinctive in that it places a very small weight on real output and a very high weight on the lagged interest rate.

Of course, the policy rules in table 1 do not exhaust all possible policy rules. Table 1 omits rules for the money supply, such as constant growth rate rules. Moreover, two policy rules for the interest rate proposed in this volume—the rule that reacts to exchange rates examined by Ball and the inflation-forecast rules examined by Batini and Haldane and by Rudebusch and Svensson—could also be subjected to robustness analysis. They were not part of this robustness exercise because many of the models do not have exchange rates and because the inflation-forecast rules are themselves model specific, making robustness tests more difficult, as explained in the comment by James Stock in this volume. Although it is quite possible that another policy rule would do better than any of the five policy rules listed in table 1, these rules represent the degree of disagreement that currently exists about the most appropriate form for policy rules.

Assessing the robustness across models is difficult because different models have different absolute measures of performance. One model might show that all the rules work much better—have smaller fluctuations in inflation or real output—than another model shows. In fact, this is the case for the models in this robustness study. For example, the Batini-Haldane model and the Fed's small model (MSR) imply that much better economic performance can be achieved by following an optimal rule than the Fuhrer-Moore model implies.

Moreover, these performance differences across models are fairly arbitrary, because the size of the variances of the shocks in u_t (or more generally the magnitude of each element in the covariance matrix Σ) is assumed in some models. Even in the models where the covariance matrix of the shocks is estimated using formal econometric methods, the estimates depend on arbitrary choices about specification—such as how many lagged endogenous variables or exogenous variables are placed in the model. This lack of uniformity in absolute performance measures means that one must focus on *rankings* of rules across different models. An analogy with expert evaluation in other areas is useful. Consider wine tasting (an analogy pointed out to me by Orley Ashenfelter). A panel of experts is asked to evaluate different wines. But some tasters tend to give high ratings and some tasters tend to give low ratings. Looking at the average rating across tasters will be a mistake because the tasters who give high scores will have greater influence on the average than tasters with low scores. However, by first converting the scores of each taster into a simple ranking of each taster and then adding up ranks, one can eliminate this scale effect. Similarly, one can consider pairwise rankings between two wines that differ in a key characteristic. Of course, in this book we have policy rules rather than wine and models rather than tasters, but the principle is the same.

Consider using this approach to determine the robustness of policy rules that are more responsive to output in comparison to rules that are less responsive. In other words, is the finding that one policy rule is better than another policy rule a robust finding that stands up against the different models in this book? Consider rule III and rule IV, for example. As stated earlier, several researchers have suggested that rule IV is better than rule III in the sense that the variability of inflation and real output is less with rule IV than with rule III. Is this finding robust? Table 2 shows the standard deviations of inflation rate, real output, and interest rate for rule III and rule IV. These standard deviations are obtained from the covariance matrix of the endogenous variables. Several conclusions can be drawn from table 2. First, it is clear that a finding that rule IV dominates rule III is not robust across models. For all models, rule IV gives a lower variance of output than rule III does, which is not surprising with the higher weight on output in rule IV. But for six of the nine models rule IV gives a higher variance of inflation. Raising the coefficient on real output from 0.5 to 1.0 represents a trade-off between inflation variance and output variance. The change in average standard deviations across all the models shown in table 2 indicates such a trade-off, but rule IV's increase in average inflation variability is small compared with the decrease in average output variability. (To be sure, this average change may be influenced by the lack of uniformity in absolute performance levels discussed above.) If we also consider the variability of the interest rate, then the finding that rule IV is better than rule III is even less robust: rule III is higher than rule IV in seven of the eight models that reported interest rate variances. (The average interest rate variance across models is higher with rule IV, though that result is also affected by the arbitrariness of a cardinal

Table 2 **Comparative Performance of Two Conference Policy Rules**

	Standard Deviation		
Model	Inflation	Output	Interest Rate
		Rule III	
Ball	1.85	1.62	–
Batini-Haldane	1.38	1.05	0.55
McCallum-Nelson	1.96	1.12	3.94
Rudebusch-Svensson	3.46	2.25	4.94
Rotemberg-Woodford	2.71	1.97	4.14
Fuhrer-Moore	2.63	2.68	3.57
MSR	0.70	0.99	1.01
FRB	1.86	2.92	2.51
TMCM	2.58	2.89	4.00
Average	2.13	1.94	2.82
		Rule IV	
Ball	2.01	1.36	–
Batini-Haldane	1.46	0.92	0.72
McCallum-Nelson	1.93	1.10	3.98
Rudebusch-Svensson	3.52	1.98	4.97
Rotemberg-Woodford	2.60	1.34	4.03
Fuhrer-Moore	2.84	2.32	3.83
MSR	0.73	0.87	1.19
FRB/US	2.02	2.21	3.16
TMCM	2.36	2.55	4.35
Average	2.16	1.63	3.03

Table 3 **Three Conference Rules That React to Lagged Interest Rates**

	Standard Deviation		
Model	Inflation	Output	Interest Rate
		Rule I	
Ball	2.27	23.06	–
Batini-Haldane	0.94	1.84	1.79
McCallum-Nelson	1.09	1.03	5.14
Rudebusch-Svensson	∞	∞	∞
Rotemberg-Woodford	0.81	2.69	2.50
Fuhrer-Moore	1.60	5.15	15.39
MSR	0.29	1.07	1.40
FRB/US	1.37	2.77	7.11
TMCM	1.68	2.70	6.72
		Rule II	
Ball	2.56	2.10	–
Batini-Haldane	1.56	0.86	0.99
McCallum-Nelson	1.19	1.08	4.41
Rudebusch-Svensson	∞	∞	∞
Rotemberg-Woodford	1.35	1.65	2.53
Fuhrer-Moore	2.17	2.85	8.61
MSR	0.44	0.64	1.35
FRB/US	1.56	1.62	4.84
TMCM	1.79	1.95	5.03
		Rule V	
Ball	∞	∞	∞
Batini-Haldane	∞	∞	∞
McCallum-Nelson	1.31	1.12	2.10
Rudebusch-Svensson	∞	∞	∞
Rotemberg-Woodford	0.62	3.67	1.37
Fuhrer-Moore	7.13	21.2	27.2
MSR	0.41	1.95	1.31
FRB	1.55	6.32	4.67
TMCM	2.06	4.31	4.24

scale.) One could formalize these ranking calculations by putting weights on the three standard deviations and then ranking the rules in terms of the values of the objective function in each model. Rule III would rank above rule IV for relatively high weights on inflation and interest rate variability, while rule IV would rank better for high weights on output variability.

Now consider the relative robustness of the three rules that respond to the lagged interest rate (rules I, II, and V) as shown in table 3. Each of these three rules has exactly the same functional form as the others. Hence, this robustness analysis considers the appropriate size of the response coefficients for rules having this functional form. The sum of the ranks of the three rules shows that rule I is most robust if inflation fluctuations are the sole measure of performance; it ranks first in terms of inflation variability for all but one model for which there is a clear ordering. For output, rule II has the lowest (best) sum of the ranks, which reflects its relatively high response to output. However, regardless of the objective function weights, rule V has the highest (worst) sum of the ranks for these three policy rules, ranking first for only one model (the Rotemberg-Woodford model) in the case of output. Comparing these three rules with the rules that do not respond to the lagged interest rate (rules III and IV, in table 2) shows that the lagged interest rate rules do not dominate rules without a lagged interest rate. Note that the variance of the interest rate is highest for the rules that react to the lagged interest rate according to many of the models. Table 3 also indicates a key reason why rules that react to lagged interest rates work well in some models and poorly in others in comparison with the rules without lagged interest rates. For a number of models the rules with lagged interest rates are unstable or have extraordinarily large variances. Observe that the models that give very poor performance for the lagged interest rate rules are the non–rational expectations models. These rules rely on people's forward-looking behavior: if a small increase in the interest rate does not bring inflation down, then people expect the central bank to raise interest rates by a larger amount in the future. But in a model without forward looking, it is obviously impossible to capture this forward-looking behavior. Because rule V has a lagged interest rate coefficient greater than one, it greatly exploits these expectations effects and is less robust than the other rules when evaluated with non–rational expectations models. These results illustrate the importance of forward-looking behavior. In his comment on the McCallum and Nelson paper, Mark Gertler reports on some preliminary estimation results that may help determine whether models are too forward looking or not forward looking enough.

Many more robustness findings can be found in the individual papers. Although this robustness analysis is very informative, I think it just touches the surface of what can now be done. It would be useful to do this type of robustness analysis for many more policy rules, including rules with the exchange rate, the forecast of inflation, or even more complex rules. There are also important statistical issues, such as measures of significant differences across models arising from the use of rank orders in robustness analysis. In fact, the subject of robustness arose in many of the comments and the discussions at the conference.

For example, in his comment on Ball's paper, Thomas Sargent calculates an alternative policy rule that is robust to changes in the serial correlation structure of the model. In effect, Sargent looks for rules that are robust if the u_t in the notation of equation (1) were serially correlated rather than uncorrelated. Sargent finds that in his robust version of Ball's policy rule, the interest rate responds even more aggressively than the relatively aggressive rule IV above.

Stock's comment on the paper by Rudebusch and Svensson also calculates a robust policy rule. In contrast to Sargent's focus on robustness to different serial correlation assumptions, Stock's policy rule is meant to be robust to different values of the parameters in the IS equation and the price adjustment equation in the Rudebusch-Svensson model. Stock's robust rule is a minimax policy with respect to this parameter uncertainty. Like Sargent, Stock finds that the optimal policy should be more aggressive in responding to inflation and output than the simple rules III and IV. Sargent's and Stock's findings that robust policy rules are more aggressive generated much discussion at the conference.

The Usefulness of Simple Rules Compared with Complex Rules

All five conference rules have a simple functional form, so the results in tables 2 and 3 are not helpful in determining how useful simple rules are compared to complex rules. But several of the papers in the volume address this question. Rudebusch and Svensson find that simple rules perform nearly as well as the optimal rule in their model. Levin, Wieland, and Williams show that simple rules are more robust across models than more complex optimal rules. Their paper reports on a robustness analysis of simple rules versus optimal rules in four models. They find that optimal rules from one model perform much worse than the simple rules when simulated in other models. Evidently, the optimal rule exploits properties of a model that are specific to that model, and when the optimal rule is then simulated in another model those properties are likely to be different and the optimal rule works poorly.

Role of the Exchange Rate

What is the appropriate role for the exchange rate in a monetary policy rule? This question is obviously very important for small open economies that operate under a flexible exchange rate system, but it may be an important issue for larger areas such as the European Central Bank.

The paper by Laurence Ball uses a small open economy model to assess the role of the exchange rate in a monetary policy rule. Ball shows that adding the exchange rate to simple policy rules, such as rule III and rule IV, can improve macroeconomic performance in his model. He adds the exchange rate to the simple policy rules in two places: (1) the monetary conditions index—a weighted average of the interest rate and the exchange rate—replaces the interest rate as the policy instrument, and (2) the lagged exchange rate is added to the right-hand side of the policy rule along with the inflation rate and real output. Alternatively stated, Ball adds both the current and lagged exchange rate to the right-hand side of the policy rule for i_t. Holding inflation variability constant, Ball finds that the standard deviations of output can be reduced by about 17 percentage points by giving the exchange rate a role in the simple policy rule. It would be interesting to see whether this result is robust. Because many of the models in this book are closed economy models, a robustness study will have to be the subject of future research.

Role of Inflation Forecasts

The papers by Batini and Haldane and by Rudebusch and Svensson focus on another key policy issue. They examine whether policy rules in which the interest rate adjusts to forecasts of future inflation perform better than simple rules, such as rule III and rule IV, that respond to current inflation and real output. Rules that respond to the forecast of inflation rather than actual inflation are frequently referred to as "forward-looking" rules, but since forecasts are based on current and lagged data, these rules are no more forward looking than "backward-looking" rules. Inflation-forecast rules implicitly respond to other variables in addition to output and inflation if such variables are useful predictors of future inflation; hence, these rules could in principle work better than rules such as rule III and rule IV.

The papers by Rudebusch and Svensson and by Batini and Haldane examine a number of inflation-forecasting rules with different forecast horizons and parameters. Both papers report that for the appropriate forecast horizon (usually greater than one year) and for the appropriate response coefficient, inflation-forecast rules can improve performance slightly compared with other simple rules. Batini and Haldane report that an inflation-forecast rule with a six-quarter forecast horizon reduces the standard deviation of inflation by 0.1 percentage points (from 1.4 to 1.3 percent) and the standard deviation of output by 0.2 percentage points (from 1.1 to 0.9 percent) compared with rule III.

Importance of Information Lags

Another policy question addressed by the models in this book is the effect of information lags on monetary policy rules. For example, Bennett McCallum has argued that it is not realistic to assume, as in equation (3), that policy can respond to current-quarter values, and that estimated performance would deteriorate if policymakers could only react to the most recently available data. To investigate this problem the researchers were asked to evaluate the performance of the following lagged version of the policy rule in equation (3):

$$(4) \qquad i_t = g_\pi \pi_{t-1} + g_y y_{t-1} + \rho i_{t-1}.$$

To be sure, it is not clear that equation (4) is any more realistic than equation (3) because policymakers have some current-period information available when they make interest rate decisions. In any case, there is virtually unanimous agreement among the models in the book that this one-quarter lag has little effect on economic performance. The variances of inflation and output increase by only a small amount when equation (3) is replaced by equation (4). Hence, it appears that this kind of information lag does not have major implications for policy rules.

Uncertainty about Potential GDP and the Natural Unemployment Rate

In his comments on the Batini and Haldane paper, Donald Kohn emphasizes that economic uncertainty—especially about potential GDP—poses a serious problem for monetary policy rules. Of course, assessing the effects of general model uncertainty, and the robustness of different policy rules to this uncertainty, is a major aim of this book. Two papers in the book specifically address the issue of uncertainty about potential GDP or the natural rate of unemployment. McCallum and Nelson examine the impact of making gross errors in estimating the trend in real GDP. They find that big errors lead to a big deterioration in performance. Similarly, Estrella and Mishkin show that errors in measuring the natural rate of unemployment lead to a worsening of performance. However, Estrella and Mishkin also show that uncertainty about the natural rate of unemployment or potential GDP is additive uncertainty; therefore, the form of the policy rule should not be affected by such uncertainty. Only in the case of multiplicative uncertainty would the policy rule itself be different.

Historical Evidence

Historical analysis of policy rules complements the evidence about the interest rate response to inflation and output found in the simulations. As I show in my paper the estimated response coefficients of monetary policy were much larger in the 1980s and 1990s in the United States than they were during the late 1960s and 1970s. Moreover, the response coefficients appear to have been even lower during the international gold standard period from 1880 to 1914 when inflation and real output were less stable. For example, the estimated inflation response coefficient is about 0.8 for the 1960s and 1970s compared to about 1.5 for the 1980s and 1990s, nearly twice as large. Since the inflation rate and real output were much more stable in the 1980s and 1990s than in the late 1960s and 1970s, or than in the international gold standard period, the result supports the model simulations that predict that such a change would take place. Similar results for the later two periods are reported in recent papers by Judd and Rudebusch (1998) and by Clarida, Galí, and Gertler (1998) as discussed in the comment on my paper by Richard Clarida.

Conclusion

Of the many important findings in this volume several seem particularly important to me. First, the model simulations show that simple policy rules work well; their performance is surprisingly close to that of fully optimal policies. Second, the simulations show that the gains reported in earlier research from using rules with high response coefficients are not robust to the variety of models considered in this volume; however, new approaches to robustness discussed in the volume suggest that rules that are robust to certain kinds of uncertainty may be more aggressive. Third, simulation results show that simple policy rules are more robust than complex rules across a variety of models. Fourth, introducing information lags as long as a quarter does not affect the performance of the policy rules by very much. Fifth, the historical analysis finds a significant correlation between policy rules and economic performance.

The areas of disagreement are also important. First, there is disagreement about whether central banks should react to the exchange rate when setting interest rates, or whether they should use a monetary conditions index. Second, there is disagreement about whether policy should respond to the *lagged* interest rate. Third, there is disagreement about whether the interest rate should respond solely to a measure of *expected future* inflation, rather than actual observed values. In these cases of disagreement, the papers are useful in determining what features of the models lead to the differences. This will be helpful in future research.

These remaining uncertainties and disagreements indicate that there is more work to do in this area. There is much to be learned from studying the many simulations already performed for this volume. The robustness analysis in this book, which is the focus of so many of the papers, comments, and discussions, makes a good start, but it has only scratched the surface. Improving the models, considering additional models, expanding the analysis to other countries, and examining more rules are all essential.

In the meantime, it is wise for policymakers to work with a collection or portfolio of policy rules as mentioned by Martin Feldstein in his comment on the Rotemberg and Woodford paper. Such a portfolio might include the rules of the type examined in table 1. When I proposed a specific simple policy rule in 1992 I suggested that the rule be used as a guideline along with several other policy rules. In his comment on the King and Wolman paper, Benjamin Friedman mentions the distinction between using a monetary policy rule as a guideline and using the rule mechanically. Although all the rules in this book can be written down algebraically—indeed that is one of their main advantages—at least for the near future they will probably be more useful as guidelines than as mechanical formulas for policymakers to follow exactly. By carefully studying the results in this volume, I hope that researchers and policymakers can make monetary policy rules even more useful in the future.

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1

Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model

Bennett T. McCallum and Edward Nelson

1.1 Introduction

In a series of studies on monetary policy rules, McCallum (1988, 1990, 1993, 1995) has utilized and promoted a research strategy that emphasizes *operationality* and *robustness*. The first of these properties intentionally limits consideration to policy rules (i) that are expressed in terms of instrument variables that could in fact be controlled on a high-frequency basis by actual central banks and (ii) that require only information that could plausibly be possessed by these central banks. Thus, for example, hypothetical rules that treat, say, M2 as an instrument or that feature instrument responses to current-quarter values of real GDP are ruled out as nonoperational. The second property focuses on a candidate rule's tendency to produce at least moderately good performance in a variety of macroeconomic models rather than "optimal" performance in a single model. The idea behind this criterion is that there exists a great deal of professional disagreement over the appropriate specification of crucial features of macroeconomic models, and indeed even over the appropriate objective function to be used by an actual central bank.

Most of the models used in McCallum's own studies have, however, been nonstructural vector autoregression or single-equation atheoretic constructs that are quite unlikely to be policy invariant. Even the so-called structural models in McCallum (1988, 1993) are essentially small illustrative systems that are not based on well-motivated theoretical foundations. Thus these studies have

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not contributed any proposed models of their own to be used in a profession-wide exploration of the robustness of candidate rules' properties.

In the present study, accordingly, we formulate, estimate, and simulate two variants of a model of the U.S. economy that is intended to have structural properties. The model is quite small—following in the line of work previously contributed to by Fuhrer and Moore (1995), Yun (1996), Ireland (1997), and Rotemberg and Woodford (1997) among others—but is based on aggregate demand and supply specifications that are designed to reflect rational optimizing behavior on the part of the economy's private actors. Our formulations pertaining to demand are rather orthodox, but in terms of aggregate supply—that is, price adjustment behavior—we consider two alternatives, one of which is not standard. In particular, we begin with the formulation of Roberts (1995), which is based on the well-known models of Calvo (1983), Rotemberg (1982), and Taylor (1980). In addition, however, we develop a modification of the Mussa-McCallum-Barro-Grossman "P-bar model," whose theoretical properties are arguably more attractive. Although we consider only two simple variants of our macroeconomic model, we suggest that its design makes it an attractive starting point for a more extensive robustness study. Our estimation is conducted by instrumental variables and utilizes quarterly U.S. data for 1955–96.

With our estimated model we carry out stochastic and counterfactual historical simulations not only with the class of policy rules promoted in McCallum's previous work but also with rules that are operational versions of the Taylor (1993) type and others with an interest rate instrument. Some of the issues that we explore in these simulations are the following:

Is it true that response coefficients in a rule of the Taylor type should be much larger than recommended by Taylor (1993)?

Is there any tendency for adoption of a nominal GDP target rule to generate instability of real GDP and inflation?

In studying questions such as these, how important is it quantitatively to recognize that actual central banks do not have complete information when setting instrument values for a given period?

How sensitive to measures of "capacity" output are rules that feature responses to output gaps?

Do interest rates exhibit extreme short-run volatility when base money rules are utilized?

Organizationally, we begin in section 1.2 with a discussion of several important background issues. Then sections 1.3 and 1.4 are devoted to specification of the macroeconomic model to be utilized, with the former pertaining to the model's aggregate demand sector and the latter to aggregate supply. Section 1.5 describes data and estimation and reports estimates of the model's basic structural parameters. Simulation exercises with various policy rules are then conducted in sections 1.6 and 1.7 for the two variants of the model, and conclusions are summarized in section 1.8.

1.2 Monetary Policy Rules: Alternatives and Issues

We begin by discussing various forms of possible monetary policy rules and some issues raised by the differences among them. In the previous research by McCallum, quarterly data have been utilized and the principal rule specification has been

$$(1) \qquad \Delta b_t = \Delta x^* - (1/16)(x_{t-1} - b_{t-1} - x_{t-17} + b_{t-17}) + \lambda(x^*_{t-1} - x_{t-1}),$$

with $\lambda \geq 0$. Here b_t and x_t denote logarithms of the (adjusted) monetary base and nominal GNP (or GDP), respectively, for period t. The variable x^*_t is the target value of x_t for quarter t, with these targets being specified so as to grow smoothly at the rate Δx^*. This rate is in turn designed to yield an average inflation rate that equals some desired value—for example, a value such as 0.005, which with quarterly data would represent roughly 2 percent per year.[1] Whereas a growing-level target path $x^{*1}_t = x^{*1}_{t-1} + \Delta x^*$ was used in McCallum's early work (1988), his more recent studies have emphasized growth rate targets of the form $x^{*2}_t = x_{t-1} + \Delta x^*$ or weighted averages such as $x^{*3}_t = 0.8x^{*2}_t + 0.2x^{*1}_t$. In equation (1), the rule's second term provides a velocity growth adjustment intended to reflect long-lasting institutional changes, while the third term features feedback adjustment in Δb_t in response to cyclical departures of x_t from the target path x^*_t, with λ chosen to balance the speed of eliminating $x^*_t - x_t$ gaps against the danger of instrument instability.

More prominent in recent years has been the rule form proposed by Taylor (1993), which we write as

$$(2) \qquad R_t = r^* + \pi^{av}_{t-1} + \mu_1(\pi^{av}_{t-1} - \pi^*) + \mu_2\tilde{y}_t .$$

Here R_t is the quarter t value of an interest rate instrument, π^{av}_{t-1} is the average inflation rate over the four quarters prior to t, π^* is the target inflation rate, and $\tilde{y}_t = y_t - \bar{y}_t$ is the difference between the (logs of) real GDP y_t and its capacity or natural rate value $\bar{y}_t$. The policy feedback parameters μ_1 and μ_2 are positive—each of them equals 0.5 in Taylor's (1993) example[2]—so that the interest rate instrument is raised in response to values of inflation and output that are high relative to their targets.

There are two major reasons for the greater prominence of Taylor's rule (2) as compared with rule (1). First, it is specified in terms of an interest rate instrument variable, which is much more realistic.[3] Second, from several studies including Taylor (1993), Stuart (1996), Clarida, Galí, and Gertler (1998), among others, it appears to be the case that actual policy in recent years, say after

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[1] Whatever the desired quarterly inflation rate, Δx^* is set equal to that value plus an estimated long-run average rate of growth of real output, a number assumed to be independent of the policy rule adopted.

[2] When annualized values of inflation and the interest rate are used.

[3] Virtually all central banks of industrialized countries use some short-term (nominal) interest rate as their instrument or "operating target" variable. For an extensive recent discussion, see Bank for International Settlements (1997).

1986, has been rather well described by a formula such as Taylor's with coefficients quite close to his for some major countries.

As specified by Taylor (1993), however, rule (2) is not fully operational since it assumes unrealistically that the central bank knows the value of real GDP for quarter t when setting the instrument value R_t for that quarter. In fact, there is considerable uncertainty regarding the realized value of real GDP even at the end of the quarter in actual economies.[4] In addition, it is far from obvious how $\tilde{y}_t$ should be measured—even in principle—as is emphasized in McCallum (1997), and different measures can imply significantly different instrument settings.[5] The first of these objections can be easily overcome by using the value of y_t expected to prevail at the start of period t. Also, in the same spirit, some more rational representation of expected future inflation could be used in place of π^{av}_{t-1}. Overcoming the second objection, regarding the measurement of $\tilde{y}_t$, could be more difficult.

Alterations in rule (1) could also be considered, such as using the expectation of x^*_t (or of x^*_{t+1}) rather than actual x^*_{t-1} as the basis for feedback adjustments. More generally, the target values in rules (1) and (2) could be exchanged, to provide rules with (i) a base instrument and π^* and $\tilde{y}$ targets and (ii) an interest instrument plus a Δx_t target. In the work that follows, we shall explore several such variants of policy rules.

In this regard, some analysts might suggest that the monetary base instrument be discarded since actual central banks are not inclined even to consider the use of a b_t instrument.[6] Several academics have hypothesized that policy could be made more effective if a base instrument were utilized, however,[7] and there are clearly some disadvantages of the interest rate scheme. In particular, there is an observable tendency for an interest instrument to become something of a target variable that is thus adjusted too infrequently and too timidly (see Goodhart 1997). In any event, the question of the comparative merits of b_t and R_t instruments is one that seems to warrant scientific study—indeed, more than is provided below.

The foregoing paragraphs have been concerned with policy rules from a normative perspective. In estimating and evaluating a macroeconomic model, however, it is useful to consider what policy rule or rules have in fact been utilized during the sample studied. In that regard, it might be argued that no rule has been in place, that the Federal Reserve has instead behaved in a discretionary manner. But we believe that there has clearly been a major component of Fed behavior that is *systematic*, as opposed to random, and this component can be expressed in terms of a feedback formula.[8] Of course, there can be little doubt but that there have been changes during our 1955–96 sample in the systematic component's specification, with prominent dates for possible changes including October 1979, late summer 1982, August 1987, and a few others.[9] Thus we have experimented with both slope and constant-term dummy variables. After considerable empirical investigation we have ended with an estimated rule of the form

$$(3) \qquad R_t = \mu_0 + \mu_1 R_{t-1} + \mu_2 E_{t-1}\Delta x_t + \mu_3 E_{t-1}\tilde{y}_t + \mu_4 d_{1t} + \mu_5 d_{2t} E_{t-1}\Delta x_t + e_{Rt},$$

where $\tilde{y}_t$ is the output gap (the log deviation of output from its flexible-price level), d_{1t} and d_{2t} are dummy variables that take on the value 1.0 in 1979:4–82:2 and 1979:4–96:4 respectively, and e_{Rt} is a serially independent disturbance. Thus our estimated rule for 1955:1–96:4 is one that combines the interest rate instrument from rule (2) with a nominal GDP target as in rule (1), as well as an extra countercyclical term. The rule is operational because the monetary authority responds to period $t - 1$ forecasts of Δx_t and $\tilde{y}_t$, not their realized values. The inclusion of dummies in equation (3) allows for shifts in the policy rule occurring in late 1979, presumably due to the change in operating procedures and anti-inflationary emphasis that was announced on 6 October. Of these, the dummy d_{1t} captures a possible intercept shift occurring during the period of nonborrowed reserves targeting, and the interactive dummy $d_{2t}E_{t-1}\Delta x_t$ reflects a permanent shift in the Federal Reserve's objectives after 1979. The empirical results of our investigation are reported below in section 1.5.[10]

Returning to the normative topic of effective rule design, several prominent issues concerning target variables will be studied in sections 1.6 and 1.7. One of these involves the claim, expressed by Ball (1997) and Svensson (1997), that targeting of nominal GDP growth rates (or growing levels) will tend to induce undesirable behavior of inflation and output gap variables. It is not difficult to show that Ball's drastic result of dynamic instability of π_t and $\tilde{y}_t$ holds only under some highly special model specifications, but it is possible that much greater volatility would obtain than with alternative target variables, so a quantitative examination of the issue is needed.

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[4] In the United States, e.g., the recent study of Ingenito and Trehan (1997) indicates that the "forecast" error for real GDP at the end of the quarter is about 1.4 percent, implying that annualized growth rates for the quarter would have a 95 percent confidence interval of about ±2.8 percent, thereby possibly ranging from boom to deep recession values. This result is based on revised data, so it abstracts from the problem of data revision.

[5] These two objections to rule (2) should not be understood as criticisms of Taylor's (1993) paper, which was written mainly to encourage interest in monetary rules on the part of practical policymakers—and was in that regard extremely successful.

[6] Goodhart (1994) has claimed that tight monetary base control is essentially infeasible.

[7] Among these academics are Brunner and Meltzer (1983), Friedman (1982), McCallum (1988), and Poole (1982).

[8] On this topic, see Taylor (1993), McCallum (1997), and Clarida et al. (1998).

[9] The study by Clarida et al. (1998) considers one possible break, in October 1979, and finds significant differences in estimated policy rule coefficients before and after that date.

[10] As the experiments in this paper are concerned with counterfactual policy rules, we do not use rule (3) in our simulations in sections 1.6 and 1.7. Our reason for nevertheless estimating and reporting eq. (3) is to demonstrate that rulelike behavior is a reasonable characterization of postwar data and to indicate the importance of the regime dummies d_{1t} and d_{2t}, which we include in our instrument set when estimating our structural model in section 1.5.

1.3 Aggregate Demand Specification

This section describes the aggregate demand side of our model; what follows is essentially a condensed presentation of the derivations in McCallum and Nelson (forthcoming). We assume that there is a large number of infinitely lived households, each of which maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, M_{t+j}/P^A_{t+j}), \tag{4}$$

where C_t denotes the household's consumption in period t and M_t/P_t^A denotes its end-of-period real money holdings, M_t being the nominal level of these money balances and P_t^A the general price level. Real money balances generate utility by facilitating household transactions in period t. The instantaneous utility function $U(C_t, M_t/P_t^A)$ is of the additively separable form:

$$\begin{aligned} U(C_t, M_t/P_t^A) &= \sigma(\sigma - 1)^{-1} C_t^{(\sigma-1)/\sigma} \exp \omega_t \\ &\quad + (1 - \gamma)^{-1} (M_t/P_t^A)^{1-\gamma} \exp \chi_t, \end{aligned} \tag{5}$$

with $\sigma > 0$ and $\gamma > 0$. Here ω_t and χ_t are both preference shocks, whose properties we specify below.

Each household also acts as a producer of a good, over which it has market power. To this end, it hires N_t^d in labor from the labor market, paying real wage W_t/P_t^A for each unit of labor. With this labor and its own capital stock K_t (which depreciates at rate δ) it produces its output Y_t via the technology $Y_t = A_t K_t^\alpha (N_t^d)^{1-\alpha}$, where A_t is an exogenous shock that affects all households' production. The household sells its output at price P_t. Each household consumes many goods, consisting of some of the output produced by other households; the C_t that appears in the household's utility function is an index of this consumption, and P_t^A indexes the average price of households' output.

As is standard in the literature, we assume that the demand function for good i is of the Dixit-Stiglitz form, and that also the producer is obliged to set production equal to this demand:

$$A_t K_t^\alpha (N_t^\alpha)^{1-\alpha} = (P_t/P_t^A)^{-\theta} Y_t^A, \tag{6}$$

with $\theta > 1$ and Y_t^A denoting aggregate output.

The household is also endowed with one unit of labor each period, and supplies N_t^S of this to the labor market. The household's budget constraint each period is then

$$\begin{aligned} &(P_t/P_t^A)^{1-\theta} Y_t^A - C_t - K_{t+1} + (1 - \delta) K_t + (W_t/P_t^A) N_t^S \\ &\quad - (W_t/P_t^A) N_t^d + TR_t - M_t/P_t^A + M_{t-1}/P_t^A \\ &\quad - B_{t+1}(1 + r_t)^{-1} + B_t = 0. \end{aligned} \tag{7}$$

In equation (7), B_{t+1} is the quantity of government bonds bought by the household in period t; each of these is purchased for $(1 + r_t)^{-1}$ units of output and redeemed for one unit of output in period $t + 1$. TR_t denotes lump-sum government transfers paid to the household in period t. Letting ξ_t denote the Lagrange multiplier on constraint (6) and λ_t the multiplier on (7), the household's first-order conditions with respect to C_t, M_t/P_t^A, K_{t+1}, and B_{t+1} are

$$C_t^{-1/\sigma} \exp \omega_t = \lambda_t, \tag{8}$$

$$(M_t/P_t^A)^{-\gamma} \exp \chi_t = \lambda_t - \beta E_t \lambda_{t+1} (P_t^A/P_{t+1}^A), \tag{9}$$

$$\lambda_t = \beta(1 - \delta) E_t \lambda_{t+1} + \alpha \beta E_t \xi_{t+1} A_{t+1} K_{t+1}^{\alpha-1} (N_{t+1}^d)^{1-\alpha}, \tag{10}$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + r_t). \tag{11}$$

Because leisure does not enter its utility function, the household's optimal labor supply is $N_t^S = 1$ each period, although, since we assume below that the labor market does not clear, this desired labor supply will not be the realized value of labor utilized.

As an employer of labor, the household's first-order condition with respect to N_t^d is

$$\lambda_t (W_t/P_t^A) = (1 - \alpha) \xi_t A_t K_t^\alpha (N_t^d)^{-\alpha}. \tag{12}$$

Equation (12) indicates that, as in Ireland (1997), the markup of price over marginal cost is equal to λ_t/ξ_t. The household has one more first-order condition, pertaining to its optimal choice for P_t. We defer the analysis of this decision until section 1.4.

We now construct a log-linear model of aggregate demand from the above conditions. While we use equation (10) in our calculations of the implied steady state level of investment, $\bar{I}$, we do not use an approximation of equation (10) to describe quarter-to-quarter fluctuations in capital or investment. Instead, we treat capital as exogenous and, for tractability, let the movements of log investment around its steady state value be a random walk. Thus we have

$$i_t = g_k + i_{t-1} + e_{it}, \tag{13}$$

where $g_k \geq 0$ is the average growth rate of capital, $E_{t-1} e_{it} = 0$, and $E(e_{it}^2) = \sigma_{ei}^2$. In equation (13) and below, lowercase letters denote logarithms of variables.

It would be standard practice to complete our specification of technology with the usual log-linear law for capital accumulation,

$$k_{t+1} = \frac{1 - \delta}{1 + g_k} k_t + \frac{\delta + g_k}{1 + g_k} i_t, \tag{13a}$$

along with a law of motion for the (log) technology shock a_t. But since we are treating capital movements as exogenous, and since leisure does not appear in