

O. Nelles

Nonlinear System Identification



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Oliver Nelles

Nonlinear System Identification

From Classical Approaches
to Neural Networks and Fuzzy Models

With 422 Figures



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Dr. Oliver Nelles
UC Berkeley / TU Darmstadt
Friedrichstraße 86
D-61476 Kronberg
Germany
e-mail: Oliver.Nelles@gmx.de

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Preface

The goal of this book is to provide engineers and scientists in academia and industry with a thorough understanding of the underlying principles of nonlinear system identification. The reader will be able to apply the discussed models and methods to real problems with the necessary confidence and the awareness of potential difficulties that may arise in practice. This book is self-contained in the sense that it requires merely basic knowledge of matrix algebra, signals and systems, and statistics. Therefore, it also serves as an introduction to linear system identification and gives a practical overview on the major optimization methods used in engineering. The emphasis of this book is on an intuitive understanding of the subject and the practical application of the discussed techniques. It is not written in a theorem/proof style; rather the mathematics is kept to a minimum and the pursued ideas are illustrated by numerous figures, examples, and real-world applications.

Fifteen years ago, nonlinear system identification was a field of several ad-hoc approaches, each applicable only to a very restricted class of systems. With the advent of neural networks, fuzzy models, and modern structure optimization techniques a much wider class of systems can be handled. Although one major characteristic of nonlinear systems is that almost every nonlinear system is unique, tools have been developed that allow the use of the same approach for a broad variety of systems. Certainly, a more problem-specific procedure typically promises superior performance, but from an industrial point of view a good tradeoff between development effort and performance is the decisive criterion for success. This book presents neural networks and fuzzy models together with major classical approaches in a unified framework. The strict distinction between the model architectures on the one hand (Part II) and the techniques for fitting these models to data on the other hand (Part I) tries to overcome the confusing mixture between both that is frequently encountered in the neuro and fuzzy literature. Nonlinear system identification is currently a field of very active research; many new methods will be developed and old methods will be refined. Nevertheless, I am confident that the underlying principles will continue to be valuable in the future.

This book offers enough material for a two-semester course on optimization and nonlinear system identification. A higher level one-semester graduate course can focus on Chap. 7, the complete Part II, and Chaps. 17 to 21 in

Part III. For a more elementary one-semester course without prerequisites in optimization and linear system identification Chaps. 1 to 4 and 16 might be covered, while Chaps. 10, 14, 18, and 21 can be skipped. Alternatively, a course might omit the dynamic systems in Part III and instead emphasize the optimization techniques and nonlinear static modeling treated in Parts I and II. The applications presented in Part IV focus on a certain model architecture, and will convince the user of the practical usefulness of the discussed models and techniques. It is recommended that the reader should complement these applications with personal experiences and individual projects.

Many people supported me while I was writing this book. First of all, I would like to express my sincerest gratitude to my Ph.D. advisor, Professor Rolf Isermann, Darmstadt University of Technology, for his constant help, advice, and encouragement. He gave me the possibility of experiencing the great freedom and pleasure of independent research. During my wonderful but much too short stay as a postdoc at the University of California in Berkeley, Professor Masayoshi Tomizuka gave me the privilege of taking utmost advantage of all that Berkeley has to offer. It has been an amazing atmosphere of inspiration. The last six years in Darmstadt and Berkeley have been the most rewarding learning experience in my life. I am very grateful to Professor Isermann and Professor Tomizuka for giving me the chance to teach a graduate course on neural networks for nonlinear system identification. Earlier versions of this book served as a basis for these courses, and the feedback from the students contributed much to its improvement.

I highly appreciate the help of my colleagues in Darmstadt and Berkeley. Their collaboration and kindness made this book possible. Big thanks go to Dr. Martin Fischer, Alexander Fink, Susanne Töpfer, Michael Hafner, Matthias Schüler, Martin Schmidt, Domink Füssel, Peter Ballé, Christoph Halfmann, Henning Holzmann, Dr. Stefan Sinsel, Jochen Schaffnit, Dr. Ralf Schwarz, Norbert Müller, Dr. Thorsten Ullrich, Oliver Hecker, Dr. Martin Brown, Carlo Cloet, Craig Smith, Ryan White, and Brigitte Hoppe.

Finally, I want to deeply thank my dear friends Martin and Alex and my wonderful family for being there, whenever I needed them. I didn't take it for granted. This book is dedicated to you!

Kronberg, June 2000

Oliver Nelles

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